A Three-Circle Goniometer.

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THE object of goniometrical measurement is to determine in the simplest and quickest manner possible the geometrical constants of crystals and the indices of their faces. The ideal instrument should enable the observations to be conveniently and quickly made, and present them in such a form that the indices of the faces may be easily calculated. It is claimed for the form of goniometer described in this paper that it does approximate to such an instrument. Before entering into the description it will not be out of place to discuss briefly the two types at present in existence, and point out their advantages and disadvantages.

I. *The One-Circle Goniometer.*

This instrument, which owes its conception to Wollaston, first made possible the accurate measurement of the angles between the faces of a crystal. In the original form the graduated circle was vertical, but Babinet afterwards introduced a goniometer with a horizontal circle. Improvements in details—such as the employment of telescope and collimator, mechanical means for adjusting the crystal, convenient object-slits, &c.—followed in due course, until finally we have the perfected instrument of the present day.

The grave objection to this form of goniometer is the necessity for readjusting the crystal for each zone, and consequently for identifying at least two faces in this zone. This identification is frequently by no means easy even in the case of fair-sized crystals, and becomes exceedingly difficult when the crystal is small and rich in faces; yet the smaller the crystal the better has been its chance of undisturbed growth.

Faces often occur which do not lie exactly in the zone under measurement, but there is no way of accurately determining the deviation from the zone. Further, if several images occur close together, but not in the same zone, it will very probably be impossible to determine the precise position of each.

Since in this method measurements are, as far as possible, always made in zones, the indices of the faces may be readily calculated by means of the law of zonality.

II. *The Two-Circle or Theodolite Goniometer.*

Since we are concerned with the relative and not the absolute position of crystalline faces, we may refer the position of any face to that of a pole on a sphere, and denote it by means of two co-ordinates—the polar distance and azimuth, corresponding to the N.P.D. and R.A. of a star in astronomy. The theodolite goniometer gives at once such co ordinates for any face.

An instrument of this character was employed many years ago (in 1874) by Miller to determine the faces presented by a platinum bead which had been strongly heated before the blowpipe. He merely fixed a goniometer with a vertical circle on another with a horizontal circle. His method and results were published after his death by Prof. W. J. Lewis,[1] but do not seem to have attracted much attention. Rather more than a decade later Prof. V. Goldschmidt[2] described a goniometer with two circles, which is practically the same as Miller's. Almost at the same time Herr Czapski[3] gave a description of his form of goniometer, in which the crystal has motion about a vertical axis, while the telescope and collimator, inclined to one another at a small angle, move about a horizontal axis. Shortly afterwards Prof. Fedorow[4] pointed out that he had in 1889 published in the proceedings[5] of the St. Petersburg Mineralogical Society a description of a two-circle goniometer. This again is, except for the use of an auto-collimating telescope, of the same type as Miller's.

In employing a goniometer of this kind we adjust the face or one of the faces of greatest symmetry accurately parallel to the circle carrying the crystal. For any other face the reading of this circle gives the azimuth, and of the second circle the polar distance, after the subtraction of constants in the two cases, if necessary.

It is evident that the chief difficulty involved in the older method is in

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[1] *Proc. Camb. Phil. Soc.*, 1882, IV, 236. Abstract in *Zeits. Kryst. Min.* 1883, VII, 619.

[2] *Zeits. Kryst. Min.* 1893, XXI, 210.

[3] *Zeits. für Instrumentenk.*, 1893, 1.

[4] *Zeits. Kryst. Min.* 1893, XXI, 574.

[5] XXVI, 458. Abstract in French in *Bibliothèque géologique de la Russie*, 1890, VI, 97

this type of instrument entirely overcome. No readjustment of the crystal is required. Each face is observed once only, and the position corresponding to every image is accurately defined.

Did we denote a face by means of these co-ordinates, no exception could be taken to this method. We, however, have to deduce from them the indices of the faces, and this is in general a tedious and troublesome process. Unless a zone includes the face of reference, or its edge is perpendicular to that face, no immediate use can be made of the law of rational indices on which depends the anharmonic ratio of four poles in a zone, and, consequently, Miller's simple logarithmic formulæ cannot be employed. The formulæ devised by Profs. Goldschmidt and Fedorow[1] are cumbersome, and not readily adapted to logarithmic computation.[2]

Further it is impossible to determine without a troublesome calculation whether three faces lie in a zone, unless the zone be one of those mentioned above. Indeed, much of the time saved in the observations is lost in the calculations.

It may be remarked that, if an unknown crystal be measured of which the symmetry is not obvious, the pole of a face of symmetry may not have been chosen as pole of reference, in which case it is exceedingly difficult to identify the crystal.

III. *The Three-Circle Goniometer.*

The advantages of both methods may readily be combined by the addition of a third circle. The crystal is adjusted once for all, and measurements may be made in any desired zone, the orientation of which is at the same time determined.

For crystals small enough to be firmly held by the wax in all positions by far the most convenient arrangement is to give all three motions to

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[1] *Loc. cit.*

[2] *Cf.* the very suggestive remarks by Signor C. Viola, *Zeits. Kryst. Min.*, 1898, XXX, 423:—

"Ich bin der Meinung, dass, sowohl was die Messung der Krystalle, als auch was die Signatur der Flächen anbetrifft, man hauptsächlich auf die Zonen Rücksicht haben muss, denn nur dadurch kann eine Erleichterung herbeigeführt werden, obwohl die meisten Flächen von den Zonen ein wenig abweichen.

"Das Princip der Zonen, oder was gleich ist, das Princip der Rationalität darf nicht bei Seite gelassen werden, wiewohl es in der Natur nicht mit absoluter Strenge zutrifft. Ein solches Princip ist eben der Leitfaden der Krystalle, wie das Newton'sche Gesetz, das auch nicht genau in der Natur gilt, für die Bahnberechnung der Planeten jedoch das Fundament bildet.

"Erfüllt ein Theodolithgoniometer solche Bedingungen nicht, so hat es besseren Instrumenten Platz zu machen."

the crystal, but, if necessary, the telescope and collimator might have one or more.

(*a*) *The Instrument.*

This form of goniometer consists essentially of three circles :—

(A)[1] The horizontal circle,
(B) The vertical circle,
(C) The third circle.

These must be so arranged that the axes of A and C are always at right angles to B, though inclinable at any angle to each other. At the same time the axis of A must be perpendicular to the bisectrix of the angle between the telescope and collimator, and all three axes must, at any rate approximately, intersect in the optic centre.[2]

The last condition must be fulfilled to avoid the necessity for re-centering any but very small crystals during the course of observations. If a collimator be used, re-centering is not essential to the accuracy of the measurements.

The axis of B is adjusted in the same way as for a theodolite goniometer.[3] A face is adjusted parallel to this circle, so that no movement of the reflected image takes place on rotation, and the axis adjusted until this image coincides with the cross-wires. B is now turned about the axis of A through a right angle, and the axis of C is adjusted at right angles to that of B in the same way as the latter axis has been adjusted to that of A.

As an experiment a Fuess Model II goniometer has been converted into this form by the substitution of a two-circle arrangement in place of the original crystal-holder. Plate II, which is reproduced from a photograph of the actual instrument, shows the general construction. Since it is of the utmost importance that all the parts should be absolutely rigid, some more efficient connection between the extra circles and A (retaining the nomenclature used above) is required than that already provided. The horizontal plate of the additional apparatus is rigidly attached by means of three clamps to the rabbet underneath it, which, in its turn, is firmly screwed to A. Underneath this plate is a short cylindrical piece, which fits into the hole already in A and serves to keep

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1 Throughout this paper reference will be made to the circles by these letters.

2 The intersection of the axes of the telescope and collimator.

3 *Vide* Fedorow, *loc. cit.*

the additional apparatus in position when rotating independently of this circle. In this way the reading for a face parallel to B may be made a convenient quantity, such as 0°0′. On the top of the plate is fixed the counterweight, and opposite this the vertical piece carrying the cone of B. This piece has from a side aspect an L-shaped termination, and is attached to the horizontal plate from underneath by four screws. Mechanical means for raising or lowering the axis of B have been omitted, as their provision would have weakened this important junction. This axis may, however, be raised a small amount by packing with paper, which is easily cut to shape and practically incompressible. By slotting the lower holes, through which the screws pass, B may be moved to or from the axis of A; an adjustment required if the axis of C does not, when adjusted in direction, pass through the optic centre. The axis of B has three adjustment screws: two (visible in the figure) to bring it into the horizontal plane, and one (at the back) to make it intersect the axis of A. In the present case it was found to be appreciably too low, and was raised, in the manner described, about 1 mm. The axis of C has also three adjustment screws: two to bring it into a plane parallel to B and one to make it intersect the axis of this circle.[1] It can, if necessary, be made to pass through the optic centre in the manner described above. Both circles have clamps and slow motions and fixed verniers reading to minutes. The circle C for the sake of lightness has been constructed of aluminium; the remainder of the apparatus of bronze. In setting off the graduations on B it was arranged that, where the axis of C is vertical, the reading should be as nearly as possible 0°0′. This condition has not quite been fulfilled. The axes of B and C more nearly intersect when the reading is 0°30′. The extra apparatus is clamped to A in such a position that the reading of this circle for a face parallel to B is exactly 0°0′. The crystal-holder is of the ordinary type, and has the usual adjustments. The counterweight to C and the crystal holder is attached on the other side of B, to prevent any shearing stress on the cones of the latter. Springs are added to keep B and C in position. The circles B and C are about 11 and 9 cm. in diameter respectively. This permits of the former being turned about the axis of A through 190° approximately,[2]

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[1] The latter adjustment may be employed to make the reading of B a convenient quantity when the axis of C is vertical.

[2] That this angle could not be conveniently made larger is one of the disadvantages of adapting an existing goniometer. It might, however, in an instrument entirely new from the beginning be easily arranged that B could be turned round the axis of A through nearly 300°.

when the telescope and collimator are inclined to one another at an angle of 70°, and of the latter through 240° approximately, when the axis of B bisects the interior angle between the telescope and collimator. The tubes of the telescope and collimator are covered with india-rubber, to prevent jarring in the event of either B or C coming into violent contact with them.

(*b*) *How to use the instrument.*

If we adjust the edge of a zone parallel to the axis of C, and rotate the crystal about this axis, we bring in turn all the faces in this zone parallel to B. Rotating A[1] until it reads 0°0′, in which case the image reflected from a face parallel to B is on the cross-wires, and for convenience making the axis of C vertical (though this is not essential), we note the readings when the different faces are parallel to B. If we then proceed to clamp C when some face is in this position and rotate B, we bring any zone containing the face parallel to A, and find the readings of the latter circle for all faces lying in the zone. In other words, we can take two-circle measurements with the pole of any face in a particular zone as pole of reference.

It is, therefore, easy to see that for any zone adjusted by means of B and C parallel to A,

A gives the angles in the zone,

B the angle between this zone and the zone of reference,[2] and

C the pole, whether representing an existing face or not, in which the zones intersect.

Since A can be rotated through 190° at the most, only half a zone can be measured at a time. To complete the zone we must rotate C through 180°, and B back through the zero position till the axis of C makes the same angle with the vertical as before, but on the other side.

In setting up any crystal for measurement we should choose the zone of greatest symmetry as zone of reference; [111] and [010] in the case of hexagonal and monoclinic crystals respectively. In this way, except on triclinic crystals, there will be a possible face parallel to C. By keeping this circle vertical we may take measurements with the pole of the face parallel to it as pole of reference, and thus get a general idea of

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[1] If the axis of C be vertical, this zone may be measured whatever be the reading of A, but it is only when the latter reads 0°0′ that the reading of C for any particular face indicates the position in which it is parallel to B.

[2] The zone whose edge is parallel to the axis of C.

the positions of the faces on the crystal. We can see at once from a stereographic projection in which zones it would be best to take measurements.

Although convenient, it is by no means essential that the zone of reference should be one of symmetry. If an unknown crystal, whose symmetry is not obvious, be measured for identification, the most prominent zone would naturally be chosen for reference, and measurements made to determine the positions of the remaining faces with respect to it.

The extra apparatus has been constructed by Messrs. Troughton and Simms, the well-known opticians and mathematical instrument makers, of London. The author is glad to have this opportunity of putting on record his gratitude to them for the able and careful manner in which they have carried out his ideas.

The following measurements of a crystal of anorthite from Monte Somma have been made with the three-circle goniometer here described, and are added as an example of the readings obtained when a crystal is measured according to this method :—

OBSERVED READINGS.

			A	B	C	C (from c)	Dana.
Zone of Reference [100]	1	$c(00\bar{1})$	0° 0′	0°30′	198°32′		
	2	$n(02\bar{1})$	,,	,,	151 47	46°45′	46°46′
	3	$b(010)$	,,	,,	104 31	85 55	85 50
	4	$e(021)$	,,	,,	61 14	42 38	42 38½
	5	$c(001)$	,,	,,	18 36		
	6	$n(0\bar{2}1)$	,,	,,	332 5	46 31	46 46
	7	$M(\bar{1}10)$	290 36	302 10	198 32		69 20
	7	$M(\bar{1}10)$	62 23	64 45	104 31		62 26½
	8	$z(\bar{1}30)$	30 54	,,	,,		30 58
	9	$y(\bar{2}01)$	90 30	99 31	,,		90 32½
	10	$w(\bar{2}41)$	38 40	,,	,,		38 41½
	11	$p(\bar{1}11)$	297 45	309 8	,,		62 13
	12	$o(\bar{1}\bar{1}1)$	64 46	,,	,,		64 58
	11	$p(\bar{1}11)$	54 16	58 54	18 36		54 17
	9	$y(\bar{2}01)$	81 8	91 44	,,		81 14
	11	$p(\bar{1}11)$	315 53½	277 22	61 14		
	9	$y(20\bar{1})$	275 44½	,,	,,		

The values of the angles given in the last column have been taken from Dana's *System of Mineralogy*, Sixth Edition. Comparison must be made with the readings of A (after subtraction from 360°, if necessary), except in the case of the zone of reference, when the angular values must be compared with those given in the fourth column.

The last two observations were taken to verify the tautozonality of *p*, *y* and *e*.