On a Question relative to Extinction-angles in Rock-slices.[1]

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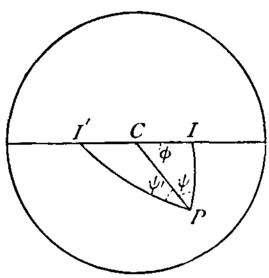
MANY minerals of the rhombic system are of markedly columnar habit, with the axis of elongation, or vertical axis, as one of the bisectrices of the angles between the optic axes. Such a crystal, in a section parallel to the vertical axis, gives straight extinction. It is natural to expect, and is perhaps generally assumed, that a section slightly oblique to the vertical axis will give nearly straight extinction. The object of the present note is to inquire how far this is true, and in what possible cases a serious error may arise.

Let P in the figure be the pole of the plane of section; C, I, I' the points corresponding with the vertical axis and the two optic axes.



Let $CI = V = CI'$; $CP = \frac{\pi}{2} - \theta$, so that θ is the obliquity of the section; the angle $ICP = \phi$; and the angles CPI and $CPI' = \psi$ and ψ' respectively.

Then by solving the triangles CPI, CPI', we have—

$$\tan\psi = \frac{\sin V \sin\phi}{\cos V \cos\theta - \sin V \sin\theta \cos\phi},$$

$$\tan\psi' = \frac{\sin V \sin\phi}{\cos V \cos\theta + \sin V \sin\theta \cos\phi}.$$

Taking for the extinction-angle (ϵ) the angle which the internal bisector of IPI' makes with PC, we have—[2]

$$2\epsilon = \psi - \psi'.$$

There are two cases to be distinguished, according as θ is less or greater than $\frac{\pi}{2} - V$. We take the former and more important case first.

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[1] Compare "Extinction-Angles in Cleavage-Flakes." By Alfred Harker. Min. Mag. 1893, Vol. X, p. 239.

[2] The extinction-angle is here reckoned from the projection of the vertical axis. In practice it would be measured from the trace of some pinacoid or prism-plane, but for a small degree of obliquity the difference is negligible.

If then θ is less than $\frac{\pi}{2} - V$, and therefore CP greater than CI, $\epsilon = 0$ both for $\phi = 0$ and for $\phi = \frac{\pi}{2}$, and reaches a maximum for some intermediate value of ϕ, θ being supposed given. From the above relations we obtain without difficulty

$$\tan^2\phi(\sin^2 V + \cos^2 V\cos^2\theta) - 2\tan\phi\sin^2 V\sin\theta\cot 2\epsilon + (\cos^2\theta - \sin^2 V) = 0,$$

an equation giving two values of ϕ between 0 and $\frac{\pi}{2}$ for a given value of ϵ less than the maximum. To find this maximum we make the two values of ϕ equal. The condition for this is

$$(\sin^2 V\sin\theta\cot 2\epsilon)^2 = (\sin^2 V + \cos^2 V\cos^2\theta)(\cos^2\theta - \sin^2 V),$$

which reduces to

$$\sin 2\epsilon = \frac{\sin V \tan V}{\cos\theta \cot\theta}.$$

The accompanying table is calculated from this simple relation. It gives the highest extinction-angles that can occur in sections of different degrees of obliquity. It is to be observed that $2V$ is here the angle between the optic axes measured over C, *i.e.* so that the vertical axis bisects it. The table shows that sections with a small obliquity will not give any considerable departure from straight extinction, except when $2V$ is a wide angle. Indeed it is a rough general rule that the maximum extinction-angle is less or greater than the obliquity of the section, according as $2V$ is less or greater than 130°.

It would be easy, if desired, to find the chance that a random section of given obliquity would have an extinction-angle higher than a given value (ϵ); for this chance is evidently the ratio of $\phi_1 - \phi_2$ to $\frac{\pi}{2}$, where $\tan\phi_1$ and $\tan\phi_2$ are the roots of the equation given above, and it can be shown that

$$\tan(\phi_1 - \phi_2) = \frac{2\sqrt{(\sin^4 V\sin^2\theta\operatorname{cosec}^2 2\epsilon - \cos^2 V\cos^4\theta)}}{(1 + \cos^2 V)\cos^2\theta}.$$

Finally, if θ is greater than $\frac{\pi}{2} - V$, and therefore CP less than CI, it is easy to see that there is no limit to the extinction-angle. As ϕ changes from 0 to $\frac{\pi}{2}$, ϵ changes from $\frac{\pi}{2}$ to 0, passing through all intermediate values.

Values of $2V$.	Values of θ.								
	1°	2°	3°	4°	5°	10°	15°	20°	30°
20°	0°	0°	0°	0°	0°	0¼°	0¼°	0¼°	0½°
30	0	0	0	0¼	0¼	0¼	0½	0¾	1¼
40	0	0¼	0¼	0¼	0¼	0¾	1	1½	2½
50	0	0¼	0¼	0½	0½	1	1½	2¼	3¾
60	0¼	0¼	0½	0½	0¾	1½	2¼	3¼	5½
70	0¼	0½	0½	0¾	1	2	3¼	4½	7¾
80	0¼	0½	0¾	1	1¼	2¾	4¼	6	10½
90	0¼	0¾	1	1½	1¾	3½	5¼	8	14
100	0½	1	1¼	1¾	2¼	4¾	7¼	10¼	18¾
110	0½	1¼	1¾	2¼	3	6	9½	13½	25¾
120	0¾	1½	2¼	3	3¾	7¾	12¼	17¾	—
130	1	2	3	4	5	10¼	16¼	24½	—
140	1¼	2½	4	5¼	6½	16	22¾	—	—
150	1¾	3½	5½	7¼	9¼	20	—	—	—
160	2¾	5¾	8½	11½	14¾	—	—	—	—
170	5½	11¾	18¼	26½	—	—	—	—	—