The Gnomonic Net.

(*With Plate II.*)

By HAROLD HILTON, M.A.

Fellow of Magdalen College, Oxford.

[Communicated by Professor Miers, and read February 2, 1904.]

MR. G. F. HERBERT SMITH [1] has pointed out the advantages of the gnomonic projection for some special crystallographic problems. It seems to me that much of the labour in drawing the projections could be saved by a device similar to that employed by Fedorow, Wulff, Penfield, &c., in drawing stereographic projections [2].

Corresponding to the 'stereographic net' described by these authors (which is the stereographic projection of lines of latitude and longitude on a plane through a line of longitude), we have the *gnomonic net*, which is the projection of the lines of latitude (l) and of longitude (m) on the tangent plane at the point whose latitude and longitude are zero. The lines m are parallel straight lines, the lines l are hyperbolae having double contact with each other at two fixed imaginary points. The projection of a point whose longitude is μ and whose latitude is λ is the intersection of the lines $x = a \tan \mu$ and $y^2 \cot^2 \lambda - x^2 = a^2$ (taking suitable rectangular Cartesian axes through O, the centre of the net) [3].

It is not difficult (though rather tedious) to construct a gnomonic net; for the positions of the lines m and their intersections with the lines l are readily obtained by the help of the table given by Mr. Herbert Smith (loc. cit.). An intersection of $x = a \tan \mu$ and $y^2 \cot^2 \lambda - x^2 = a^2$ is the point ($a \tan \mu$, $a \sec \mu . \tan \lambda$), and these coordinates are given in the table for every degree from 0° to 65° inclusive. A net showing every other degree [4] is given (Plate II); this should be cut out and pasted on a drawing-board. It is surrounded with a protractor [5] showing every

[1] Min. Mag., 1903, vol. xiii, pp. 309–321.

[2] See, for instance, Zeits. Kryst. Min., 1902, vol. xxxvi, p. 14; Phil. Mag., 1903, ser. 6, vol. vi, p. 67.

[3] In the diagram a = 1·56 inches = 39·64 mm.

[4] As in the stereographic net given by Professor G. Wulff in plate II, Zeits. Kryst. Min., vol. xxxvi, which I have found useful.

[5] Unlike the stereographic net, the gnomonic net does not itself serve as a protractor.

degree. The net is square, and the outer pair of lines m (separating the net from the protractor) correspond to a distance of 65° from the centre of the net[1]. The gnomonic projection of a crystal is drawn on tracing-cloth (or 'Pauspapier'), the centre of the projection being always placed over the centre (O) of the net.

The use of the gnomonic net is so similar to that of the stereographic net that I shall not describe it in detail; it is only necessary to take the account given by Professor G. Wulff[2] of the use of the stereographic net and to apply the necessary alterations. For instance, to find the projection of the pole of a given great circle, turn the tracing-cloth till the projection of the circle is parallel to the lines m and read off the position of the projection of its pole by means of the net. (The projections of a great circle and its pole are polar and pole with respect to the imaginary circle $x^2+y^2+a^2=0$.)[3] Similarly, given the projection of the pole we may find the projection of the corresponding great circle.

To change the plane of a gnomonic projection so that a point whose projection is P may become the centre of the projection, turn the tracing-cloth so that OP is perpendicular to the lines m, and move all points of the projection along the lines l through a distance corresponding to PO. It is best to mark the new positions of the points on another piece of tracing-cloth laid over the first (the net is readily seen through two pieces of good cloth).

To find the angle between two great circles whose projections are PQ and PR, change the plane of the projection so that P becomes the centre of the net and measure the angle between the new positions of the projections of the great circles; or else find the projection (h) of the great circle perpendicular to the two given great circles, produce PQ and PR to meet h in Q' and R', turn the tracing-cloth till h is parallel to the lines m, and read off the angle corresponding to $Q'R'$ on the net.

We can readily find by aid of the net the gnomonic projection of the poles of a crystal measured by means of a two-circle goniometer[4] on

[1] The corners of the net correspond to a distance of 71° 45′ from the centre.

[2] Zeits. Kryst. Min., 1902, vol. xxxvi, p. 15; cf. Phil. Mag., 1903, ser. 6, vol. vi, p. 68.

[3] If we project figures on a sphere from any point V on to any plane cutting the tangent cone from V to the sphere in the conic j, then all circles on the sphere project into conics having double contact with j. It follows that the tangents at the intersection of the projections of two orthogonal spherical curves are conjugate with respect to j. If V is on the sphere, j degenerates into a pair of points. In the projection shown in the diagram j is $x^2 + y^2 + a^2 = 0$.

[4] The method may also be applied to crystals measured with a one-circle goniometer,

a plane parallel to the face through which the measured zones pass. We can then find (by the net) the projection of the poles on any other plane. The zones of the projection are then perpendicular to the orthographic projection of the crystal-edges on the same plane (as explained by Mr. Herbert Smith), so that the orthographic projection of the crystal can be readily drawn. It should be noticed, however, that we can find the directions of these edges by means of the stereographic net thus:—Find the stereographic projection on the same plane, and obtain the projections of the poles of the zones; the lines joining these points to the centre of the projection are parallel to the crystal-edges of the orthographic projection.

It should be noticed that the gnomonic projections of the poles of four cozonal faces form a range whose anharmonic ratio is that of the four faces[1]; hence, if we can find the projections of the poles, we can find the anharmonic ratio of the faces (by measuring the actual distances between the projections), even if we do not know the angles between the faces. This is useful sometimes when one of the projections is at infinity[2].

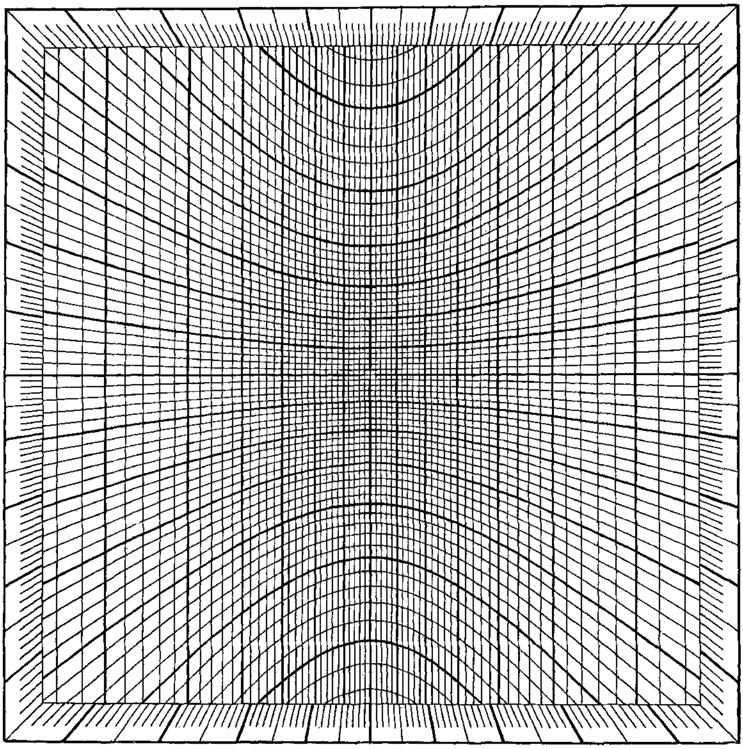
The gnomonic net may be used to solve graphically many problems in cartography or astronomy which can also be solved by the stereographic net. For example:—To graduate a sun-dial. Let P and Z be the gnomonic projections of the celestial north pole and zenith on the plane of the dial. (These points are readily found by means of the net when the position of the plane of the dial is given.) Produce PZ to cut the projection (h) of the celestial equator in R. Turn the projection till the line h is parallel to the lines m of the net, plot off points along h whose distances from R are 15°, 30°, 45°, 60°, &c.[3], and join these points to P; we thus get lines on the dial corresponding to each hour of local apparent solar time. The problem may also be solved (but not quite so readily) by the stereographic net or by the orthographic net (which is the orthographic projection of the lines of latitude and longitude on the plane of a line of longitude).

but the stereographic net is more convenient. This will be clear on a careful reading of Wulff, loc. cit., p. 16, problem 3.

[1] For the pencil of lines joining the poles of the faces to the centre of the sphere is cut by any plane in a range whose anharmonic ratio is that of the four faces.

[2] See Min. Mag., 1903, vol. xiii, p. 315.

[3] If we wished the dial to register apparent Greenwich time in longitude d°, we should take points whose distances from R are $(15 + d)$°, $(30 + d)$°, &c.



HAROLD HILTON: THE GNOMONIC NET.