

*On the method of minimum deviation for the  
determination of refractive indices.*

*With a diagram (Plate IV).*

By G. F. HERBERT SMITH, M.A., F.G.S.

Assistant in the Mineral Department of the British Museum.

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A DETERMINATION of the refractive power, which need be accurate to the second place of decimals only, is of considerable service for the discrimination of transparent substances, such as, in particular, faceted gem-stones. If the substance be so highly refractive that a total-reflectometer be not available, the method of minimum deviation is the simplest that can be used. Although the well-known formula

$$\mu = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}},$$

where  $A$  is the angle of the prism,  $D$  the minimum deviation, and  $\mu$  the index of refraction, is only strictly true when the plane bisecting the prism-angle is one of symmetry with regard to the optical indicatrix, yet the value so given is sufficiently correct for the purpose. It is the large errors which have to be avoided. When there are many faces in the same zone, the incident and emergent faces corresponding to the refracted image under observation are not easily identified, and mistakes may be made in the arithmetical computation even though the formula given above is so simple. Any process or device, therefore, which will add to the certainty of arriving at a correct result may be of service.

In the form of goniometer which is employed for this work, the telescope and graduated scale can rotate independently about the same axis. The procedure should be as follows:—The telescope is clamped in the position directly opposite to the collimator, and the scale rotated until it reads zero. The latter is now clamped and the telescope turned in the direction of increasing readings until it reaches the position of minimum deviation for some refracted image; the corresponding reading of the scale gives at once the angle  $D$  required. The stone and scale both remaining fixed, the telescope is turned to the position in which

the image reflected from the face of incidence is bisected by the cross-wires. This face should, if possible, always be one readily recognized, for instance the table facet of a cut stone; but in any case there should be little difficulty in identifying the face, since, if the telescope be rotated so that the images from consecutive faces are brought on to the cross-wires, it turns through double the angle between the faces in question, and therefore only one face will as a rule lie anywhere near a possible position. The telescope is in its turn clamped and the scale is read. The latter with the stone is rotated until the reading is that found for  $D$ . There should be in this position an image reflected from the face of emergence. The coincidence may not be exact because at minimum deviation a comparatively large rotation of the stone has no apparent effect on the position of the refracted image, apart from the fact that the formula already mentioned is not in general quite true. The difference between the last two readings gives the supplement of  $A$ . If this method be followed there will be no difficulty in correctly identifying the faces of the prism.

When the telescope is in the position for receiving the image reflected from the face of incidence the reading of the scale is theoretically  $180^\circ + A + D$ . When the scale is rotated so that the image reflected from the face of emergence is on the cross-wires the reading is increased by  $180^\circ - A$ , and thus we have  $360^\circ + D$  or  $D$ . Had the angle of minimum deviation been determined in the negative direction so that the reading of the scale was  $360^\circ - D$ , we should have had instead  $180^\circ - (A + D)$  less  $(180^\circ - A)$  or  $-D$ , i. e.  $360^\circ - D$ . The rule thus still holds good that the reading for the face of emergence is that previously found for the position of minimum deviation; and it remains true in the more general case, when the direct reading of the object-slit is any arbitrary angle and not zero.

In order to minimize the risk of errors in the computation, the author has prepared the diagram given on Plate IV. It is really a graphical interpretation of the formula

$$\mu = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$$

put into the form  $r = a \cos \theta + a \sin \theta \cot \frac{A}{2}$ ,

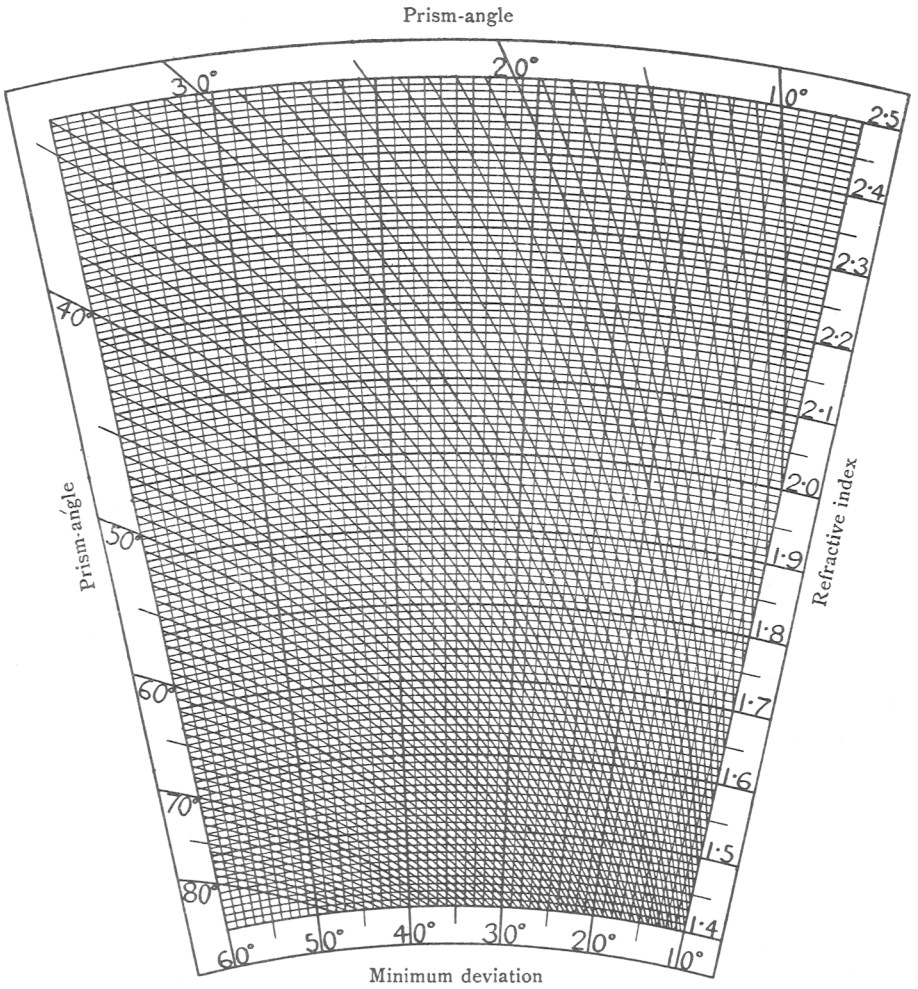
where  $\frac{r}{a} = \mu$  and  $\theta = \frac{1}{2}D$ .

This may also be written in Cartesian co-ordinates

$$x^2 + y^2 = ax + ay \cot \frac{A}{2},$$

whence we see that the formula thus expressed represents a family of circles, which pass through the origin and cut the axis of  $X$  at a distance  $a$  from the origin, and whose centres lie on the line  $x = \frac{1}{2} a$ . The family of circles are the curves running skew-wise across the diagram and have been drawn for every degree from  $A = 10^\circ$  to  $A = 80^\circ$ . The radial lines are also drawn for every degree from  $D = 10^\circ$  to  $D = 60^\circ$ . It may be noticed that the refractive index corresponding to any observed prism-angle and angle of minimum deviation within the given limits may be read off directly from the diagram to at least the second place of decimals. The range of the diagram includes refractive indices lying between 1.40 and 2.50.

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G. F. HERBERT SMITH : REFRACTIVE INDEX DIAGRAM.