Note on the thirty-two classes of symmetry.

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CONSIDERABLE confusion exists with regard to the nomenclature of the thirty-two possible classes of symmetry and their classification into systems. Each class is usually denoted by a name and a symbol; but as every author invents his own names and symbols (the latter being often merely arbitrary numbers) the whole nomenclature is hopelessly confused. This is due to the fact that a different classification into systems, and consequently a different nomenclature is suggested according as the subject is approached (1) from the geometrical aspect based on the law of rational indices alone, (2) from the aspect of the structure-theory, (3) from the aspect of the practical crystallographer. Thus, for example, Dr. A. Schoenflies[1] finds two distinct classifications necessary according as the subject is approached from aspect (1) or (2).

The question arises whether a simple nomenclature and classification are possible, which will reconcile the differences between the practical and theoretical crystallographer. It is usual to take as fundamental symmetry-elements n-al axes of the first and second sort; where by the former is meant a rotation-axis l such that the crystal is brought to self-coincidence by a rotation through $2\pi \div n$ about l (but through no smaller angle), and by the latter is meant an axis l of rotatory-reflexion (Drehspiegelung) such that the crystal is brought to self-coincidence by a rotation through $2\pi \div n$ about l followed by a reflexion in a plane perpendicular to l. Now we can avoid many difficulties if we define an n-al symmetry-axis of the second sort as an axis l of rotatory-inversion (Drehinversion[2]) such that the crystal is brought to self-coincidence by a rotation through $2\pi \div n$ about l followed by inversion about a point in l. Then any line through a centre of symmetry is a 1-al axis of the

[1] 'Krystallsysteme und Krystallstructur,' 1891, pp. 146 and 148.

[2] As is done by T. Liebisch in his 'Grundriss der physikalischen Krystallographie' (1896), though he does not point out the logical conclusion of his method.

second sort and a line perpendicular to a symmetry-plane is a 2-al axis of the second sort. An n-al axis of the second sort is equivalent to an n-al rotation-axis plus a centre of symmetry if n is odd, and to an $\frac{1}{2}n$-al rotation-axis plus a perpendicular symmetry-plane if n is even and $\frac{1}{2}n$ odd.

The thirty-two classes will then be grouped naturally into seven systems containing the classes with (1) only 1-al axes (of either sort), (2) one 2-al axis, (3) three perpendicular 2-al axes, (4) a 3-al axis, (5) a 4-al axis, (6) a 6-al axis, (7) more than one n-al axis ($n>2$). These systems coincide exactly with the triclinic, monoclinic, orthorhombic, rhombohedral, tetragonal, hexagonal, and regular systems of the structure-theory and practical crystallography.

As an instance of the simplicity gained by the proposed definition we notice that the same proof of the fact that every n-al symmetry-axis is parallel to a possible edge applies to axes both of the first and second sort ($n=3$ being an exception in both cases).

The classes with a single n-al axis of the first or second sort may be denoted by the symbols C_n and c_n respectively; those with a single n-al axis of the first or second sort and a perpendicular 2-al rotation-axis by D_n and d_n respectively (it is convenient, however, to write C for C_1, D for D_2); and those with an n-al axis of the first sort and a perpendicular 2-al axis of the second sort by δ_n. The tetrahedral and octahedral classes may be denoted by T and O. The addition of a centre of symmetry is conveniently shown by changing C, D, T, O into Γ, Δ, Θ, Ω respectively. These symbols have the advantage of being both simple and descriptive. For comparison they are tabulated below with the corresponding numbers from Professor Miers's 'Mineralogy' (1902, p. 280).

Triclinic	C, Γ	1, 2
Monoclinic	C_2, c_2, Γ_2	4, 3, 5
Orthorhombic	δ_2, D, Δ	7, 6, 8
Rhombohedral	C_3, c_3, δ_3, D_3, Δ_3	9, 15, 12, 10, 19
Tetragonal	C_4, c_4, Γ_4, δ_4, D_4, d_4, Δ_4	21, 22, 24, 25, 23, 26, 27
Hexagonal	C_6, c_6, Γ_6, δ_6, D_6, d_6, Δ_6	14, 11, 17, 18, 16, 13, 20
Regular	T, Θ, θ, O, Ω	28, 30, 31, 29, 32

If the suggestion here thrown out is adopted, all diagrams illustrating crystallographic theory should be drawn on the gnomonic projection. Diametrically opposite points on the sphere of reference are represented by the same point on the plane; the projection of any point being

marked with a × or a O according as the point lies on the half of the sphere nearer to or remote from the plane of projection. Thus the projections of all co-zonal poles lie on one straight line instead of on one or other of two circles as in the usual representation by the stereographic projection. The main objection to the use of the gnomonic projection (namely that the projection cannot be contained within convenient limits) does not apply to illustrative figures; for we may always choose the position of the crystal poles so that the gnomonic diagram occupies a convenient space.

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