The graphical determination of angles and indices in zones.[1]

(*With Plate XI.*)

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NINE years ago (in 1904) I described a diagram[2]—the moriogram, as I called it—which served for the graphical determination of angles and indices in the case of zones in which the principal poles included a right angle. It was, therefore, restricted in its application to zones in the cubic, tetragonal, orthorhombic, hexagonal, and rhombohedral systems, and to zones in the monoclinic system passing through the pole of symmetry. Although the moriogram may be used with sufficient accuracy for practical purposes in the case of many substances with triclinic symmetry, in which the angles between the principal poles differ little from right angles, there necessarily remain many instances in which it cannot be applied with any approximation to accuracy. Two methods have occurred to me of dealing with the perfectly general case, in which the angles between the principal poles may have any arbitrary value; the first of them is an immediate deduction from a property of the gnomonic projection, and the second is an extension of the moriogram referred to.

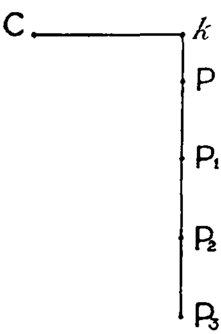

FIG. 1.

First Method.—In fig. 1 C is the central pole of the gnomonic projection, and $PP_1P_2P_3$ is a zone meeting the perpendicular zone through C in k. Then, if Ck, Pk, PP_1, PP_2, PP_3, represent respectively angles

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[2] Mineralogical Magazine, 1904, vol. xiv, pp. 49–53.

ϕ, θ, A, B, C, the linear distances kP, kP_1, kP_2, kP_3, on the projection are as follows :—

$$kP = a\tan\theta\,.\sec\phi,$$
$$kP_1 = a\tan(\theta+A)\,.\sec\phi,$$
$$kP_2 = a\tan(\theta+B)\,.\sec\phi,$$
$$kP_3 = a\tan(\theta+C)\,.\sec\phi,$$

a being the radius of the sphere to which the projection corresponds.

The anharmonic ratio of four poles in a zone is equal to

$$\frac{\sin A}{\sin(C-A)}\cdot\frac{\sin(C-B)}{\sin B}=\frac{\sin\{(\theta+A)-\theta\}}{\sin\{(\theta+C)-(\theta+A)\}}\cdot\frac{\sin\{(\theta+C)-(\theta+B)\}}{\sin\{(\theta+B)-\theta\}}$$

$$=\frac{\tan(\theta+A)-\tan\theta}{\tan(\theta+C)-\tan(\theta+A)}\cdot\frac{\tan(\theta+C)-\tan(\theta+B)}{\tan(\theta+B)-\tan\theta}$$

$$=\frac{kP_1-kP}{kP_3-kP_1}\cdot\frac{kP_3-kP_2}{kP_2-kP}=\frac{PP_1\,.\,P_2P_3}{P_1P_3\,.\,PP_2}$$

If the pole P_3 be taken on the equatorial zone, which is at infinity on the projection, the anharmonic ratio reduces to

$$\frac{PP_1}{PP_2}.$$

Supposing P, P_1, P_2, P_3 have respectively the indices $10l$, $11l$, hkl, $01l$. We have

$$\frac{PP_1}{PP_2}=\frac{h}{k}.$$

Hence the distances from P of the various poles in the zone will between P and P_1 be sub-multiples, and beyond P_1 multiples of PP_1; thus if d be the distance between the poles $10l$ and $11l$ on the projection, we have the distances between $10l$ and other poles in the zone as under :—

$21l$	$\frac{1}{2}d$	$12l$	$2d$
$31l$	$\frac{1}{3}d$	$13l$	$3d$
$h1l$	$\frac{1}{h}d$	$1kl$	kd

The straight line in the projection representing the zone in question is therefore spaced out in simple multiples and sub-multiples of the unit. Now $PP_1 = a\sin A\sec\phi\sec\theta\sec(\theta+A)$, and varies with θ and A even if a and ϕ remain constant. We must, therefore, be able to vary the unit in the drawing. This may very simply be done by drawing a pencil of straight lines corresponding to the various poles, and by applying to it, as for instance in fig. 2, a double tangent scale at right angles to the central straight line, in such a way that the latter lies on the angle included by the principal poles, for instance $ab(100:010)$, and the unit line 1 to the unit angle, for instance $am(100:110)$. The angles between $a(100)$ and the other poles, positive or negative, or the poles corresponding to the angles may be at once read off, if less than 30°, beyond which limit it is not convenient to draw a tangent scale. To obtain the angles beyond the limit, readings must be taken from the other principal pole, for instance $b(010)$, and should the unit angle be less than 30°, the angle corresponding to some other pole, for instance (210), within the limits of the scale must be used instead.

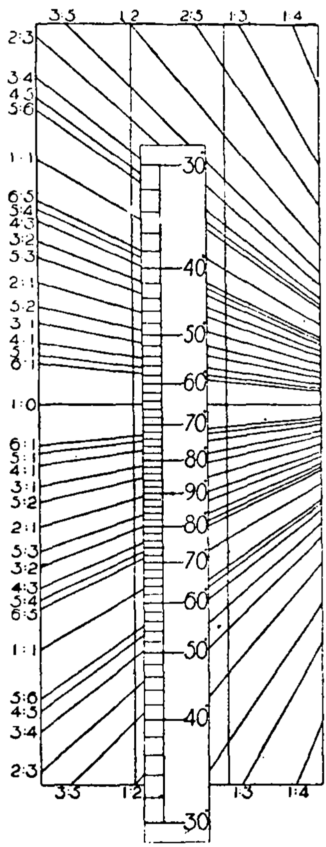


FIG. 2.

Second Method.—This is an extension of the moriogram, and the diagram (Plate XI) is a graphical expression of the equation

$$\frac{\cot\psi - \cot\psi_0}{\cot\psi_1 - \cot\psi_0} = \frac{\cot\theta}{\cot\theta_1} \cdot \frac{n}{m},$$

where ψ_0 and ψ_1 are the constants for the zone, and stand, for instance, for the angles $ab(100:010)$ and $am(100:110)$, and ψ is the angle required corresponding to any ratio of indices m/n, for instance $mn0$.

The diagram is in two parts. That on the right hand is the simple moriogram, corresponding to zones in which the principal poles include a right angle. It is in a slightly different form, which is perhaps a little more convenient in the general case, both coordinates being tangent scales extending as far as 45°. The graphs, which have, as before, been drawn for all indices including numbers not larger than 10, are straight lines; they are inverted at the straight lines corresponding to 45°, and become rectangular hyperbolae. The diagram is symmetrical about the diagonal running upwards from left to right.

The left-hand part of the diagram represents the equation

$$\cot\psi - \cot\psi_0 = \cot\theta.$$

Putting $x = \cot\psi$, $y = \cot\theta$, $c = \cot\psi_0$ we have

$$x - c = y,$$

which represents a series of parallel straight lines. In the diagram the origin is at the bottom on the right, the positive direction of y is vertically upwards, and of x horizontally from right to left; the coordinates are tangent scales. To avoid negative values of y, the lines are reflected at the axis of x, and run upwards from left to right. On reaching the line $x = 1$, corresponding to 45°, the lines are inverted, and we have a second series represented by

$$x'(c+y) = 1,$$

where $x' = 1/x$. The graphs are now rectangular hyperbolae. Carrying these as far as $y = 1$, and inverting with respect to y, i.e. putting $y = 1/y'$, we have

$$cx'y' = y' - x',$$

a series of rectangular hyperbolae passing through the origin.

To obviate hopeless confusion in the diagram, the graphs for values of ψ_0 between 90° and 180° have not been drawn, but they are precisely the same as those shown if the axes of x and y be interchanged.

In using the diagram, the inclined line corresponding to the angle, ψ_0, between the principal poles is followed to its intersection with the vertical line corresponding to the unit angle, ψ_1, and the resulting value of θ_1 is read off on the right-hand vertical scale. If ψ_1 is less than 45°, the second intersection, i.e. where the ordinate meets the curved graph, is taken, and, if it meets the inverted curve, the corresponding reading less than 45° on the vertical scale must be taken as θ_1.

If ψ_0 be greater than 90°, the procedure is similar, but we start from the vertical scale and read off the values of θ_1 on the horizontal scale. If ψ_1 be less than 45°, we take the first intersection of the ordinate with the graph, and if greater, the second; and, if it be greater than 90°, we must take the intersection with the inverted straight line running upwards from left to right.

From the left-hand diagram the values θ corresponding to other poles in the zone are read off, and the corresponding values ψ are obtained from the left-hand diagram by reversing the procedure given above.

Let us consider some examples on the diagram :—

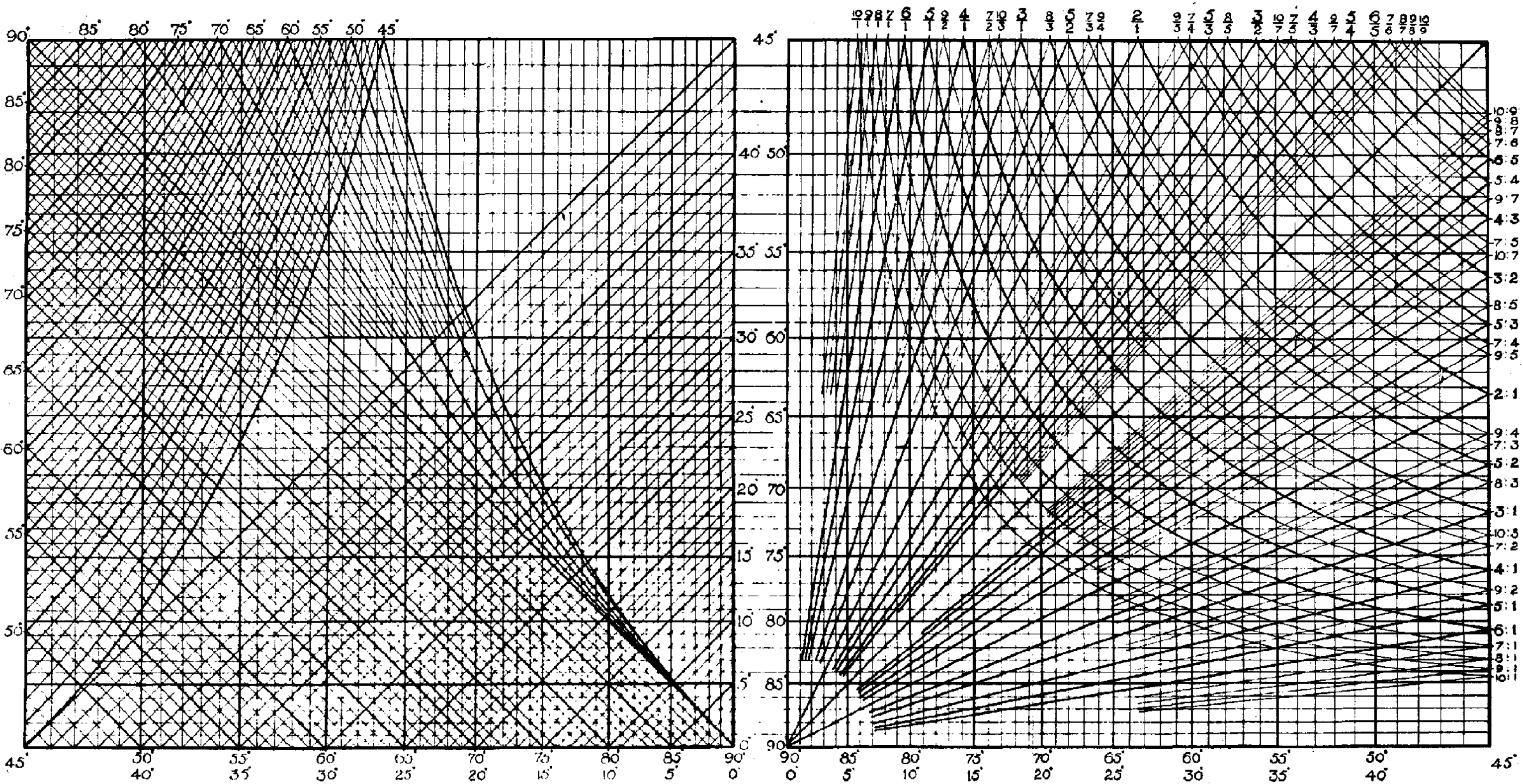
ψ_0	ψ_1	θ_1
80°	50°	56½°
"	40	44½
"	25	27
100	50	44½
"	40	36
"	25	23½
110	98	77½

As an example of the use of the moriogram we will take the zone *ac*(100 : 001) of epidote, the measurements given being from *c*(001).

Epidote.

Form.	θ	ψ	Calculated.[1]
105	11½°	10½°	10° 39′
102	27¼	22½	22 31
101	45¾	*	34 43
201	64	46	46 12
301	72	51½	51 26
100	90	*	64 37
$\bar{3}01$	72	98½	98 37
$\bar{2}01$	64	89½	89 26
$\bar{3}02$	57	80½	80 16½
$\bar{4}03$	54	76	75 51
$\bar{1}01$	45¾	63¾	63 42
$\bar{3}04$	38	50¾	50 44½
$\bar{2}03$	34¾	45½	45 37
$\bar{1}02$	27¼	34¼	34 21
$\bar{1}03$	19	22½	22 21
$\bar{1}04$	14½	16½	16 23

[1] Extracted from Dana's 'System of Mineralogy,' Sixth edition.



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