The use of the stereographic protractor for the interpretation of Laue crystal-photographs.

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THE spots of a Laue crystal-photograph may be regarded as due to the reflection of X-radiation by the planes of the crystal and the interpretation of these photographs involves the determination of the index of the plane by which any given spot is produced. This interpretation can be readily accomplished by two distinct though related methods. We may, in the first place, utilize the stereographic projection and determine the indices of the planes by means of intersecting zone-circles. This method, the earliest in point of time, was explained by W. L. Bragg[1] in 1913. In recent years, however, it has become usual to employ for this purpose the gnomonic projection which affords a solution in some respects simpler and more direct than the stereographic method used by Bragg, and detailed illustrations of its application in this field have been given by R. W. G. Wyckoff,[2] and E. Schiebold.[3]

It is the object of this communication to show how the protractor described to the Society by the author[4] in 1908 may be used with advantage for finding rapidly, and without calculation, the gnomonic projections of the planes producing the Laue spots.

The nature of the problem to be solved is readily seen from fig. 1, where XN represents the path of a narrow beam of X-rays passing through a crystal situated with its centre at O and impinging normally at N on a photographic plate RG. This beam causes the large intense

[1] W. L. Bragg, Proc. Roy. Soc. London, ser. A, 1913, vol. 89, p. 248.

[2] R. W. G. Wyckoff, Amer. Journ. Sci., 1920, ser. 4, vol. 50, p. 319. [Min. Abstr., vol. 1, p. 194.]

[3] E. Schiebold, Die Naturwissenschaften, 1922, vol. 10, p. 399.

[4] A. Hutchinson, Min. Mag., 1908, vol. 15, pp. 93-112.

central spot of the Laue photograph. Part of the X-radiation is reflected to R by a crystal plane PP', passing through O normal to the paper, and making an angle $P'OX = \theta$ with the incident beam. OG drawn perpendicular to PP' gives the gnomonic projection at G of the plane to which the spot produced at R is due. If ON, the distance of the centre of the crystal to the photographic plate, and NR, the distance between the centres of the central spot and the spot R, are measured, the angle 2θ can be found from a table of tangents and the distance NG can then readily be calculated, for $NG \div NO = \tan(90° - \theta)$.

It has been pointed out by Wyckoff and also by Schiebold that time and trouble can be saved if a standard distance is adopted for ON (distance from crystal to the photographic plate) and a table is constructed

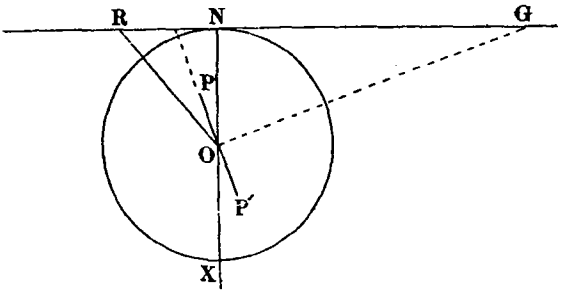

FIG. 1. Gnomonic projection of X-ray reflection.

giving corresponding values of NR and NG. Such tables have been prepared by Wyckoff (loc. cit.) for $ON = 3$, 4, and 5 cm. Further, Wyckoff has found it convenient to solve the problem graphically by employing rulers bearing graduations corresponding to the tabular distances mentioned above. Fig. 2 represents one of Wyckoff's rulers, the finer graduations of the original having been omitted. The short arm is graduated in centimetres subdivided to millimetres and numbered from 0–7. On the long arm, at distances taken from the appropriate table, are placed graduations which give the gnomonic projection of the plane producing a Laue spot, the distance of which from the centre is read on the short arm. If the ruler is placed on the Laue photograph with its zero-point C on the middle of the central spot, the gnomonic projection of the plane causing a spot lying opposite division 2 (say) on the short arm is found opposite division 2 on the long arm.

The stereographic protractor for a sphere of radius 5 cm. described by the author in 1908 can be used for this determination in a precisely

similar way, the distance from crystal to the photographic plate being 5 cm. The zero-point O of the protractor shown in fig. 3 (the finer graduations of the original have been omitted) is placed on the middle of the central spot and the position of a Laue spot read on the lower scale of the short arm OS. This reading, divided by four, gives the position, as read on the lower scale of the long arm OT, of the gnomonic projection of the plane producing the spot. Thus, if a Laue spot is opposite 60° as read on the lower scale of the short arm, the projection

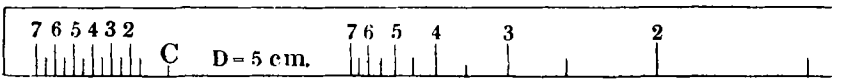

Fig. 2. Wyckoff's ruler for Laue photographs.

of the corresponding plane is found opposite 15° read on the lower scale of the long arm.

Since, however, the protractor as made for students' use, and intended especially for the construction of stereographic projections, is too short for finding the gnomonic projections of planes lying at large angles from the centre of the projection, the author has had some protractors specially constructed of greater length. These protractors are made exactly in the

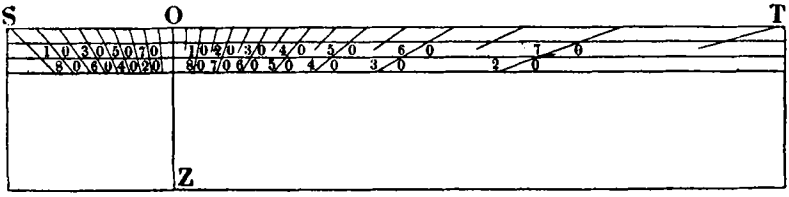

Fig. 3. Hutchinson's stereographic protractor.

same way as those previously described, that is to say, a long strip of boxwood or celluloid 5 cm. in width is adjusted so that a point in one edge about 9 cm. from one end coincides with the centre of a circular dividing engine. The opposite edge is then graduated in degrees. The strip is then turned round and the opposite edge graduated in the same way. The scales are then appropriately numbered. In the new protractors the scales on one edge are numbered exactly as heretofore, that is to say, as shown in fig. 3. These scales are used for making stereographic and gnomonic projections and for measuring angles between projected planes and zones as described in 1908 (loc. cit., p. 100 et seq.).

To make the instrument more conveniently applicable to the interpretation of Laue photographs the numbering of the scales on the other edge has been slightly modified, as shown diagrammatically in fig. 4, the finer graduations being omitted. Thus, as regards the scale on the long arm, only the lower row of numbers extending from 30° to 7° is given, and as the portion of the scale from the zero-line to 30° is not required, it has been omitted. Similarly, as regards the short arm, only the lower row of numbers of fig. 3 is given but the scale has been extended, and to obviate the necessity of dividing by four, the numbering is so arranged that an angle of twenty degrees as graduated is numbered ten degrees. Now, since in fig. 3 twenty degrees as graduated is numbered forty degrees on the short arm, the new numbering is equivalent to dividing by four the scale-reading on the short arm of the protractor shown in

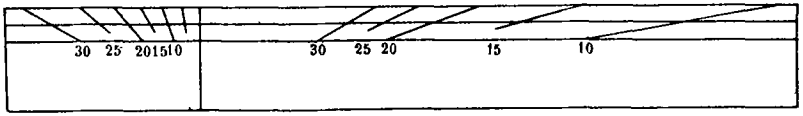

Fig. 4. Hutchinson's scale for Laue photographs.

fig. 3. From this it follows that a Laue spot and the gnomonic projection of the plane causing it have identical scale-readings on opposite sides of zero on the short and long arms respectively. A further and important advantage of this method of numbering is that the scale-reading of a Laue spot gives directly the value of θ, the glancing-angle at which the X-rays strike the plane producing the spot. The sine of this angle can then be taken from the tables without further calculation.

The reason for adopting the numbering described above becomes evident on comparing the scales with fig. 1. The graduations on the long arm are at distances proportional to $\tan(90° - \theta)$. The distance of the Laue spot is proportional to $\tan 2\theta$. By assigning to any graduation on the short arm a number of half its real value in degrees the position of the gnomonic projection of the plane inclined at the glancing-angle θ has the same numerical value.

As the protractors can be graduated to a sufficient degree of accuracy on a circular dividing engine with but little attention on the part of the operator, they can be produced at a minimum amount of trouble and expense. They can be obtained from Mr. W. H. Harling, 117 Moorgate, London, E.C. 2.