On the advantages of the face-adjustment for two-circle goniometry.

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IN two-circle goniometry as usually practised, the zone-adjustment is almost invariably used. It would be difficult to discover the reasons which have led to this unfortunate neglect of the face-adjustment; actually the face-adjustment has all the advantages of the zone-adjustment without its defects.

In the orthorhombic and higher systems, the zone-adjustment as generally practised (adjustment of the zone *ab* [001]) is identical with a face-adjustment of *c* (001), and there is therefore no distinction between the two methods; it is only necessary, therefore, to consider the monoclinic and anorthic systems.

The principal advantages which two-circle goniometry, using the zone-adjustment, can claim over single-circle methods are: reduced handling of the crystal; increased speed of measurement; that an accurate projection is quickly and easily prepared, facilitating indexing and the detection of possible symmetry obscured by a distorted habit; and the possibility, by using the methods devised by V. Goldschmidt and based on the gnomonic projection, of using the whole of the measured angles in the computation of the elements. An advantage frequently quoted, that a complete table of angles is more concise than a complete single-circle table of angles, is, in the author's opinion, of no importance. The only proper functions of a table of angles in an original paper are to record the actual measurements[1] and to compare them with the calculated values, and with the corresponding angles of isomorphous substances. Formerly, a table of angles was regarded partly as a collection of determinative data, but the introduction of a rational determinative system based

[1] This primary function is not fulfilled by the tables of angles sometimes given, containing calculated angles only.

on crystal morphology[1] by T. V. Barker renders this function obsolete (except in mineralogy, where there is generally a strong suspicion as to the identity of a crystal before the actual determination is begun), so that a complete table of angles is no longer a desideratum in an original description.

It will be obvious that the advantages of reduced handling and increased speed of measurement remain with two-circle goniometry whatever form of adjustment is used—indeed, the crystal may even be measured without adjustment, a useful facility in the case of highly deliquescent or very unstable substances. The face-adjustment, therefore, retains these advantages. The objections that are generally urged against it are that the preparation of a projection is not so simple as with the zone-adjustment, and that computation is not so straightforward. Both objections are quite fallacious.

In anorthic crystals, with the zone-adjustment (the zone adjusted being *ab* [001]), the method of projection on a plane normal to the adjusted zone is illustrated by fig. 1; since each face-pole has its own ϕ and ρ values, it is necessary in projecting each pole to draw a line OP at an angle ϕ with OP_0 (for which $\phi = 0°$), and measure off $OP = r.\tan\rho$, where r is the radius of projection.

It is usually stated that an equally simple method of projection is not available for the face-adjustment, and if the face adjusted is a terminal face, as *c* (001), this is true; but if the face adjusted lies in the prism zone, say *b* (010), the method of projection is quite as simple and often even quicker. Instead of separate ϕ and ρ values for each face, we shall have, since the faces usually all lie in relatively few zones through *b* (010), a few interzonal angles at *b*, and an interfacial angle from *b* to each other face. To project on a plane normal to zone *ab* [001], a line OM (fig. 2) is drawn to represent the trace of *b* (010); normal to this is the gnomonic *b*-axis, OB. Lay off along OM distances $OR = r.\cot\Phi$, where Φ is the interzonal angle $[ab]:[bPQ\ldots]$ and, in the opposite direction, $OT = r.\tan\Phi/2$, and draw RPQ perpendicular to OM. Then RPQ is the gnomonic zone-line $[bPQ\ldots]$, and T is the stereographic pole of this zone. To project all the faces in the zone, it is only necessary to draw lines TP, TQ, &c., making with RT angles $90° - bP$, $90° - bQ$, &c. (bP, bQ being the inter-

[1] The author would add a fifth independent sub-division of crystallography to the four given by A. F. Rogers (Amer. Min. 1933, vol. 18, p. 538); determinative crystallography differs sufficiently in its aims and methods from pure morphology or from structural crystallography to form a separate sub-division.

facial angles between b (010) and the faces P, Q, &c.), to cut RPQ at the points P, Q, &c., which are then the gnomonic poles required.[1] The same proceeding is followed with each other zone through b (010), and the projection is quickly and accurately made. The distance of the 'angle-point', T, from O cannot exceed r, the radius of projection. If the faces are numerous and concentrated in a few zones through b (010), the advantages of the method become most evident.

With a monoclinic substance, face-adjustment of b (010) is identical with zone-adjustment of ac [010], and much preferable to the adjustment of any other face or zone-axis.[2] If the crystal is to be projected on a plane perpendicular to the zone ab [001], the above procedure serves, while a projection on the face b (010) is readily made by the usual method.[3]

Turning now to the matter of computation, it will readily be seen that with b (010) in face-adjustment, x and y, the Goldschmidt rectangular co-ordinates [4] of a face-pole F in the gnomonic projection are: $x = \cot [ab] : [bF]\ (= \cot \Phi)$, and $y = \cot bF . \operatorname{cosec} [ab] : [bF]$ $(= \cot bF . \operatorname{cosec} \Phi)$. Hence the normal Goldschmidt method of computation can be used quite straightforwardly.

1 The formal proof of this construction is simple. Let OB extend to infinity, to meet RPQ in B, which will then be the gnomonic projection of b (010). Draw PY parallel to TR to cut OB in Y. Then BPY is the gnomonic projection of a right-angled spherical triangle, the right angle being at Y. And the angle YBP of this triangle $= 90° - [ab] : [bP] = 90° - \Phi$. Hence $\cot BY = \cot bP . \operatorname{cosec} \Phi$. But in the gnomonic projection, $RP = OY = \cot BY$. And $RP = RT . \cot bP$ by construction. Hence if $RT = \operatorname{cosec} \Phi$, the construction is correct. Now by construction, $RT = \cot \Phi + \tan \frac{1}{2}\Phi = \operatorname{cosec} \Phi\ (\cos \Phi + \sin \Phi . \tan \frac{1}{2}\Phi) = \operatorname{cosec} \Phi$ $(1 - 2 \sin^2 \frac{1}{2}\Phi + 2 \sin \frac{1}{2}\Phi . \cos \frac{1}{2}\Phi . \tan \frac{1}{2}\Phi) = \operatorname{cosec} \Phi$.

2 The face b (010) can almost always be adjusted, even if it is not developed. Thus the zone ac [010] may be adjusted, which comes to the same thing, since the face b is perpendicular to this zone-axis; or the zone ab [001], or any other zone containing b (010), may be adjusted parallel to the axis of the horizontal circle, the zone measured, and the position of b(010) calculated. Then any face is brought into the reflecting position, the horizontal circle turned through the angle which the normal to b(010) has been found to make with the axis of the vertical circle, and the face brought back into the reflecting position by means of the crystal-adjusting segments. With the three-circle goniometer, this adjustment is made with the greatest ease by means of the third circle.

3 The principal advantage of projection on b (010) lies in the assistance given in selecting the most suitable axes. Cf. T. V. Barker, Systematic crystallography. London, 1930, p. 44.

4 Projection is assumed to be on a plane perpendicular to the zone ab, and the y-axis is parallel to the zone-line bc. The radius of projection is taken as unity. With the zone ab in zone-adjustment, $x = \tan \rho . \sin \phi$, $y = \tan \rho . \cos \phi$.

It is thus evident that the face-adjustment retains all the advantages over single-circle goniometry possessed by the zone-adjustment. And it possesses several distinct advantages over the zone-adjustment. Not the least is the fact that the measured angles are all interfacial or interzonal angles, while with the zone-adjustment, the measured angles are of no direct crystallographic significance, being neither interfacial nor interzonal angles. Further, the measurement proceeds

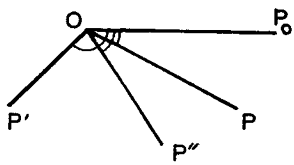

FIG. 1.

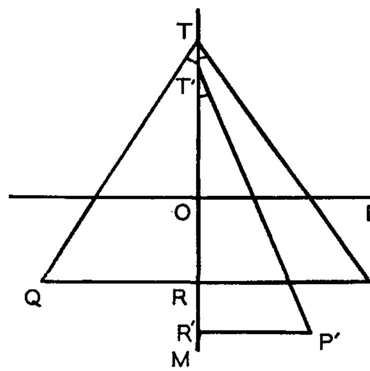

FIG. 2.

FIG. 1. The preparation of a projection, after measurement with the zone-adjustment, on a plane perpendicular to the adjusted zone. $OP = r \,.\, \tan \rho$, angle $P_0OP = \phi$.

FIG. 2. The preparation of a projection after measurement with the face-adjustment, on a plane perpendicular to the adjusted face, b(010). Radius of projection 2·5 cm. $OR = r.\cot\,[bP]:[ba]$, $OT = r.\tan\frac{1}{2}\,[bP]:[ba]$, angle $OTP = 90° - bP$.

on zonal lines, one advantage of which is immediately apparent in that the theoretically equal interzonal angles (Φ-angles) of all faces in any zone through b(010) can be averaged directly without computation.

A less apparent advantage of the zonal course of measurement when the face-adjustment is used lies in the fact that adjustment can be made more truly, though this statement may at first sight appear open to some doubt. When the zone-adjustment is used, the faces in the adjusted zone all have $\rho = 90°$, and one is arbitrarily selected as reference-face, with $\phi = 90°$. Naturally, there will be a certain error in the determination of the actual position of this face, and hence a process of calculation from the other measured angles is necessary to correct this error (cf. L. Borgström and V. Goldschmidt, Zeits. Kryst. Min., 1905, vol. 41, p. 73, or A. L. Parsons, Amer.

Min., 1920, vol. 5, pp. 190, 198). On the other hand, no correction of the ρ values is possible, nor should such a correction be necessary, seeing that in actual practice $\rho = 90°$ will be the average reading for all the prism faces; the adjustment will have been made so that on rotating the vertical circle the departures from the vertical cross-wire of the signals from the several faces of the zone *ab* [001] as they pass across the field of view are evenly distributed—the departures to the left balance the departures to the right, after full allowance has been made for the perfection of the various reflections.

When the face-adjustment is employed, it is usual to consider that adjustment is achieved when the signal reflection from the chosen face does not move appreciably as the vertical circle is rotated. If this means of adjustment, depending solely on one face, is used, then indeed the face-adjustment is less accurate than the zone-adjustment.

But when a face has been truly adjusted, then it will be possible by rotating the vertical circle to bring every zone through that face into adjustment. Hence the correct and accurate method of making the face-adjustment is, first to adjust until the signal image from the selected face does not move as the vertical circle is rotated. Next each of the several zones [1] through the adjusted face is brought into the plane of the horizontal circle in turn; then on rotating the horizontal circle, the signal images from each face in a given zone should pass across the field of view at the same level. Any maladjustment will be apparent and can be corrected and the calculation of a correction such as is needed for the zone-adjustment is avoided. When correctly adjusted, the movement of the vertical circle should bring each zone in turn into accurate adjustment. In this way, any departure of the selected face from its true position, or any diffuseness in the signal image it gives, is eliminated. Sometimes it may prove possible to adjust a face which is not actually developed. Indeed, it would be theoretically possible, though hardly practicable (except with the three-circle goniometer, where it is quite feasible), to bring into face-adjustment the possible face lying at the intersection of any two developed zones of a crystal.

It may be noted that when the face-adjustment is made in the above manner, we abandon the direct attempt to bring the face-normal into coincidence with the axis of the vertical circle, which

[1] It is necessary that a zone shall contain at least two faces other than the face selected for adjustment, if it is to serve as a test of the accuracy of adjustment.

cannot always be done with any great accuracy owing to vicinal development, diffuse reflections, &c.; instead, the same end is attained by bringing into coincidence with the plane of the vertical circle the axes of all sufficiently developed zones through the selected face.

The advantages of the face-adjustment over the zone-adjustment may not be so great as the advantages both can show over single-circle goniometry, but the two points above noted are of some importance. That the measured angles are all either interfacial or interzonal renders the special symbols ϕ and ρ superfluous, and the table of angles can be given in the form of interfacial angles $bF = (010):(hkl)$ and interzonal angles $ab\,[001]:bF\,[l0\bar{h}]$. It is usual to make a projection on a plane perpendicular to the zone-axis $ab\,[001]$; if this is to be done, measurement with any face in the zone $ab\,[001]$ adjusted will give results from which a projection can readily and easily be prepared, but computation is greatly simplified if the adjusted face is[1] $a\,(100)$ or $b\,(010)$. If the above method of effecting the adjustment is employed, the face need not give particularly good reflections. Alternatively, any face may be adjusted, regarded temporarily as $b\,(010)$, a projection made and indexing and computation carried out. A 'correct' setting can then be sought on determinative or structural lines, or to correspond with some isomorphous compound, and the indices transformed and the elements for the new setting computed without difficulty. There is not the slightest need to re-compute the measured angles to correspond to a conventional selection of adjusted face for the new setting, for, as emphasized above, the primary function of a table of angles in an original paper is merely to record the actual measured values, and, if desired, to show their departure from a set of rigorously consistent values.

[1] The face $c\,(001)$ can very well be adjusted, provided projection is made on a plane normal to one of the zone axes $ac\,[010]$ or $bc\,[100]$.