The orientation of Widmanstetter figures in a random section.

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THE problem of determining the orientation of Widmanstetter figures has recently arisen in a paper by Dr. L. J. Spencer on the Gibeon shower of meteoritic irons in South-West Africa:[1] the method of attacking it to be described here will be applied to the crystals illustrated in plate II of his paper.

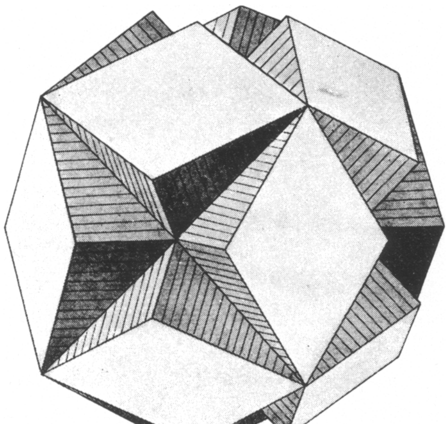

FIG. 1.

FIG. 1. Cluster of six rhombic prisms obtained by extending the faces of the regular octahedron. The basal planes (unshaded) are the faces of the rhombic-dodecahedron.

By producing the faces of a regular octahedron in all possible directions a set of six congruent prisms is formed, as shown in fig. 1. The Widmanstetter figures are the traces of these prisms in a plane of section. The section across any one of the prisms by a plane normal to its axis is a rhomb whose angles are those of a face of the rhombic-dodecahedron, that is to say, 70° 31′ and 109° 28′. If we imagine each of the prisms

[1] L. J. Spencer, Min. Mag., 1941, vol. 26, p. 19, pls. I, II.

to be a right prism closed at one end by such a face, and at the other end by the plane of section, then the problem resolves itself into finding the angle between the terminal plane $d(110)$ of a prism and the section plane; and the angle between one of its lateral faces $o\,(111)$ and the section plane, given their traces on this latter plane.

To do this, the Widmanstetter figures are traced in ink on tracing-paper, and from this transferred in pencil on to a plate of ground glass. Next, a rhomb having the shape of a rhombic-dodecahedral face is cut out of cardboard and set up normal to a parallel beam of light, conveniently done on an optical bench: the glass is then held so as to receive the shadow of the rhomb. By tilting and turning the glass, the edges of the shadow can be brought into approximate coincidence with two of the traces. When this is so, the plate is lowered parallel to itself until its bottom edge becomes embedded in a lump of plasticine: the glass is now self-supporting, and final adjustments are easily made. The edges of the shadow are measured with a pair of dividers. The size of the cardboard rhomb which can be used will depend on the aperture of the collimating lens employed. The edges of the one I used measured 2 cm., and the drawings which have subsequently to be made were done on a scale of one inch to a centimetre.

Suppose only two concurrent straight lines are drawn on the glass, then it may be set in an infinite number of positions, and an infinite number of shadow-parallelograms can be formed having the same angle. If now a third line be introduced forming a diagonal, then for a given cardboard rhomb the size and shape of the shadow become uniquely determined and it remains only to measure the edges of the shadow. Having got the unique parallelogram, it will be found to fit the prism in two equivalent positions (compare s_1 and s_2 of fig. 3).

On applying the method to the crystal occupying the left-hand half of plate II of Dr. Spencer's paper, the edges of the shadow were found, as a mean of three determinations, to be 2·00 and 2·64 cm. In fig. 2, draw *abcd* to represent the cardboard rhomb with sides 2 inches in length. Produce *ab* to serve as the ground line: draw lines perpendicular to this from the vertices *c* and *d*. Since the length of the shadow of *ab* is the same as *ab* itself, viz. 2 cm., its vertical and horizontal projections coincide. To find the vertical projection of *bc*, mark off *be* on the ground line equal to *bc*: through *e* draw a perpendicular to *be* and on it locate the point e', where be' equals the true length of the shadow of *bc*, viz. 2·64 inches: through e' draw $e'c'$ parallel to the ground line, intersecting the perpendicular from *c* in c'. Complete the parallelogram *abc′d′*. We have now got the horizontal and vertical projections of the section plane: from these the angle which it makes with the horizontal plane *d*(011) and with an adjacent face *o*(111) can be found by the usual methods of descriptive geometry. Thus, denoting the section plane by *s*; to find the angle *s*:(011). Since the plane containing the dihedral angle is perpendicular to both faces, through *b* draw *bh* per-

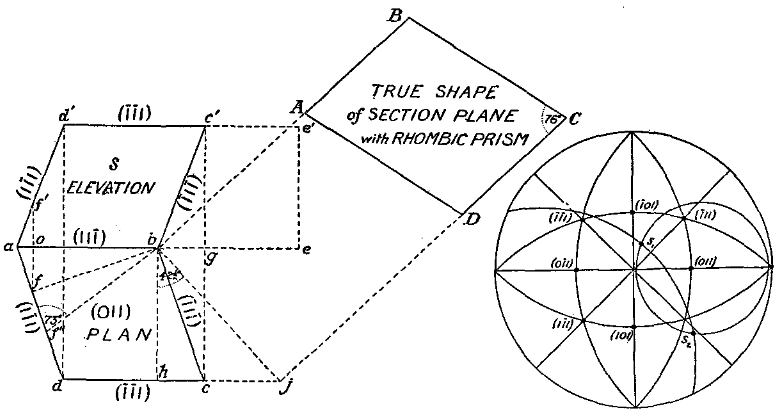

FIG. 2. FIG. 3.

FIGS. 2 and 3. Geometric and stereographic projections of the section plane in the first crystal.

pendicular to *dc* and intersecting it in *h*. From *h* lay off *hj* along *dc* produced, equal to *cg*. Then *hbj* is the angle required. Similarly, to find *s*:(111); from *b* draw *bf* perpendicular to *ad*, meeting it in *f*. From *f* draw a line of correspondence intersecting the ground line in *o* and *ad′* in *f′*. Along *fd* mark off *ff″* equal to *of′*. Then *ff″b* is the angle required. The true shape of the section plane can be got by erecting perpendiculars to *bj* through its extremities; from any point *A* on one of them mark off *AB* 2 inches in length and draw *AD* equal to 2·64 inches. The completed parallelogram *ABCD* corresponds to that in the top right-hand corner of Dr. Spencer's fig. 5.

Having found the dihedral angles s:$d(011) = 42\frac{1}{2}°$; s:$o(111) = 73°$, draw a stereographic representation of the octahedron-dodecahedron combination projected on a plane parallel to (001), as in fig. 3. About a dodecahedral pole, say (011), draw a small circle of radius $42\frac{1}{2}°$; and about an octahedral pole at right angles to (011), say $(1\bar{1}1)$, draw a small circle of radius 73°: these small circles intersect in the points s_1 and s_2 either of which may be the pole of the section plane. From this projection I find graphically $s_1 : d(101) = 64°$; $s_1 : c(010) = 83\frac{1}{2}°$. From Dr. Spencer's projection (fig. 6 of his paper) I find the comparable values $s : d(0\bar{1}1) = 63\frac{1}{2}°$; $s : c(001) = 84\frac{1}{2}°$.

Two matters call for special comment, first the choice of traces in the optical part of the work; and secondly the choice of poles in the stereographic projection. It is sometimes possible to bring the shadow into coincidence with two traces which ought not thus to be associated. If the choice has been rightly made, then the data obtained enable us to determine (by drawing) the traces yielded by another prism, say one at right angles to the first, and these also will agree with actual traces in the photograph. In this connexion the model shown in fig. 1 is very helpful; by turning it about and comparing its various aspects with the photograph a proper choice of traces can soon be made. In the case of the left-hand crystal this difficulty scarcely arises. The model is also of great use in the choice of poles in the stereographic projection; by considering the direction in which the long diagonal of the rhomb runs, the range of choice is reduced from that between six poles to that between four.

The crystal on the right-hand side of plate II will now be considered. Ten measurements were made of shadow edges brought to coincidence with a pair of traces, and using the mean of these the following angles were found: $s : d(011) = 31\frac{1}{2}°$; $s : o(111) = 78°$. These two angles form the basis on which the synthesis of the Widmanstetter figures was effected, the results being shown in fig. 5. A stereographic projection was first made similar to fig. 3, in the manner already described, the basal faces being assigned the symbols (011) and $(1\bar{1}1)$ respectively. The projection was then redrawn on the plane of section, as in fig. 4. From this other interfacial angles were taken as required and the parallelogram of traces due to any particular dodecahedral face determined by the descriptive methods. The parallelograms thus derived are shown in fig. 5, the symbol of the zone plane to which each is due being indicated. It will be seen that all the octahedral traces dealt with by Dr. Spencer are accounted for. Turning to the model shown in fig. 1, it is easily seen that the diagonals of the cardboard rhomb may correspond to an edge of the form $c(100)$. Thus the diagonal marked in the parallelogram [101] may be the trace of $c(\bar{1}00)$, agreeing exactly with the vertical line in Dr. Spencer's plate which he also assigns to that form. No other diagonals have their counterpart in the photograph, and two conspicuous traces remain unaccounted for: these are the short band forming a hook at the top of the vertical band $(\bar{1}00)$, and a series of parallel striations at right angles to it near the boundary of the two crystals.

At this stage some difficulty was experienced in finding faces to which these lines might possibly be assigned. The traces of the icositetrahedron (211) were determined, but they failed to meet the requirements

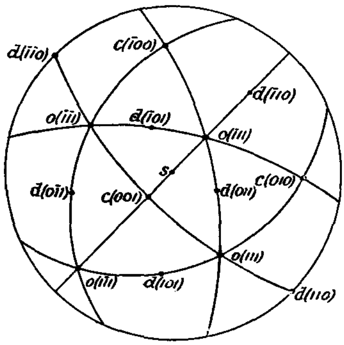

FIG. 4.

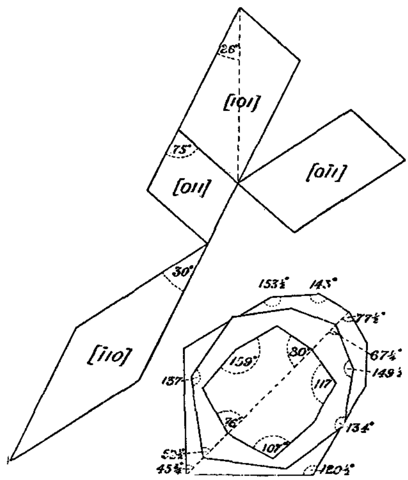

FIG. 5.

FIGS. 4 and 5. Stereographic and geometric projections of the section plane in the second crystal.

of the case. The traces of the hexakisoctahedron (321) were also investigated, the form being conveniently dealt with by resolving it into three pyramids ($hk1$), ($hk2$), and ($hk3$), and finding the traces of each one separately: the polygons in the right-hand bottom corner of fig. 5 represent them. On drawing these to a convenient scale on tracing-paper and moving them about over plate II they will be found to account for the lines mentioned at the end of the last paragraph. They also account for those short bands in the centre and half-way down the right-hand margin of the plate, which make a very acute angle with the octahedral traces; and for the striations running horizontally at the top of the plate. Thus the hexakisoctahedron (321) affords a complete geometrical solution. But Dr. Spencer informs me that (321) is not likely to occur: this being so, the mineralogical solution is yet to seek.

In conclusion, I wish to express my thanks to Dr. Spencer for much valued encouragement.