The 'superposition error' in the micrometric analysis of rocks.

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Introduction.—It is a well-known method in the measurement of birefringence to utilize a crystal's 'bevelled edge', on which appears a sequence of polarization colours, from that colour representing small relative retardation where the edge is thin, to a higher colour corresponding to the full thickness of the mineral in the microsection. In dolerites these bevelled edges on augites are extremely common and are very apparent between crossed nicols. In ordinary light, however, the majority of them are undiscernible and a grain appears to have uniform thickness over its entire area. The reason for the non-appearance of the bevelled edge in ordinary light is straightforward: the augites have excellent relief, that of the adjacent felspars is low; relief is a surface phenomena and independent of thickness, so that a thin edge of augite overlying felspar has the relief of augite, and the felspar is unnoticed.

It follows that in the measurement of the width of a grain of a high relief mineral, e.g. augite, surrounded by a low relief mineral, e.g. felspar, along any traverse we measure the maximum diameter of the high relief mineral, that is, from the tip of the bevelled edge on one side to the tip of the bevelled edge on the other. This will produce an inaccuracy, for we ought to measure from the centre of the bevel on one side to the centre of the bevel on the other. As a result of this inaccuracy we constantly overestimate the proportion of the high-relief mineral at the expense of the low-relief mineral.

In a normal dolerite this error will not be great because the bevelled edge is narrow compared with the diameter of the grain; however, the error will be there. As the grain diameter diminishes the distance between the bevelled edges on either side of the augite grains also diminishes until that grain-size is reached when the bevelled edges are adjacent —the mineral is all bevelled edge and has no uniform thickness.

The Rosiwal method of analysis assumes that the thickness of all grains is the same, equal to the thickness of the slide, for only in this way can the volumes (areas × thickness) be proportional to areas and hence to lengths on a measured traverse. As mentioned above the thickness is not always constant, the average thickness of a small grain may be only half that of the slide and an error in the estimation of volumes from measured lengths is inevitable.

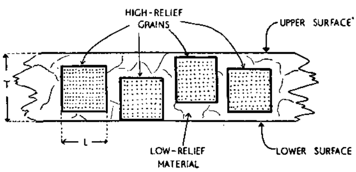

FIG. 1.

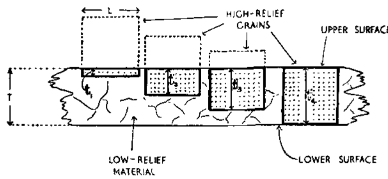

FIG. 3.

FIG. 1. Cross-section of a thin section assuming that all grains lie entirely within the slide.

FIG. 3. Cross-section assuming that grains are cut by the upper and lower surfaces of the slide.

This error is produced fundamentally because minerals of contrasted relief overlap one another in the slide, and so it may conveniently be called the 'error of superposition'. With the aim of measuring this error of superposition and its relation to grain-size the following calculations were made.

Calculations.—In order to make the subject amenable to computation, and as free as possible from mathematical intricacies, two assumptions have been made regarding the shape and orientation of the grains. These are: (1) that the grains are cubes, and (2) that they are orientated so that one face is parallel to the upper and lower margins of the thin section.

(*a*) Let us assume that the cubic grains are all completely within the thin section and that none of them cut its lower and upper surfaces (fig. 1).

In this case the assumed volume of the grain $= L^2T$ and the actual volume $= L^3$, where L is the length of one side of the cube and T is the thickness of the thin section.

$$\text{The \% error} = \frac{L^2T - L^3}{L^3} \times 100 = \frac{T-L}{L} \times 100.$$

When $L = T$, the error is zero; and as L decreases the error increases. Fig. 2 shows the relation between % error and L, when $T = 0{\cdot}03$ mm.

(*b*) The condition that all the grains fall within the thin section is never fulfilled, most of them will be cut by the surfaces of the thin section, so that the real volume will not be L^3 but L^2t, where t is the average thickness of cube within the thin section (average of t_1-t_4 in fig. 3).

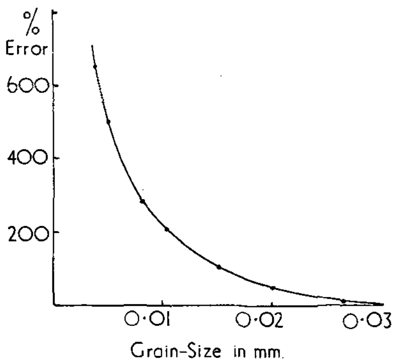

FIG. 2.

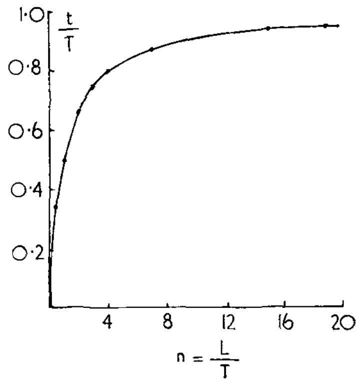

FIG. 4.

FIG. 2. Graph showing the relation between % error and grain-size on the assumption that all the grains are completely within the thin section.

FIG. 4. Graph showing the relation between t and L, both expressed as proportions of T.

This average thickness varies with the size of the cube. It is found that when $L = T, t = \frac{1}{2}T$; $L = 2T, t = \frac{2}{3}T$; $L = 3T, t = \frac{3}{4}T$; and generally

when $L = nT$, then $t = \dfrac{n}{n+1}T$.

It can be seen that as the size of the grain, and n, increases, so t approaches T in value and the computed volume L^2T approaches the true value L^2t.

Fig. 4 shows the relationship between t and L, both expressed in terms of T, and illustrates how t/T approaches unity as L/T becomes large.

(*c*) Taking into consideration the relations:

$$L = nT \qquad \text{(i)}$$

and

$$t = \frac{n}{n+1}T \qquad \text{(ii)}$$

it is possible to relate grain-size and error.

The assumed volume of a grain $= L^2T$; the real volume $= L^2t$; the

$$\% \text{ error} = \frac{L^2T - L^2t}{L^2t} \times 100 = \frac{T-t}{t} \times 100.$$

Substituting $\frac{n}{n+1}T$ for t (ii),

$$\% \text{ error} = \frac{T - \frac{n}{n+1}T}{\frac{n}{n+1}T} \times 100 = \frac{100}{n}.$$

Substituting L/T for n from (i),

$$\% \text{ error} = \frac{100T}{L}; \qquad \text{(iii)}$$

$\%$ error expressed in mm. $= \frac{3}{L}$ in slides of normal thickness. (iv)

This error is plotted against L in fig. 5.

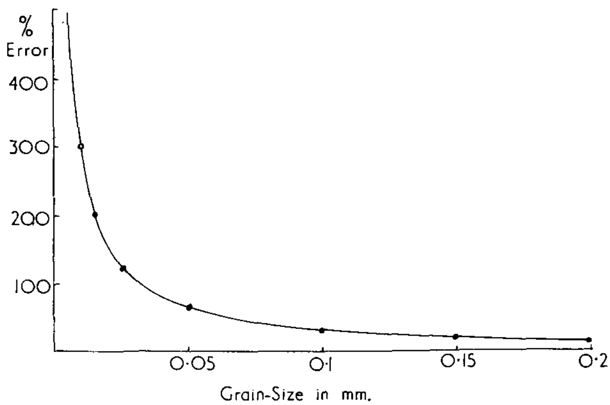

FIG. 5. Graph showing the relation between the % error and grain-size when the assumed volume is L^2t.

Conclusions.—1. The accuracy of the modal analyses of rocks with minerals of contrasted relief decreases rapidly with diminution of the size of the grains and in such a way that the superposition error in the estimation of the amount of high-relief mineral is a hyperbolic function of the grain-size.

The results show that for orientated cubes of constant size the error is 100 % for grains 0·03 mm. in diameter, 10 % for grains 0·3 mm. in diameter, and is not reduced to 1 % until the grains reach 3 mm. in diameter. Since the grains are not cubic and not orientated the exact figures above have limited application, but the generalization of decreasing accuracy with decreasing grain-size remains independent of shape.

2. Where rocks are fine grained and have minerals of contrasted relief, modal analyses should be used for comparative purposes only and should not be used for the recasting of chemical composition.

3. Comparisons should be made with other rocks only when the rocks are of comparable grain-size. If not, some correction must be made.

4. The superposition error is proportional to T, the thickness of the microsection; it can therefore be reduced by carrying out modal analyses on sections thinner than normal.

5. The percentage of high-relief mineral present will influence the amount of the superposition error, for when it is very high small grains will pack underneath one another and increase t.