XVIII.—*Experiments on the Elasticity of Crystals.*

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WHEN we look at a collection of crystals we see that they can be divided into groups according to their symmetry. Thus, one group the members of which form the cubical system, have a symmetry in three directions at right angles to each other; that is to say we shall find similarly arranged planes at the extremity of three properly chosen axes. In other groups the symmetry is not so perfect. Thus, in the tetragonal system we find that the symmetry in the direction of the principal axis is different to the symmetry in the direction of the lateral axes.

When we see a crystal which seems to shew a similar symmetry in three directions, we naturally imagine that the molecules in their arrangement along these directions followed the same law. If, however, we have two directions in a crystal which as compared with each other shew differences in symmetry, we should expect that the molecules in these two directions had been arranged differently.

From what we know of the physical characters presented to us by minerals when examined in various directions, this view seems to be borne out. Thus Mitscherlich has shewn us that the dilatation of a crystal by heat has an intimate connection with its symmetry; if the symmetry is the same in three directions, the dilatation is the same in three directions, but along directions where the dilatation is different the symmetry is also different. De Senarmont has shewn us a similar relation for the conductivity of heat. The manner in which light is transmitted in different directions through a crystal depends on its symmetry, and in accordance with the laws which govern the passage of light through a crystal,we have the laws for electric conductivity. Tyndall and Knoblauch have shewn us that the magnetic and diamagnetic properties of a crystal have a connection with its symmetry. Or in short we find similar phenomena exhibited in a crystal along similar directions. Now if we accept the popular hypothesis of a crystal, that it is a body formed by the mutual attractions of molecules which are separated by an elastic medium, in order to more fully appreciate the above coincidences, we should be greatly assisted if we could shew that along the directions which in any crystals exhibit different phenomena, that there was also a difference in material elasticity, which probably means a difference in intermolecular space or a difference in density. The chief experiments which have a direct connection with the material elasticity of crystals, are those made by Savart, who shewed that the figures formed upon vibrating plates of crystals were directly connected with their optical axes. Lately, I have endeavoured to shew that the material elasticity in a crystal was different in different directions, and at the same time to give some idea of its relative values by bumping together spheres cut out of calcite and quartz.

As these experiments have not given definite results, because of my not being able to obtain proper materials to be experimented on, and subsequently in not being able to get these turned into true spheres, the results which I now give must only be regarded as indicating what may be obtained when similar experiments are made under more favourable circumstances.

The experiments were as follows :—

I *Series of Experiments.*

The first experiments were made with a small ball of Iceland Spar having a diameter varying between 23·7 and 23·8 millimeters, and a ball of pure quartz with diameters between 26·0 and 26·1 millimeters. The ball of calcite was placed at the lower part of a cycloidal groove. The quartz ball was then allowed to roll from a fixed point down the groove and strike the ball of calcite. This was repeated, striking the calcite each time at a different point. Although the calcite ball was observed in consequence of being struck to roll different distances, indicating that these might be due to differences of elasticity in different directions, the experiments were too crude to be worth noting.

II *Series of Experiments.*

These experiments were made with the same balls as the first, and were very kindly carried out for me by the students of the Physical Laboratory, under the superintendence of my late colleague Prof. Ayrton.

The calcite ball was fixed in a piece of wood. The quartz ball was suspended by a silk fibre as a pendulum 6 ft. $5\frac{3}{4}$ in. long, in such a manner that when it was at rest it just touched the calcite ball.

The quartz ball was next drawn back $3\frac{1}{2}$ inches and allowed to swing against the fixed ball of calcite.

In consequence of this the swinging ball was reflected back a certain distance. This distance was observed. By repeating this experiment on several points of the fixed ball the reflection was found to be different. On the fixed ball fifteen different points were struck by the pendulum and each of these six times. The following table gives the mean result obtained for each of these points, arranged in order according to the deflection—

1.	....	1·54 in.	9.	....	1·28 in.
2.	...	1·46 ,,	10.	....	1·25 ,,
3.	....	1·42 ,,	11.	....	1·25 ,,
4.	....	1·33⅓ ,,	12.	....	1·25 ,,
5.	....	1·33 ,,	13.	....	1·25 ,,
6.	....	1·33 ,,	14.	....	1·25 ,,
7.	....	1·33 ,,	15.	....	1·16⅔ ,,
8.	....	1·33 ,,			

The points which were struck were chosen indiscriminately, but in the above table they are arranged according to the deflection.

The following are approximately diametrically opposite to each other; 1 and 2, 12 and 14, 12 and 15, 3 and 6, 11 and 12. From these it will be seen that, as might have been expected, in diametrically opposite directions approximately similar results were obtained.

A direction of no double refraction is approximately in the direction of the diameter 1 and 2.

Although it might be possible to deduce more conclusions from these results, I will not lay stress on them, as the method of experimenting was not altogether so satisfactory as could be wished, as the calcite ball between each series of experiments was refixed, it is exceedingly probable that this was not done each time in exactly the same manner, and in consequence it would not have been always in the same position to reflect the ball of quartz.

Notwithstanding this, the results are very significant of differences in elasticity.

The chief point to observe is, that in a direction of no double refraction the greatest deflection was obtained.

III *Series of Experiments.*

In these experiments, the ball whose elasticity was to be determined was placed upon a billiard table, and was there struck by another ball suspended as a pendulum.

The pendulum was a polished ball of quartz whose diameter varied between 36·8 and 36·9 millimeters. At diametrically opposite points in this ball two small holes were bored in which two small brass pins were placed. These were used as points of attachments for two pieces of silk thread, the other ends of which were attached to two points 60 centimetres apart, affixed to a horizontal bar of iron above the billiard table. These were of such a length that the ball hung 65 centimetres below the iron bar. Being thus suspended it could be swung backwards and forwards without any movement of rotation.

The ball which was experimented on was of calcite; it varied in diameter from 35·2 to 35·3 millimeters.

In the first series of experiments the swinging ball was drawn backwards three inches, and then allowed to swing suddenly forwards and strike the ball of calcite.

The distance to which the calcite ball rolled was then measured, when the calcite ball was struck at different points it rolled different distances.

The points taken were as follows:—

First 1 and 2. These were diametrically opposite.
3 and 5 ditto ditto ditto
4 and 6 ditto ditto ditto
7 was placed half-way between 3 and 1
8 ditto ditto 4 and 1

Deflection in Centimeters.	Mean.	Mean.
1. 22 —22·5—22	22·1	21.8
2. 23·5—21·5—20	21·6	
3. 20 —21·5—	20·7	20·6
4. 20 —21 —20	20·5	
5. 21·5—22 —19	20·8	20·4
6. 20 —20 —20	20	
7. 21·5—21·5—21	21·3	21·5
8. 21·5—22 —22	21·8	

In a second set of experiments the deflection given to the pendulum was four inches.

Deflection in Centimeters.	Mean.	Mean.	Mean.
1. 37—40—42	39·6	39·8	
2. 39—39—42	40		
3. 37—36—33	35·3	36·7	36·8
4. 37—41—38	38·2		
5. 34—34—34	34	37	
6. 40—40—40	40		
7. 41—36—37	38	39·1	
8. 41—40—40	40·3		

The points 1 and 2 approximately give the direction of the optic axis, whilst the points 3, 4, 5, and 6 approximately lie in a great circle of which 1 and 2 are the two poles.

Looking at the above table we see that in all cases when the ball was struck in the direction of the optic axis, that it rolled further than when it was struck at right angles to this direction. When it was struck in the direction 5 or 6, both of which are positions intermediate to the direction of the optic axis, and the great circle 1, 2, 3, 4, the rolling was intermediate to that given by the other directions.

These series of experiments were open to several errors.

First, the ball of calcite was not perfectly spherical. However, as the points which were taken were diametrically opposite, the observations ought in part to balance each other.

Second, the line joining 1 and 2 only roughly represents the direction of the optic axis.

Third. the points 3, 4, 5, and 6, are not in the same great circle, thus 5 is nearer to 2 than to 3 and therefore we could not expect them to give the same deflection.

IV *Series of Experiments.*

These experiments were conducted in a similar manner to the third series. The balls which were used were the same as before, the calcite ball however being re-ground and being brought into a form more truly spherical. On this ball seven pairs of points diametrically placed were chosen. The diameters measured between these points and their relative distribution was as follows—

	Millimeters.	
1. and 2.	33·6 ..	Approximately the line of the principal axis of the crystal.
3. — 4.	33·8 ..	These four points were in a great circle at right angles to the line joining 1 and 2.
5. — 6.	33·9 ..	
7. — 8.	33·9 ..	8 lay between 1 and 3.
9. — 10.	33·7 ..	9 lay between 2 and the middle point of the arc. 5, 4.
11. — 12.	33·7 ..	12 lay between 1 and the middle point of the arc. 6, 3.
13. — 14.	33·7 ..	13 lay between 2 and the middle point of the arc 5, 3.

The deflections obtained were as follows—

		Mean.	Mean.
1.	22 —22·5—23	22·5	23·05
2.	25 —21 —25	23·6	
3.	21 —21 —21	21	20·9
4.	21 —21 —20½	20·8	
5.	20·5—22·5—22	21·6	21·6
6.	22 —21 —22	21·6	
7.	21 —24 —21	22	21
8.	21 —19 —20	20	
9.	21 —21 —22	21·3	22·3
10.	23 —23 —24	23·3	
11.	20 —22 —22	21·3	21·6
12.	22 —22 —22	22	
13.	25 —24 —21	23·3	22·3
14.	22 —22 —20	21·3	

In a repetition of these experiments it was found—First when the quartz ball was deflected three inches, the roll of the calcite ball was as follows:—

		Mean.	Mean.
1.	25—25—25	25	24·7
2.	23—25—25	24·4	
3.	20—21—21	20·6	22·1
4.	23—23—25	23·6	
5.	23—23—24	23·3	23·3
6.	25—22—23	23·3	

Secondly, when the quartz ball was deflected four inches—

1.	29—26—26	27	28·1
2.	28—29—31	29·3	
3.	27—27—27	27	26·6
4.	26—26—27	26·3	
5.	25—25—28	26	26·1
6.	28—25—26	26·3	

For the total roll in diameters of the calcite ball in all these experiments, 9 must be added to each of the above numbers.

It may be remarked that points like 11, 12, 13, and 14 were the most dented by being struck with the ball of quartz, whilst the other points did not shew any signs of the blows they had received.

In these experiments it will be observed that the same general results have been obtained as before, namely, that when the calcite ball was struck in the direction of the principal axis of the crystal there was the greatest deflection.

In any direction at right angles to this there was the least deflection, and at intermediate points deflections of intermediate values were obtained.

V *Series of Experiments.*

These experiments were made upon two balls of rock crystal, the one which was used as a pendulum being the same as the one in the last series of experiments.

The ball which was struck had diameters 36·7 and 36·9 millimeters.

On it a number of points were taken marked from 1 to 11 consecutively. The pendulum was deflected three inches, and each of the above points were struck three times.

The distances rolled by the ball which was struck are given in the following table—

								Mean.
1.	40	—	42	—	42	—		41·3
2.	42	—	43	—	43	—		42·6
3.	40	—	40	—	41	—		40·3
4.	44	—	42	—	42	—		42·6
5.	43	—	43	—	43	—		43·0
6.	40	—	41	—	43	—		41·3
7.	40	—	42	—	43	—		41·6
8.	39	—	42	—	41	—		40·6
9.	42	—	39	—	45	—		42·6
10.	43	—	40	—	44	—		42·6
11.	40	—	44	—	43	—		42·3

The following points were approximately diametrically placed, 1 and 4, 2 and 11, 3 and 6, 8 and 4.

On looking through the quartz ball at a pin hole in a card in diametrical directions through the points 1, 3, 4, 5, and 6, no double refraction was observed. In other diametrical directions through the remaining points, double refraction was observable.

All that can be said with regard to these experiments in a ball of quartz, is that it apparently shews different elasticities in different directions.

CONCLUSION.

From the above experiments it would seem to be shewn that crystals have different material elasticities in different directions. In the case of a ball of calcite, the greatest rolling effect was obtained when it was struck parallel to the principal axis of the crystal.

In a direction at right angles to this the least effect was obtained, and in intermediate directions, intermediate effects.

Before endeavouring to shew what relations numbers such as those which I have obtained hold to the elasticity of a crystal, it would be better that such experiments were repeated by persons who have instruments, good materials, and skilful workmen at their command, which I am sorry to say I have found it impossible to obtain in Japan.