

*The morphology of the baryte group.*¹

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Summary. The observed order of importance of the forms of baryte, anglesite, and celestine is first determined by a modification of the methods used by Donnay and Harker. Their hypothesis is applied to determine the space-groups of these minerals by morphological means, and a comparison is made of the observed and theoretical orders of importance of forms in these minerals. It is concluded that agreement is poor between the observed and predicted frequencies of forms in the baryte group. The rank distribution of some of the more important forms is presented graphically.

A NEW method of calculating the reticular area of lattice planes which allows for the operation of screw axes and glide planes has been introduced by J. D. H. Donnay and D. Harker.² This, in conjunction with Donnay's work on zonal development,³ makes it possible to extend the Bravais hypothesis to cover the 230 space-groups.⁴ They call their work a 'Law of Observation', and say that it only takes into account the geometry of space-groups and does not claim to embrace all the factors governing the morphological development of crystals. They add, however, that the effects of the lattice and *motif* on the habit of a crystal are very rarely masked by other factors.

Donnay has published many morphological analyses of minerals, and one of an artificial compound, some of which are based on relatively scanty data. Three further papers have been published by his research students, E. D. Taylor and J. A. Tremblay.⁵ The author believes it is better to use as examples minerals for which more data are available.

In some papers Donnay uses different methods to determine the relative importance of forms. It seems desirable to find one method which

¹ 'The morphology of baryte' forms part of a thesis approved for the degree of Ph.D. by the University of London, 1953.

² J. D. H. Donnay and D. Harker, *Amer. Min.*, 1937, vol. 22, p. 446 [M.A. 7-241].

³ J. D. H. Donnay, *Ann. Soc. géol. Belgique*, 1938, vol. 61, p. B 260 [M.A. 7-242].

⁴ J. D. H. Donnay and D. Harker, *Naturaliste Canadien*, 1940, vol. 67, p. 33 [M.A. 8-4].

⁵ J. D. H. Donnay, *Bull. Soc. franç. Min.*, 1946, vol. 69, p. 152. A bibliography of papers by Donnay and others (1937-46) on the morphology of crystals.

can be applied consistently to all minerals. In this paper a new method of determining the observed morphological aspect is suggested, which consists, in part, of a synthesis of several of Donnay's methods.

Method of determining the observed morphological aspect.

The 'observed morphological aspect' is a list of forms observed on crystals of a substance in order of decreasing importance. There is no simple criterion by which the importance of a form can be assessed or calculated. The information from which the observed morphological aspect has to be derived is usually a number of drawings of crystals from various localities. Donnay regards the 'locality frequency', or number of localities at which the form has been found, as the most important factor. He separates forms that have equal, or nearly equal, locality frequencies by their 'mean rank', which indicates the degree of development of a form relative to other forms on the crystal. Sometimes Donnay uses the 'mean rank of the locality' and at other times the 'mean rank of the figures'.

In morphological studies the best index of form importance would probably be the mean velocity of growth, and the observed morphological aspect would then consist of a sequence of forms in order of increasing growth velocity. Great labour would be involved in making accurate determinations of growth velocities from several hundred drawings, and it appears that sufficiently accurate results can be obtained for the morphological analysis by much simpler means. The size of a face is closely related to its growth velocity, but is also affected by the disposition of surrounding faces. The mean rank of the locality gives a rough measure of growth velocities, and is included in the formula for calculating the observed morphological aspect.

It would be possible to base the observed morphological aspect only on the locality frequency or frequency of occurrence of forms (form frequency). The chief disadvantage of the latter lies in the possibility of giving undue importance to less common forms when, as sometimes happens, a large number of crystals of unusual habit are described from one locality.

For the baryte group it is found that the locality frequency, form frequency, and mean rank when used alone each give a similar, but slightly different, order for the observed morphological aspect.

When preparing observed morphological aspects it seems advisable to make use of all information that is numerically assessable and to combine it into a single figure which may be called the *form rank index*. This information comprises the relative development of the form at each

locality, its frequency of occurrence at each locality, and the number of localities at which it occurs. In this paper these factors have been assessed in the following manner.

Let the number of crystals recorded from any locality be n , and let the number of crystals from any locality showing a particular form x be n_x ; let the number of localities from which the mineral has been studied be N , and let the locality frequency or number of localities at which it shows the form x be N_x ; and let the general symbol for the rank of a form x on a particular crystal be r .

The rank of forms is found in the manner suggested by Donnay. On each crystal all the forms present are listed in order of decreasing size, the largest, the second largest, . . . , r th largest are given the corresponding ranks, 1, 2, . . . , r . The mean rank of a form at any locality is then $\Sigma r/n_x$. The mean rank of a form is given by $\bar{R} = \Sigma(\Sigma r/n_x)/N_x$. The actual frequency of a form x is Σn_x and the percentage frequency is $F\% = 100\Sigma n_x/\Sigma n$. The relative frequency of a form x at a particular locality is n_x/n . The locality form rank index ρ_l , which assesses the importance of a form at any locality, takes into account the relative development of the form and its frequency of occurrence at the locality; it is obtained by multiplying the reciprocal of the mean rank at the locality by the relative frequency, $\rho_l = n_x^2/n\Sigma r$. The final assessment of the importance of a form is made by the form rank index ρ , which is obtained by summing all the locality form rank indices and dividing by the number of localities from which the mineral has been studied, $\rho = \Sigma \rho_l/N$. Its range is from zero to unity. The locality frequency, of which so much use is made by Donnay, receives indirect expression in the formula; the greater the locality frequency the larger will be the numerator in the expression for the form rank index. This index effectively reduces the importance of rare forms which, owing to unusual conditions of growth, may be common at one or a few localities. The form rank index may be used to compare directly the importance of forms on different minerals as, for example, in isomorphous series. While not suggesting that the form rank index is necessarily the best index of form importance that can be devised, it does appear to be an improvement, as it incorporates all the values which are numerically assessable in one index figure.

Morphology of the baryte group.

The chief source of morphological data is the 'Atlas der Krystallformen' compiled by V. Goldschmidt. Some of the drawings in the

Atlas represent incomplete crystals, some contain errors, while others are illegible. All drawings of crystals the locality of which is unknown were also rejected.

The 'Handbuch der Mineralogie' by C. Hintze was used as an auxiliary source of information to check the relative form frequencies found from the Atlas. The Handbuch gives the habit of crystals from various localities or a list of forms found at a locality. Each list of forms has been counted as one crystal. Some of the longer lists, over about eighteen forms, were omitted from the calculations, as they would give undue importance to the rarer forms. The form frequencies derived from the Handbuch substantially corroborated those derived from the Atlas and gave useful additional information on the occurrence of rarer forms.¹ Since little information is given about the rank of forms quoted in the Handbuch, form rank indices cannot be calculated from these data.

It is not possible to quote very precisely the number of localities at which a common mineral occurs showing a given form. For example, one author may describe a crystal from a particular district, while another quotes a mine in that district, when it is impossible to say whether the mineral is being reported from one or two localities. In this paper each mine in a region is counted as one locality.

Table I gives summaries of form frequency and rank data for baryte, anglesite, and celestine, for forms with frequencies of occurrence of 5 % or more in the Atlas. In each section of the table columns two to five give the form rank index (ρ), locality frequency (N_x), mean rank (\bar{K}), and percentage frequency of occurrence (F %) calculated from data in the Atlas, and the last column gives the percentage frequency of occurrence calculated from the Handbuch (F' %). In each section of the table the forms are arranged in the order of decreasing importance of rank. Table II gives rank and frequency data for the rarer forms of baryte, anglesite, and celestine. There is little point in calculating the mean rank or form rank index of forms with low frequencies of occurrence, as they are not likely to be very reliable. In order to obtain more data for the assessment of the relative importance of the rarer forms the information from the Atlas and the Handbuch has been combined. In

¹ For baryte the Atlas yielded the required data for 486 crystals with an aggregate of 3373 forms from 153 localities and the Handbuch gave 505 crystals with 3527 forms from 423 localities. For anglesite the Atlas gave 359 crystals showing 2204 forms from 63 localities and the Handbuch gave 167 crystals with 1173 forms from 108 localities. For celestine the Atlas gave details of 201 crystals with 941 forms from 48 localities and the Handbuch of 159 crystals with 935 forms from 123 localities.

many cases the same crystal is recorded in both sources, but all such cases of duplication have been eliminated. The rarer forms have been listed in order of decreasing locality frequency (N_x), and when this is equal, in order of decreasing frequency of occurrence (Σn_x). This is taken as the order of decreasing rank, the observed morphological aspect. It would be unwise to place too much reliance on the order of the rarer forms, as the discovery of a few more examples of these forms could alter the order. This table does, however, serve a useful purpose in giving at least an approximate order of importance for the rarer forms. There is insufficient data to justify arranging the still rarer forms in order of rank.

Morphological analysis of the baryte group.

It has been shown by X-ray analysis of the minerals of the baryte group that the a -axis of the unit-cell¹ has twice the value indicated by the axial ratios assigned on morphological grounds. In the morphological analysis the latter values must be used unless the analysis indicates that they are incorrect. The axial ratios used are shown in table I, and the symbols of forms in this paper are always referred to these axes, unless otherwise stated.

TABLE I. Morphological analysis of the baryte group.
Summary of form rank and frequency data.

<i>Baryte.</i> <i>Axial ratios*</i> $a:b:c = 0.8152:1:1.3136$						<i>Anglesite.</i> <i>Axial ratios†</i> $a:b:c = 0.78516:1:1.28939$						<i>Celestine.</i> <i>Axial ratios‡</i> $a:b:c = 0.77895:1:1.28005$					
Form	ρ	N_x	\bar{R}	$F\%$	$F'\%$	Form	ρ	N_x	\bar{R}	$F\%$	$F'\%$	Form	ρ	N_x	\bar{R}	$F\%$	$F'\%$
001	0.708	149	1.77	91	96	110	0.333	56	2.91	82	86	011	0.601	47	1.95	87	89
110	0.341	145	3.25	89	87	001	0.326	51	2.85	69	72	001	0.405	37	2.05	71	80
011	0.298	137	3.70	77	74	102	0.300	51	3.31	75	77	102	0.276	41	3.36	76	88
102	0.293	137	3.82	82	82	011	0.225	51	3.88	68	61	110	0.251	40	3.20	71	80
111	0.126	114	5.70	54	47	100	0.162	37	3.41	31	45	100	0.067	19	3.96	25	26
010	0.123	106	4.81	47	45	111	0.155	51	4.85	64	66	104	0.067	17	4.17	26	32
100	0.104	95	5.65	42	40	104	0.130	29	3.91	24	31	133	0.067	8	2.48	10	8
210	0.062	54	6.49	26	18	122	0.128	39	4.90	52	50	111	0.058	20	4.83	30	35
101	0.061	67	6.37	26	32	120	0.043	25	5.36	20	26	144	0.038	8	3.25	6	10
104	0.047	64	5.92	23	30	010	0.042	30	5.31	26	35	122	0.019	11	5.16	8	15
122	0.033	57	7.59	20	21	324	0.040	22	6.47	19	29	210	0.008	6	5.57	7	6
320	0.029	38	6.46	14	10	112	0.031	22	7.04	19	25	010	0.007	4	5.30	6	15
130	0.022	37	7.31	13	12	221	0.008	11	6.54	8	10						
113	0.020	44	9.40	15	13	124	0.008	10	8.50	5	9						
112	0.017	33	9.28	8	9												
120	0.016	24	7.28	6	8												
114	0.011	27	9.71	9	6												
115	0.009	25	9.49	7	5												

* R. Helmhaecker, Denkschr. Akad. Wiss. Wien, 1872, vol. 32, Abt. II, p. 1.

† N. von Koksharov, Min. Russlands, 1853, vol. 1, p. 34.

‡ A. von Auerbach, Sitzungsber. Akad. Wiss. Wien, Math.-naturwiss. Cl., Abt. I, 1869, vol. 59, p. 549.

¹ R. W. James and W. A. Wood, Proc. Roy. Soc., 1925, ser. A, vol. 109, p. 598 [M.A. 3-345].

TABLE II. Rank and frequency data for the rarer forms.

<i>Baryte.*</i>			<i>Anglesite.*</i>			<i>Celestine.*</i>		
<i>Form.</i>	<i>N_x.</i>	<i>Σn_x.</i>	<i>Form.</i>	<i>N_x.</i>	<i>Σn_x.</i>	<i>Form.</i>	<i>N_x.</i>	<i>Σn_x.</i>
106	25	33	130	17	32	124	10	11
124	23	28	012	12	22	113	9	9
012	23	26	132	10	21	221	8	8
103	20	25	144	8	15	101	8	8
132	15	23	210	8	12	324	6	9
230	15	17	121	7	19	012	5	8
223	14	17	142	7	15	132	5	7
116	12	14	113	6	9	1.10.10	5	7
302	11	14	114	5	8	103	5	5
142	9	13	018	5	8	015	4	8
312	9	11	212	5	7	0.1.12	4	6
021	8	11	133	4	9	210	4	5
140	8	10	021	4	7			
212	7	12						
121	6	9						

* Axial ratios as in table I.

TABLE III. Development of the central zones in the baryte group. Actual form frequencies derived from the Atlas (A) and Handbuch (H).

<i>Baryte.*</i>						<i>Anglesite.*</i>					
<i>{hhl}</i>	<i>A.</i>	<i>H.</i>	<i>{hkk}</i>	<i>A.</i>	<i>H.</i>	<i>{hhl}</i>	<i>A.</i>	<i>H.</i>	<i>{hkk}</i>	<i>A.</i>	<i>H.</i>
1.1.26	—	1	1.22.22	1	1	616	1	1	116	3	2
1.1.20	—	1	1.18.18	—	1	414	2	4	114	4	4
1.1.17	—	1	188	1	—	313	4	2	113	4	5
118	3	3	177	1	3	212	8	4	112	68	42
117	—	3	166	—	1	111	263	235	111	231	110
116	6	8	155	1	2	121	8	1	133	5	4
115	36	25	144	1	3	131	3	1	221	27	16
114	42	28	133	3	4	141	3	—	122	186	83
4.4.15	—	1	255	—	1	151	1	—	331	1	—
113	73	64	122	95	108				344	—	1
337	—	1	355	—	1				111	231	110
7.7.16	—	1	344	—	1						
6.6.13	1	1	111	263	235				<i>Celestine.*</i>		
112	38	47	322	—	1				119	—	1
223	10	7	211	—	1				117	—	1
111	263	235	522	—	1				115	1	2
887	1	1	311	—	1				114	2	—
776	—	1	411	—	1				113	2	9
441	—	1							112	1	3
									111	60	55
									332	—	1
									221	2	7
									122	17	24
									111	60	55
									322	1	2

* Axial ratios as in table I.

Determination of the lattice type.

The lattice type is determined from the three zones $[1\bar{1}0]$, $[0\bar{1}1]$, and $[\bar{1}01]$, of forms $\{hhl\}$, $\{hkk\}$, and $\{hkh\}$. These are called 'central zones' by Donnay, as they all intersect in $\{111\}$. Form rank indices and other frequency data for the commoner forms in these zones are given in

table I. Frequency data for the rarer forms appear in table II. In both these tables the forms are listed in the order of decreasing importance. Table III contains the actual frequency of all the forms in these three zones derived from the Atlas and the Handbuch.

Baryte. The zone $[1\bar{1}0]$ of forms $\{hhl\}$. The observed morphological aspect already obtained for forms with frequencies over 1 % is $\{111\}$, $\{113\}$, $\{112\}$, $\{114\}$, $\{115\}$, $\{223\}$, and $\{116\}$. There is little point in discussing the remaining rarer forms. The form $\{111\}$ is clearly predominant. In the harmonic segment of the zone¹ there is a complete sequence of the primary series² from $\{111\}$ to $\{118\}$. The only secondary form² is $\{223\}$ which, as might be expected, is the simplest possible. The arithmetic segment¹ is virtually undeveloped, no form in it even attaining a frequency of 1 %.

The predominance of $\{113\}$ over $\{112\}$ suggests the presence of an *F*-lattice, but this is definitely ruled out by the relative importance of the other forms in the zone. The zonal series is of the simple type with the unit form dominant. This conclusion is confirmed by a study of the two remaining central zones.

The zone $[0\bar{1}1]$ of forms $\{hkk\}$. The observed morphological aspect for forms with frequencies over 1 % is $\{111\}$ and $\{122\}$. These are followed in importance by $\{133\}$. The primary series extends from the dominant form $\{111\}$ to $\{188\}$ in the harmonic segment and to $\{411\}$ in the arithmetic segment of the zone, although most of the forms are very rare. Three forms of the secondary series and two of the tertiary series are each recorded once. The primary series is of the simple type with the unit form dominant.

The zone $[\bar{1}01]$ of forms $\{hkh\}$. The observed morphological aspect for forms with frequencies over 1 % is $\{111\}$, $\{212\} \simeq \{121\}$. A complete zonal series extends from $\{414\}$ through $\{111\}$ to $\{151\}$. It is clear that $\{111\}$ is the dominant form of the zone, the adjacent forms $\{212\}$ and $\{121\}$ are approximately equal and very much less important than $\{111\}$, while the next pair of forms, $\{313\}$ and $\{131\}$, are less important still. This establishes the zone to be of the simple type with the unit form dominant.

In the zone of forms $\{hhl\}$ the superiority of $\{113\}$ over $\{112\}$ left a slight doubt as to the character of that zone, although the distribution of the other forms indicated that it was of the simple type with the unit form dominant. This conclusion is completely confirmed by the study

¹ M. A. Peacock, Amer. Min., 1937, vol. 22, p. 210.

² J. D. H. Donnay, Ann. Soc. géol. Belgique, 1938, vol. 61, p. B 266.

of the two other central zones; since they are both of the simple type with the unit form dominant the third central zone must also have this character. It can thus be concluded that baryte has a primitive lattice and that Helmhacker's choice of axial ratios is fully justified on morphological grounds.

Anglesite. The zone $[\bar{1}\bar{1}0]$ of forms $\{hhl\}$. For this zone the observed morphological aspect is $\{111\}$, $\{112\}$, $\{221\}$, $\{113\}$, and $\{114\}$. The unit form is clearly dominant. In the harmonic segment of the zone only forms of the primary series occur, and these decrease in importance from $\{111\}$ to $\{114\}$. The only important form in the arithmetic segment of the zone is $\{221\}$. These features indicate that the zone is of the simple type with the unit form dominant.

The zone $[0\bar{1}1]$ of forms $\{hkk\}$. In this zone the observed morphological aspect is $\{111\}$, $\{122\}$, $\{144\}$, and $\{133\}$. $\{155\}$ is less important. In this zone, in which the unit form is dominant, the harmonic segment alone is developed, and in it only forms of the primary series are of importance. Apart from the reversal of $\{133\}$ and $\{144\}$, which are not forms with high frequencies, there is a continual decrease in importance from $\{111\}$ to $\{166\}$, indicating the presence of a zone of the simple type with the unit form dominant.

The zone $[\bar{1}01]$ of forms $\{hkh\}$. This zone is poorly developed. The observed morphological aspect is $\{111\}$, $\{121\}$, and $\{212\}$. This is a zone of the simple type with the unit form dominant.

The three central zones thus prove to be of the simple type with the unit form dominant. This indicates that the lattice is primitive, and gives complete support on morphological grounds to Koksharov's choice of axial ratios.

Celestine. The zone $[\bar{1}\bar{1}0]$ of forms $\{hhl\}$. This zone is poorly developed. The observed morphological aspect is $\{111\}$, $\{113\}$, and $\{221\}$. In the harmonic segment of the zone only forms of the primary series occur, and there is a complete sequence from $\{115\}$ to $\{111\}$. $\{113\}$ is more important than $\{112\}$, as is the case with baryte. Although little information is available to indicate the nature of the zone, the presence of $\{221\}$ as the only primary form in the arithmetic segment of the zone and the series from $\{115\}$ to $\{111\}$ in the harmonic segment suggests that the zone is of the simple type with the unit form dominant.

The zone $[0\bar{1}1]$ of forms $\{hkk\}$. The observed morphological aspect in this zone is $\{133\}$, $\{111\}$, $\{144\}$, $\{122\}$, and $\{1.10.10\}$. Of the pyramidal zones this is quite the best developed. There is a primary series in the harmonic segment extending from $\{177\}$ to $\{111\}$. Only the first four

forms listed are of any importance. It is rather surprising that {133} ranks above {111}. From table I it may be seen that the latter has a much higher frequency of occurrence and locality frequency. In the fewer localities where it occurs {133} is often well developed, as indicated by its mean rank, and this raises it above {111} in the final assessment. The unusual frequency and mean rank of {133} together suggest that, where it does occur, local conditions of growth, such as the presence of particular impurities, may favour its development at the expense of other forms. Although the relative development of forms in this zone is far from ideal, the fact that the four most important forms are the first four of a primary series does suggest that this zone is of the simple type. The unit form is dominant in respect of locality frequency and frequency of occurrence, but comes after {133} in the list of form rank indices.

The zone $[\bar{1}01]$ of forms $\{hkh\}$. The unit form is dominant, and there are no others of importance. The presence of {121} may suggest that the zone is of the simple type.

The evidence for the lattice type is rather unsatisfactory. Since the zones $[\bar{1}\bar{1}0]$ and $[\bar{1}01]$ clearly have the unit form dominant, and the zones $[\bar{1}\bar{1}0]$ and $[0\bar{1}1]$ show some evidence of a primary series it may be concluded that the lattice is primitive and the choice of axial ratios is justified.

Determination of the space-group.

The space-group is determined from the development of the zones [100], [010], and [001], termed 'axial zones' by Donnay. Since it is known on morphological grounds that the point group of baryte, angle-site, and celestine is *mmm*, it is necessary to determine whether a mirror plane or a glide-plane exists parallel to each pinacoid, and in the latter case the nature of the glide-plane present.

The observed morphological aspect for the commoner and rarer forms in these three zones, with the data on which it is based, will be found in tables I and II. The actual frequency of all the forms occurring in these zones is given in table IV.

Baryte. The zone [100] of forms $\{0kl\}$. The observed morphological aspect for forms with frequencies over 1 % is {011}, {012}, and {021}. This zone is difficult to interpret. Although a complete series of forms extends from {014} through {011} to {051} most of the forms are very rare, only three having a frequency over 1 %. The unit form is completely dominant and followed, in order, by {012} and {021}, while {031} and {013} are still more rare. In view of their rarity and approximately

equal frequency no useful purpose can be served by discussing the relative importance of $\{014\}$, $\{013\}$, $\{031\}$, $\{041\}$, and $\{051\}$.

TABLE IV. Development of the axial zones in the baryte group. Actual form frequencies derived from the Atlas (A) and Handbuch (H).

<i>Baryte.*</i>						<i>Anglesite.*</i>					
{0k}. A. H.	{h0}. A. H.	{hk0}. A. H.	{0k}. A. H.	{h0}. A. H.	{hk0}. A. H.	{0k}. A. H.	{h0}. A. H.	{hk0}. A. H.	{0k}. A. H.	{h0}. A. H.	{hk0}. A. H.
0.1.12 — 1	1.0.80 — 1	1.1.10.0 — 1	— 1	1.10.0 — 1	1.1	0.1.10 — 1	1.0.24 1	2.70 1	1	1.0.24 1	2.70 1
018 — 1	1.0.30 — 1	1.170 — 1	— 1	018 7 4	1.0.22 — 1	018 7 4	1.0.22 — 1	1.130 17 18	— 1	1.130 17 18	— 1
014 1	1.0.10 1	— 150 — 1	— 1	016 — 1	1.0.14 — 1	016 — 1	1.0.14 — 1	1.120 72 44	— 1	1.120 72 44	— 1
027 — 1	1.109 — 1	1.140 7 3	— 1	0.2.11 1	1.108 1	0.2.11 1	1.108 1	— 230 2 3	— 1	— 230 2 3	— 1
013 1	1.108 1	2 3.10.0 1 1	— 1	013 4 1	1.106 1	013 4 1	1.106 1	1.340 4 3	— 1	1.340 4 3	— 1
025 1	1.106 9	24 130 64 61	— 1	012 17 7	1.105 1	012 17 7	1.105 1	1.110 294 143	— 1	1.110 294 143	— 1
012 9	19 105 3	4 120 31 39	— 1	035 1 1	1.104 87 51 320 — 1	035 1 1	1.104 87 51 320 — 1	— 1	— 1	— 1	— 1
047 1	1.8.0.33 — 1	1.350 — 1	— 1	045 1 —	1.103 1	045 1 —	1.103 1	2.740 1 1	— 1	2.740 1 1	— 1
035 4	3 104 113 152 230 10 8	— 1	— 1	011 245 102	1.102 270 128 210 7 5	011 245 102	1.102 270 128 210 7 5	— 1	— 1	— 1	— 1
023 3	4 103 9 17 450 1 —	— 1	— 1	021 4 3	1.101 — 4 520 — 2	021 4 3	1.101 — 4 520 — 2	— 1	— 1	— 1	— 1
034 — 3	7.0.20 — 1	1.110 431 438	— 1	031 1 2	21.0.2 — 1	031 1 2	21.0.2 — 1	310 1 1	— 1	310 1 1	— 1
079 1	2 308 1	5 540 — 1	— 1	041 — 1	— 1	041 — 1	— 1	410 2 2	— 1	410 2 2	— 1
089 1	— 5.0.13 — 1	1.430 — 1	— 1								
011 376 372	205 3 3 320 68 53		— 1								
065 — 1	1.102 400 415 530 2 1	— 1	— 1	0.1.20 1	1.1.0.17 — 2	0.1.20 1	1.1.0.17 — 2	140 — 1	— 1	140 — 1	— 1
032 1	1.407 1	1.740 1 1	— 1	0.1.16 — 1	1.1.0.14 — 1	0.1.16 — 1	1.1.0.14 — 1	1.130 — 1	— 1	1.130 — 1	— 1
085 — 1	1.508 1	2 210 126 90	— 1	0.1.14 — 1	1.1.0.12 — 2	0.1.14 — 1	1.1.0.12 — 2	2.120 14 9	— 1	2.120 14 9	— 1
053 — 1	1.203 2 4 310 2 4	— 1	— 1	0.1.12 3	4.1.0.10 — 1	0.1.12 3	4.1.0.10 — 1	1.350 — 1	— 1	1.350 — 1	— 1
021 6	6 304 3 2 410 — 6	— 1	— 1	0.1.10 1	2.108 1	0.1.10 1	2.108 1	2.580 — 1	— 1	2.580 — 1	— 1
083 1	1.405 1	2 910 — 1	— 1	019 1	3 107 — 1	019 1	3 107 — 1	1.110 142 127	— 1	1.110 142 127	— 1
031 — 4	23.0.24 1 —	— 1	— 1	018 1	1.106 1	018 1	1.106 1	1.650 2 1	— 1	1.650 2 1	— 1
041 1	1.101 124 161	— 1	— 1	017 — 2	2.0.11 1	017 — 2	2.0.11 1	1.750 5 2	— 1	1.750 5 2	— 1
051 — 1	1.403 1 —	— 1	— 1	015 7	2 105 — 3	015 7	2 105 — 3	3.320 4 2	— 1	3.320 4 2	— 1
	302 6 8	— 1	— 1	0.3.10 — 2	2 104 53 51 530 2 2	0.3.10 — 2	2 104 53 51 530 2 2	— 1	— 1	— 1	— 1
	905 — 1	— 1	— 1	013 2	2 207 1 1 210 — 5	013 2	2 207 1 1 210 — 5	— 1	— 1	— 1	— 1
	201 1 2	— 1	— 1	012 7	3 103 3 3 520 — 1	012 7	3 103 3 3 520 — 1	— 1	— 1	— 1	— 1
		— 1	— 1	035 — 1	7.0.15 — 1	035 — 1	7.0.15 — 1	10.3.0 — 1	— 1	10.3.0 — 1	— 1
		— 1	— 1	023 — 2	11.0.23 — 2	023 — 2	11.0.23 — 2	410 — 1	— 1	410 — 1	— 1
		— 1	— 1	067 — 1	1.102 153 140	067 — 1	1.102 153 140	— 1	— 1	— 1	— 1
		— 1	— 1	011 175 141	304 — 1	011 175 141	304 — 1	— 1	— 1	— 1	— 1
		— 1	— 1	021 1	2 101 1 8	021 1	2 101 1 8	— 1	— 1	— 1	— 1
		— 1	— 1	0.15.2 — 1	908 2 1	0.15.2 — 1	908 2 1	— 1	— 1	— 1	— 1
		— 1	— 1		705 — 1		705 — 1	— 1	— 1	— 1	— 1

<i>Baryte.</i>			<i>Anglesite.</i>			<i>Celestine.</i>		
A.	H.		A.	H.		A.	H.	
100	204	201	100	113	75	100	51	41
010	227	228	010	94	58	010	12	24
001	444	486	001	247	121	001	142	127

* Axial ratios as in table I.

Since the lattice is primitive the planes of symmetry parallel to (100) could be m -planes, b -glide planes, c -glide planes, or n -glide planes. The dominance of the unit form rules out the possibility of b - or c -glide planes. The relative order of importance

$$\{011\} > \{012\} > \{021\} > \{031\} \cong \{013\}$$

is consistent with the existence of m -planes, but the very low frequency of all the forms in the zone, except $\{011\}$, is hardly reconcilable with such

an assumption. The high frequency of $\{011\}$ and low frequency of the other forms is much more in accord with the existence of n -glide planes parallel to (100) , but in that case it would be expected that $\{013\}$ and $\{031\}$ would predominate over $\{012\}$ and $\{021\}$, but this is not so. In this case the matter must be left open, and it can only be stated that m -planes or n -glide planes exist parallel to (100) .

The zone $[010]$ of forms $\{h0l\}$. The observed morphological aspect is $\{102\}$, $\{101\}$, $\{104\}$, $\{106\}$, $\{103\}$, and $\{302\}$ for forms with frequencies over 1 %. The dominant form is not the unit form $\{101\}$, but the form with the last index even, $\{102\}$. In the harmonic segment of the zone forms with the last index even predominate over those in which it is uneven. $\{104\}$ is completely predominant over $\{103\}$, and $\{106\}$ is more important than $\{105\}$.

Parallel to (010) there could be m -planes, or a -, c -, or n -glide planes. This type of zonal development could be accounted for quite satisfactorily by the existence of c -glide planes parallel to (010) .

The zone $[001]$ of forms $\{hk0\}$. The observed morphological aspect for forms with frequencies over 1 % is $\{110\}$, $\{210\}$, $\{320\}$, $\{130\}$, $\{120\}$, $\{230\}$, and $\{140\}$. This zone is also difficult to interpret. The dominant form is indisputably $\{110\}$, which is followed by $\{210\}$. $\{320\}$ and $\{130\}$ are almost equal, and both these forms are clearly more important than $\{120\}$, the last prism with a frequency over 5 %. $\{230\}$ and $\{140\}$ have frequencies between 1 % and 5 % and $\{310\}$ has a frequency below 1 %.

In this zone the unit form $\{110\}$ is clearly dominant. This rules out the possibility of a - or b -glide planes parallel to (001) , leaving m -planes and n -glide planes for consideration. Turning attention first to the arithmetic segment of the zone the forms $\{110\}$, $\{210\}$, and $\{310\}$ form a series of decreasing importance. These have the appearance of a primary series, and this is established by the appearance of the first form of the secondary series, $\{320\}$, with an importance less than that of its adjacent primary forms, $\{110\}$ and $\{210\}$. From a study of the arithmetic segment alone the zone appears to be of the simple type with the unit form dominant.

The harmonic segment of the zone does not seem, at first sight, to support this conclusion. The undoubted superiority of $\{130\}$ over $\{120\}$ suggests the presence of a double zone. In a zone of the double type $\{110\}$ and $\{130\}$ would belong to the primary series and $\{120\}$ would belong to the secondary series, and be denoted by doubled indices $\{240\}$. The form $\{350\}$ of the tertiary series would come between $\{240\}$ and

{110}, and {460} of the quaternary series would come between {350} and {110}. If the zone is of the simple type, {110} and {120} belong to the primary series, and {230} comes between them as a member of the secondary series, while {350} comes between {230} and {120} as a member of the tertiary series. Since {230} is decidedly more important than {350} it suggests that the zone is of the simple type. This conclusion is strengthened by the fact that {140} is more important than {150}; if the zone was of the double type the reverse should be the case. Thus the evidence of the arithmetic segment and the majority of the evidence from the harmonic segment points to the zone being of the simple type with the unit form dominant, and hence to the existence of m -planes parallel to (001).

The pinacoids. In the observed morphological aspect {001} is easily predominant, as it is the most important of all forms in baryte, and {010} has a slight superiority over {100}. This is the order to be expected from the axial ratio $a:b:c = 0.8152:1:1.3136$.

The analysis now gives two possible morphological aspects¹ P^*c^* or Pnc^* . Since the point group is known these may be converted into symbols of actual space-groups, $Pn\bar{c}m$ or $Pncm$.

Anglesite. The zone [100] of forms { $0kl$ }. The unit form completely dominates all other forms in the zone; the only other forms to appear in the first 27 places of the observed morphological aspect are {012}, {018}, and {021}, and these are succeeded by {013}. The distribution of forms in this zone is very similar to that in baryte, and is also difficult to interpret. The outstanding importance of the unit form and the relative insignificance of all others suggests the presence of n -glide planes, but this is not consistent with the superiority of {012} and {021} over {013} and {031}. The only other possibility is the existence of m -planes parallel to (100), but in that case it would be expected that some of the other forms in the zone would have a higher frequency of occurrence. In view of this uncertainty it can only be stated that n -glide planes or m -planes may exist parallel to (100).

The zone [010] of forms { $h0l$ }. Only two forms, {102} and {104}, appear in the first 27 places of the observed morphological aspect, and of these {102} is dominant. The fact that {102} and not {101} is the dominant form suggests that there are c -glide planes parallel to (010), and this conclusion is strengthened by the fact that {104} and not {103} is the second most important form in the harmonic segment of the zone.

The zone [001] of forms { $hk0$ }. In the observed morphological aspect

¹ J. D. H. Donnay and D. Harker, *Naturaliste Canadien*, 1940, loc. cit.

the prisms are, in order of decreasing importance, $\{110\}$, $\{120\}$, $\{130\}$, and $\{210\}$, and no others need be considered in the study of this zone. The unit form is dominant, and there is a steady decrease in importance from $\{110\}$ to $\{130\}$ in the harmonic segment of this zone. The only form of any importance in the arithmetic segment of the zone is $\{210\}$. This zone shows the type of development to be expected when m -planes are parallel to (001) .

The pinacoids. In the observed morphological aspect the pinacoids are placed in the order of decreasing importance $\{001\}$, $\{100\}$, and $\{010\}$. This does not agree with the order of the theoretical morphological aspect, which is $\{001\}$, $\{010\}$, and $\{100\}$ for the axial ratios $a:b:c = 0.78516:1:1.28939$ found by morphological means. The analysis thus indicates two possible space groups, $Pn\bar{c}m$ or $Pncm$.

Celestine. The zone $[100]$ of forms $\{0kl\}$. The only major form in this zone is $\{011\}$, which is the most important form on celestine. The other forms of this zone in the observed morphological aspect are $\{012\}$, $\{015\}$, and $\{0.1.12\}$, in order of decreasing importance. The last three forms are of nearly equal importance and are moderately rare. The dominance of the unit form suggests that there are no b - or c -glide planes parallel to (100) , leaving m -planes and n -glide planes for consideration. With respect to the choice between the last two planes the remarks made when discussing this zone in baryte also apply here.

The zone $[010]$ of forms $\{h0l\}$. The observed morphological aspect in this zone is $\{102\}$, $\{104\}$, $\{101\}$, and $\{103\}$. The first two of these forms are important and the last two are rare. Since the dominant form is $\{102\}$ and the next most important is $\{104\}$, and both of these are very much more common than $\{101\}$ and $\{103\}$, the presence of c -glide planes is clearly indicated.

The zone $[001]$ of forms $\{hk0\}$. For this zone the observed morphological aspect is $\{110\}$, $\{120\}$, and $\{210\}$ and the first of these forms is clearly dominant. This distribution would be consistent with the presence of m -planes parallel to (001) .

The pinacoids. The observed morphological aspect is $\{001\}$, $\{100\}$, and $\{010\}$. The order to be expected from the axial ratios

$$a:b:c = 0.77895:1:1.28005$$

is $\{001\}$, $\{010\}$, and $\{100\}$.

The morphological analysis thus indicates two possible space-groups, $Pn\bar{c}m$ or $Pncm$, the weight of evidence being slightly in favour of the former.

Review of the morphological analysis.

X-ray analysis has shown that members of the baryte group, in the usual orientation, have the space-group $Pnma$, and that the a -axis of the unit-cell has twice the value indicated by the axial ratios used in the morphological analysis.¹ It seems very remarkable that the wealth of detail in the zonal development of the three central zones should point unambiguously to the wrong axial ratios. In the cases of the orthorhombic minerals brookite,² topaz,² and danburite³ morphological studies led to correct determinations of the structural parametral form.

Since the determination of the lattice type did not indicate that the a -axis should be doubled an attempt had to be made to determine the space-group while using incorrect axial ratios. It is now necessary to repeat the analysis briefly, using the doubled value of the a -axis. When the indices are modified a form $\{hkl\}$ becomes $\{2h.k.l\}$. In the following section of this paper such modified indices will be used, and it will be stated when their use is discontinued.

In the ensuing discussion it is not practicable to use the same frequency limits for the three minerals. For baryte only forms with a frequency of 1 % or over from either the Atlas or the Handbuch will be considered. For anglesite only forms with actual frequencies of occurrence over three from the Atlas or Handbuch will be used. In the case of celestine some of the zones are so poorly developed that it will be necessary to use lower frequencies to illustrate the argument. For all three minerals the actual frequencies from the Atlas are given after the form, and those from the Handbuch follow in brackets.

Determination of the lattice type.

Baryte. The zone $[\bar{1}\bar{1}0]$. This zone is greatly reduced in importance when the correct indices are used. The dominant form is $\{111\}$ 95 (108), followed by $\{112\}$ 7 (21) and $\{221\}$ 8 (1). This zone is of the simple type with the unit form dominant.

The zone $[\bar{1}01]$. The dominant form is $\{111\}$ 95 (108), and the remaining forms are $\{212\}$ 38 (47), $\{232\}$ 14 (10), and $\{121\}$ 6 (7). This zone is also of the simple type with the unit form dominant, containing the three most simple primary forms and one secondary form.

The zone $[0\bar{1}1]$. The dominant form is $\{211\}$ 263 (235), followed by

¹ R. W. James and W. A. Wood, loc. cit.

² F. C. Phillips, An introduction to crystallography, 1946, pp. 287-8 [M.A. 10-97].

³ J. D. H. Donnay, Trans. Roy. Soc. Canada, 1940, ser. 3, sect. IV, vol. 34, p. 33 [M.A. 8-306].

$\{111\}$ 95 (108). This zone is anomalous, but since two of the three central zones are of the simple type with the unit form dominant the lattice must be primitive, and the analysis of this zone is of no importance for the present purpose.

Anglesite. The zone $[1\bar{1}0]$. The dominant form is $\{111\}$ 186 (83), followed by $\{112\}$ 19 (15) and $\{221\}$ 14 (5). This zone is reduced in importance but is still of the simple type with the unit form dominant.

The zone $[\bar{1}01]$. The dominant form is $\{111\}$ 186 (83), which is followed by $\{212\}$ 68 (42), $\{232\}$ 13 (8), and $\{121\}$ 11 (4). In the primary series the forms $\{212\}$ and $\{121\}$ succeed the dominant form in importance, and the only secondary form has the simplest indices, $\{232\}$. This zone is thus of the simple type with the unit form dominant.

The zone $[0\bar{1}1]$. The dominant form is now $\{211\}$ 231 (110), and is succeeded by $\{111\}$ 186 (83), $\{122\}$ 8 (7), $\{233\}$ 5 (4), and $\{255\}$ 5 (3). This zone is anomalous, as it is in baryte, the remarks on which also apply in this case. It may be concluded that the lattice of anglesite is primitive.

Celestine. The zone $[1\bar{1}0]$. Only three forms appear in this zone, $\{111\}$ 17 (24), $\{112\}$ 0 (11), and $\{221\}$ 2 (2). Since the unit form is dominant and the other two forms which appear belong to the primary series and are adjacent to the dominant form this zone belongs to the simple type with the unit form dominant.

The zone $[\bar{1}01]$. The observed frequencies of forms germane to the discussion are $\{111\}$ 17 (24), $\{232\}$ 7 (0), $\{121\}$, $\{212\}$, and $\{414\}$ all 1 (3), and $\{313\}$ 0 (1). There is a tenuous but complete primary series of forms from $\{121\}$ to $\{414\}$, in which the unit form is dominant. The only secondary form is $\{232\}$. This zone is of the simple type with the unit form dominant.

The zone $[0\bar{1}1]$. This zone is more fully developed, and only forms with a minimum frequency of four from one of the sources will be quoted. $\{211\}$ 60 (55), $\{111\}$ 17 (24), $\{233\}$ 20 (12), $\{122\}$ 12 (16), $\{277\}$ 0 (4) and $\{155\}$ 6 (3). The same problem occurs in this zone as was found in baryte and anglesite, in which $\{211\}$ and not the unit form is dominant. The comments made on the re-analysis of this zone in baryte also apply to celestine. It may be concluded that the lattice is primitive.

Determination of the space-group.

The required frequency data can be obtained from table IV, in which the symbols of forms are referred to morphological and not structural axial ratios. Symbols of forms in the zone $[100]$ are unaffected, and the table may be used without any modification. For the zones $[010]$ and

[001] it is necessary to read the form symbols in table IV with the first index doubled, to allow for the doubled value of the a -axis used in this section of the paper.

Since the symbols of forms in the zone [100] are unaffected by the change of axial ratios it is unnecessary to reinterpret this zone. It was concluded above that n -glide planes or m -planes exist parallel to (100) in all three minerals. The other two zones will be discussed below.

Baryte. The zone [010]. The observed morphological aspect is {101}, {201}, {102}, {103}, {203}, and {301}. In this zone the unit form is now dominant. There is a complete primary series from {301} through {101} to {103}, and the importance of the forms falls off steadily on either side of {101}. There is only one secondary form, {203}. This distribution suggests the presence of m -planes parallel to (010).

The zone [001]. The observed morphological aspect is now {210}, {410}, {310}, {230}, {110}, {430}, and {120}. The dominant form is {210}, and forms with the first index even tend to predominate, indicating the presence of a -glide planes parallel to (001).

The pinacoids. The observed order of decreasing importance, {001}, {010}, and {100}, does not agree with the order calculated from the unit-cell axes, which is {100}, {001}, and {010}.

The space-group can now be read as *Pmma* or *Pnma*.

Anglesite. The zone [010]. The dominant form becomes {101}, and the only other form of importance is {102}, which suggests the presence of m -planes parallel to (010).

The zone [001]. The dominant form is {210}, followed in order of decreasing importance by {110}, {230}, and {410}. Since {210} is dominant and forms with the first index even are common, a -glide planes are probably present parallel to (001).

The pinacoids. The observed morphological aspect is {001}, {100}, and {010}, while the theoretical order for the doubled a -axis is {100}, {001}, and {010}.

The probable space-group is therefore *Pnma* or *Pmma*.

Celestine. The zone [010]. The observed morphological aspect in this zone is now {101}, {102}, {201}, and {203}. The first two forms are important, the last two are rare, and {101} is clearly the dominant form. The first three forms represent a simple primary series in which the unit form is dominant. {203} is the only secondary form, and it comes between the two most important primary forms. This distribution suggests the presence of m -planes parallel to (010).

The zone [001]. The observed morphological aspect of this zone is

{210}, {110}, and {410}. This distribution would be consistent with the presence of *a*-glide planes parallel to (001).

The pinacoids. The observed morphological aspect is {001}, {100}, and {010}, but the expected theoretical order is that given under baryte.

The space-group derived for celestine from the morphology is thus *Pnma* or *Pmma*.

It has now been shown by morphological means that the space-group of baryte, anglesite, and celestine can be reduced to a choice between *Pnma* and *Pmma*, of which the former is correct. It is interesting that, once the correct axial ratios are known, the axial zones lead to the deduction of only two possible space-groups, of which one is correct, while a study of the central zones did not lead to correct determinations of the axial ratios. It may be noted here that Taylor,¹ in his morphological analysis of columbite crystals, obtained the space-group *Pman*, whereas X-ray determination gave it as *Pcan*.

In the succeeding part of this paper the symbols of forms will be referred to the axial ratios given at the head of table I, in conformity with the earlier part.

Predictions of the Donnay-Harker hypothesis.

This hypothesis predicts that the morphological importance of a form will be proportional to its reticular density or inversely proportional to its reticular area, *S*. If this statement represents an exact law the determination of space-groups by morphological means should be as accurate as those found by X-ray analysis, provided sufficient data are available.

An examination of the above statement has been made by calculating the theoretical order of importance (theoretical morphological aspect) for the minerals of the baryte group, and comparing it with the observed morphological aspect. The theoretical morphological aspect is calculated using the axial ratios with the doubled value of the *a*-axis, corresponding to the structural unit-cell. The symbols of faces referred to these axial ratios will be called 'structural symbols', while those referred to the axial ratios in which the *a*-axis is not doubled will be called 'morphological symbols'.

For the orthorhombic system the value of S^2 is given by the formula $S_{hkl}^2 = h^2b^2c^2 + k^2c^2a^2 + l^2a^2b^2$. Structural symbols are used throughout in the calculation of S^2 , modified where necessary by the requirements of the lattice or space-group.

¹ E. D. Taylor, Amer. Min., 1940, vol. 25, p. 123 [M.A. 8-95].

For the space-group *Pnma* no condition is imposed on (*hkl*) planes, the symbols of which are unmodified. The *n*-glide planes parallel to (100) impose the restriction '*k+l* even' on all forms {*Ok**l*}, so that the symbols of forms which do not obey this criterion are doubled. In the zone [010] the *m*-planes impose no restriction, and the symbols are unmodified. In the zone [001] the *a*-glide planes impose the restriction '*h* even', and all symbols which do not conform to this criterion are doubled. The 2₁ axes normal to the pinacoids require their symbols to be doubled.

The first thirty forms of the theoretical morphological aspect, based on the values of S^2 , are shown in table V, in which the forms are referred to by their more familiar morphological symbols. The data for baryte are given in the left-hand section of the table. The first column indicates the theoretical rank of the forms (R_t), which is the order of the theoretical morphological aspect. The last four columns in this section of the table give the value of S^2 , the observed rank of the form (R_0) as shown in tables I and II, and the actual frequencies of the forms from known localities, derived from the Atlas (A) out of 486 crystals, and the Handbuch (H) out of 505 crystals. The theoretical order of importance is the same for the first thirty forms of anglesite and celestine, but it differs appreciably from the order for baryte. The theoretical rank of the forms of anglesite and celestine is given in the second section of the table and the values of S^2 , the observed rank and the frequencies of occurrence from known localities, derived from the Atlas and the Handbuch, are shown in the third and fourth sections of the table for anglesite and celestine respectively. In the case of anglesite the number of crystals derived from the Atlas was 359 and from the Handbuch it was 167; for celestine the corresponding numbers of crystals were 201 and 159 respectively.

Table V shows that the agreement between theoretical and observed ranks is very poor. The two most important forms in baryte occupy the 6th and 7th places in the theoretical list. The forms that are 7th and 11th in the observed list should be 2nd and 4th according to the theory. In the case of anglesite the agreement is little better. The two most important forms occupy the 7th and 6th places in the theoretical list and a form which is not placed among the first 27 observed forms should, in theory, occupy the 5th rank. The two most important forms in celestine occupy the 2nd and 6th ranks of the theoretical order, while forms placed 10th and 15th (equal) in the observed list should theoretically occupy the 4th and 5th ranks. Many other such anomalies may be seen for each of the three minerals in table V. It suffices to say that, in the case of the baryte group, the strict statement of the Donnay-Harker

hypothesis is not justified; morphological importance or rank is not inversely proportional to reticular area.

Form development in the baryte group.

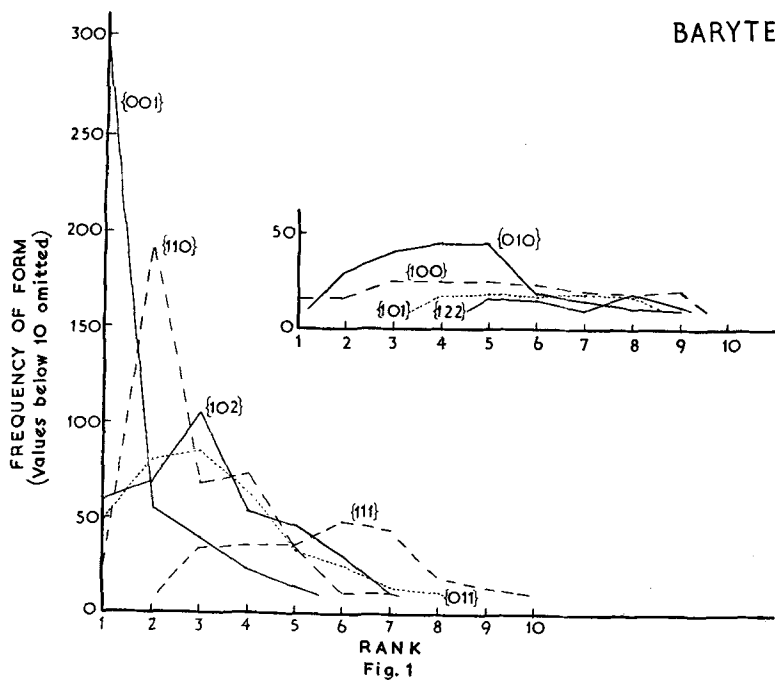
Some interesting comparisons may be made between the form developments of baryte, anglesite, and celestine. Bipyramids $\{hhl\}$ show a strong development in baryte, but they decrease in anglesite and again in celestine. In baryte forms are common in the primary series from $\{111\}$ to $\{115\}$, but in anglesite the range is not only less, but its centre has moved towards $\{111\}$, the commoner forms being from $\{221\}$ to $\{112\}$. In baryte $\{113\}$ was developed to a greater extent than $\{112\}$, but this anomaly disappears in anglesite. Of the bipyramids $\{hkk\}$ only two are well developed in baryte, there is a slightly better development in anglesite and a good series in celestine. It will be observed that some of these changes are of a serial nature, following the order of the ionic radii. The bipyramid $\{324\}$ has an interesting development; it is quite important in anglesite, scarce in celestine, and almost unknown in baryte. Of the forms $\{h0l\}$, $\{102\}$, $\{101\}$, and $\{104\}$ are very important in baryte, but $\{101\}$ suffers almost total eclipse in the other two minerals. There is a very marked serial change in the development of forms $\{hko\}$; in baryte there are five prisms of frequent or fairly frequent occurrence, in anglesite there are two, while in celestine there is only one. A progressive change also takes place in the relative importance of the pinacoids $\{100\}$ and $\{010\}$ from baryte to celestine.

Rank distribution of forms.

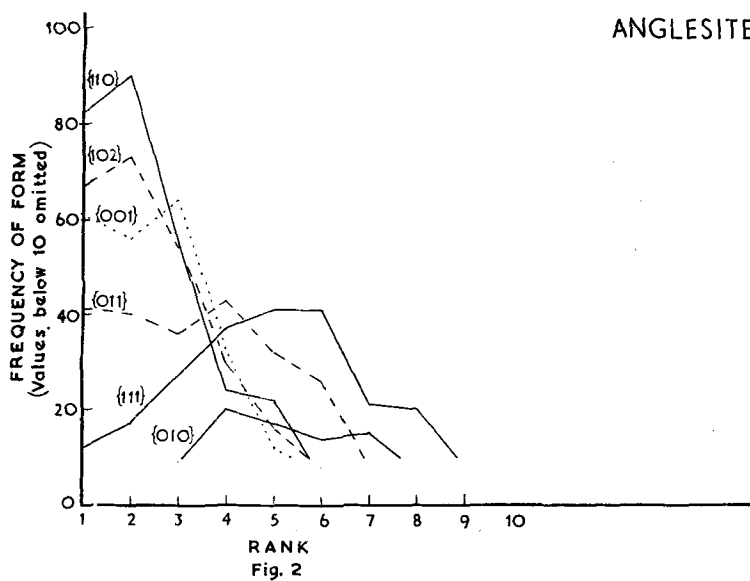
Finally, as a matter of interest, it seems worth while to record the rank distribution of some of the more important forms. For the crystals from known localities recorded in Goldschmidt's Atlas the frequency of occurrence of a form in each rank has been plotted against the rank, as shown in figs. 1, 2, and 3. Frequencies below 10 have been omitted in figs. 1 and 2, and below 5 in fig. 3, to avoid the straggling 'tails' of the distribution curves, which are of little significance.

It can be seen in fig. 1 that the dominant form on baryte, $\{001\}$, has a frequency which is very high in the first rank and falls away rapidly in the later ranks, while the second most important form, $\{110\}$, has its highest frequency in the second rank. $\{011\}$ has a broad maximum in the second and third ranks and $\{102\}$ has a peak in the third rank. In general, as the importance of forms decreases their peaks become lower and broader and are usually displaced farther to the right.

BARYTE



ANGLESITE



CELESTINE

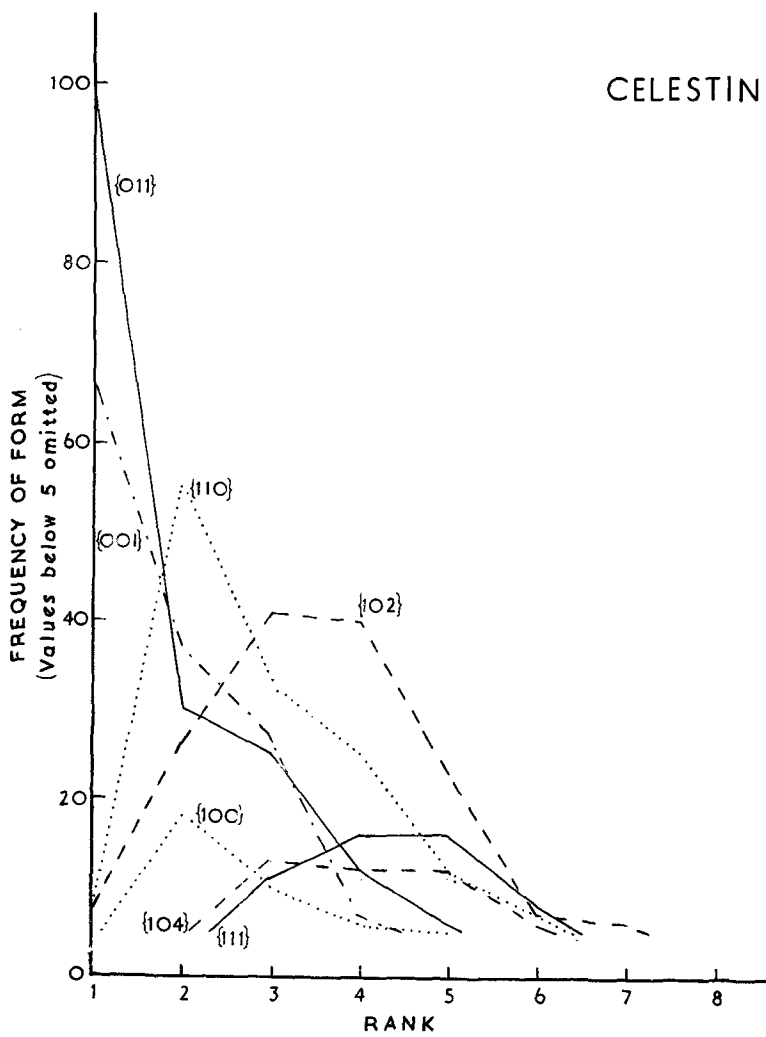


Fig. 3

In the case of anglesite fig. 2 illustrates that $\{110\}$ is somewhat more important than $\{001\}$, while for celestine (fig. 3) it shows the clear superiority of $\{011\}$ over $\{001\}$.

A study of figs. 1 to 3 and table I illustrates some interesting features of the development of bipyramids relative to other forms and to each other in the minerals of the baryte group. In table I it will be seen that

there is an approximate proportionality between the rank of a form and its mean rank, locality frequency, or frequency of occurrence. There are a number of exceptions to this broad generalization, nearly all of which concern bipyramids. In baryte {111} and {122} have high mean ranks compared with the forms adjacent to them in the table, and the bipyramids {113}, {112}, {114}, and {115} have much higher mean ranks than the prism in their midst, {120}. In anglesite the bipyramids {111} and {122} have a relatively low rank for forms with such high values of locality frequency and frequency of occurrence. The same two forms in celestine also appear to be placed low in the table, if only their locality frequency or frequency of occurrence are considered. The apparently anomalous position of these two forms in anglesite and celestine is accounted for by the relatively large value of the mean rank which they possess. It thus appears that most of the common bipyramids in minerals of the baryte group have relatively high values of the mean rank. The probable explanation of this fact may be found by considering the common habits of this group of minerals. In all three minerals there are four dominant forms, {001}, {110}, {011}, and {102}. These determine the common habits, in which crystals are elongated on the *a*- or *b*-axes or tabular on the basal pinacoid and bounded by either {110} or by {011} and a form {*h*0*l*}. Most of the bipyramids which are developed replace coigns or short edges between faces of the dominant forms, and so the faces are relatively small. Because of their geometrical position the pyramids would need a considerably reduced growth velocity to make an appreciable increase in the size of their faces, sufficient to affect their mean ranks and positions in table I.

The bipyramids {133} and {144} in celestine appear to be in a different category to all the others discussed. These forms have rather low frequencies of occurrence and locality frequencies but they are higher in the table than might be expected because of the small value of their mean ranks. The fact that these forms only occur at a few localities yet at these localities the forms are well developed suggests that some aspect of the environment encouraged their growth. The factor most likely to cause the preferential development of a form at a few localities is the presence of a specific impurity.

Postscript. The work described in this paper was completed in 1952, and a paper on the morphology of baryte was read before the Mineralogical Society in November of that year. In the interval before publication a series of papers has appeared by Hartman¹ and by Hartman and

¹ P. Hartman. Thesis, 1953. Groningen [M.A. 12-314].

Perdok,¹ on crystal morphology, some of which make special reference to the minerals of the baryte group. The particular aim of the present paper was to examine the validity of the Donnay-Harker hypothesis with respect to the baryte group. It will have been seen that this hypothesis does not give a very satisfactory prediction of the morphology of this group of minerals. In their work Hartman and Perdok estimate the importance of a form by the study of bond energies. In order to make the calculations the structure of the crystal must be known, which is not the case with the simpler method of Donnay and Harker. Hartman and Perdok are able to give a more accurate prediction of the relative importance of the commoner forms than is provided by the Donnay-Harker hypothesis.

¹ P. Hartman and W. G. Perdok, *Acta Cryst.*, 1955, vol. 8. A paper in three parts, I, p. 49; II, p. 521; III, p. 525 [M.A. 13-279]. P. Hartman and W. G. Perdok, *Amer. Min.*, 1956, vol. 41, p. 449 [M.A. 13-280].
