The stereographic projection: a procedural revision in crystallography

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Summary. Objections are raised to the custom of reversing the pole of projection when dealing with lower crystal faces, for some points so projected no longer retain their natural interangular relations. Instead, lower face normals may be replaced by their antipodal points; these *antipodal points* are then projected in the usual manner, with suitable symbols. The resultant stereographic projection not only portrays all crystal faces within the primitive circle but is also strictly angle-true.

The attention of mineralogists is called to the standard procedure of changing the pole of projection of the stereographic projection in order to depict the lower faces of crystals. The advantage of that method, or rather its object, lies in the fact that the lower faces so projected remain conveniently within the area of the primitive. In the opinion of the author the method is objectionable and might profitably be replaced. The considered objections are explained and an alternative procedure presented.

Objections to the standard method of presenting a lower crystal face by reversing the pole of projection

The crux of the dispute may be illustrated with the faces {111} of a tetragonal bipyramid. In fig. 1 the upper faces (111), $(1\bar{1}1)$, $(\bar{1}11)$, and $(\bar{1}\bar{1}1)$ are represented by crosses projected from the lower pole of projection; the single face $(11\bar{1})$ is marked by a circle projected from the upper pole of projection. It will be seen firstly that the two faces (111) and $(11\bar{1})$ occupy the same position in this projection whereas on the crystal they possess an interfacial angle of 100°. Secondly, the two parallel faces $(\bar{1}\bar{1}1)$ and $(11\bar{1})$ are separated from each other on projection.

If, by switching the pole of projection, planes which have different spatial orientation have yet to share the same position on the projection, and planes possessing the same spatial orientation are projected to different positions, then something is amiss. The very essence of the usefulness of the stereographic projection lies in its retention of measured interangular relationships. Any means adopted that blur these

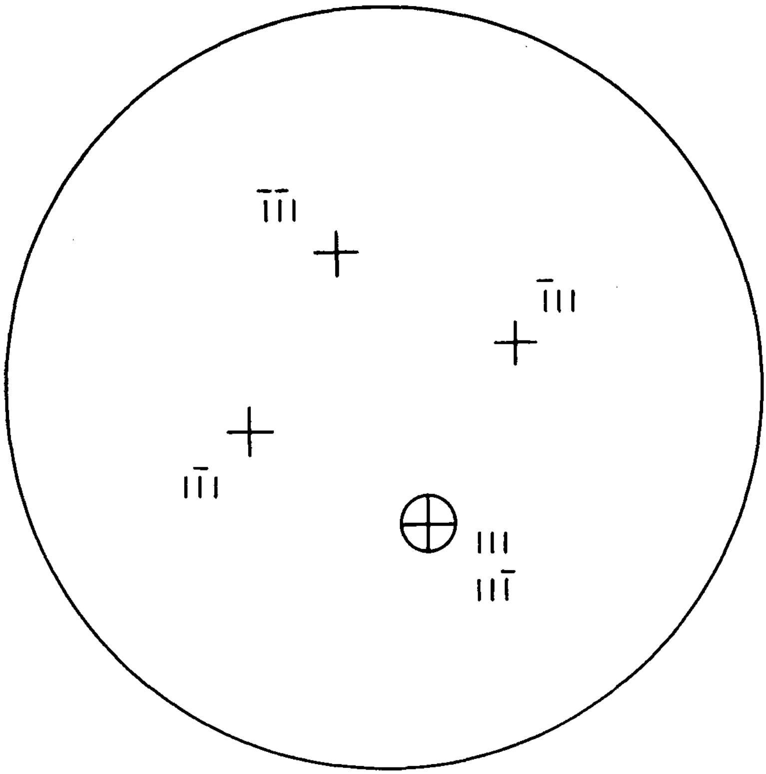

FIG. 1. Standard conventional projection: (111), ($1\bar{1}1$), ($\bar{1}11$), and ($\bar{1}\bar{1}1$) are projections from the lower pole, ($11\bar{1}$) from the upper. Face normals projected from the upper pole are shown as circles, from the lower as crosses.

relationships can only sabotage our chance of enjoying that freedom of measurement which the method might otherwise have furnished.

It may be retorted that ($\bar{1}\bar{1}1$) and ($11\bar{1}$), although parallel, need not be represented uniformly as would two sedimentary beds having equal dip and strike. The two face normals, it may be said, are in fact opposed in sense for they radiate out in opposite directions from the centre of the parent spherical projection to strike its surface at diametrically opposite points. This is certainly true, and a blind designation of ($\bar{1}\bar{1}1$) as synonymous with ($11\bar{1}$) would be erroneous. The revised procedure distinguishes such parallel faces, which although both occupying the same position on projection, are marked individually as either a dot or a cross.

It should be noted that even pole-switching cannot prevent the great circles (010) and ($0\bar{1}0$) from coinciding; and the face normals (001), ($00\bar{1}$) similarly occupy an identical position in many an orthodox pole-switched projection. The plotting of parallel faces as spatially equivalent faces cannot, therefore, be regarded as intrinsically incorrect; especially so when the opposite sense of the face normals is indicated in an unequivocal manner by appropriate symbols.

The revised procedure

It is proposed to replace a point on the lower half of the spherical projection by its antipodal point on the upper hemisphere; this latter point will be projected in the usual manner.

Where it is necessary to preserve the original lower-hemispherical provenance of such projected antipodal points they will be marked as dots, in contradistinction to the crosses representing native upper hemispherical points. Fig. 2 shows again the four upper faces of the tetragonal bipyramid with the addition of ($11\bar{1}$) projected by the new method. Measurement of the angle between any two face normals may now be performed along their common great circle. However, attention must be paid to the symbol. The angle ($\bar{1}\bar{1}1$) $\wedge$ (111) is acute whereas the angle ($11\bar{1}$) $\wedge$ (111) is obtuse. Where the two face normals belong to the same hemisphere, upper or lower, the angle between them is measured within the primitive, in the normal manner. But where, as for the angle ($11\bar{1}$) $\wedge$ (111), the face normals belong each to an opposite hemisphere then measurement must be performed across the primitive; such a measurement is indicated by dotted lines in Fig. 2. In this manner the true crystallographic angle ($11\bar{1}$) $\wedge$ (111) of 100° may be measured directly on the stereogram.

In effect, a single great circular arc, of angular length 180° within the primitive, is serving as a complete great circle of angular arc 360° because it may be traversed twice, once along the range of the upper hemisphere, and again when crossing the surface of the lower hemisphere. Such fertile uses of the stereographic projection as those published recently in connexion with rotating indicatrices (for example, D. J. Fisher, Amer. Min., 1962, vol. 47, p. 649) involve the replacement of axes reaching an alien hemisphere by their antipodal extremities.

Indeed, where axes, or the normals to most geological planes, are concerned, there is no significance to 'up' or 'down'. It is the object of this paper to demonstrate that even crystallography, in which directional sense dare not be neglected, may profitably comply with this same

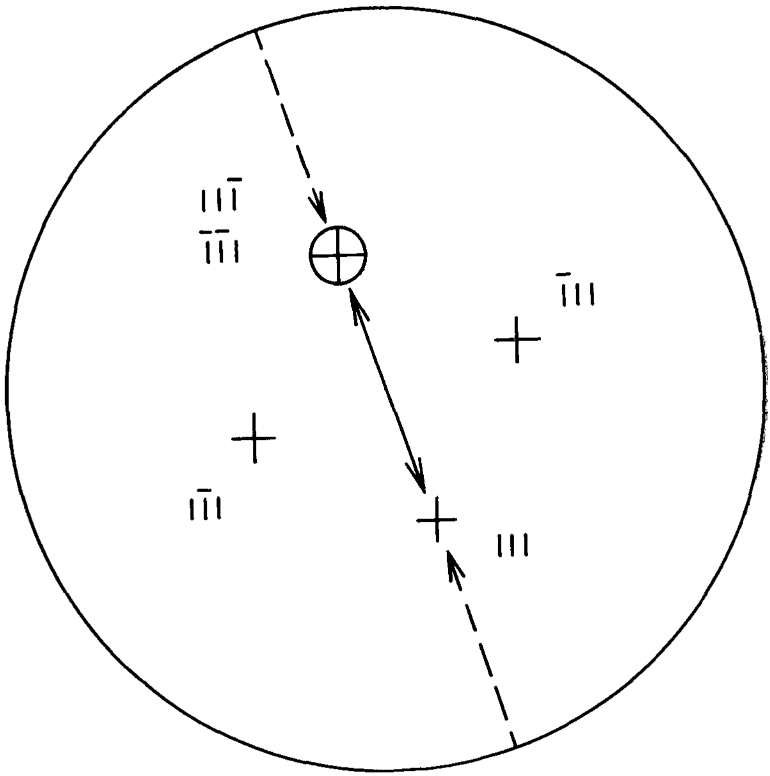

FIG. 2. Revised projection: ($11\bar{1}$) is represented by the parallel face ($\bar{1}\bar{1}1$) and distinguished as a circle, the upper hemisphere poles being shown as crosses.

treatment. Not only is uniformity of approach thereby attained in all applications of the stereographic projection—a highly significant consideration in teaching—but, in addition, advantages hitherto unattainable may be enjoyed.

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