A new universal stage extinction technique: the determination of 2V when neither optic axis is directly observable

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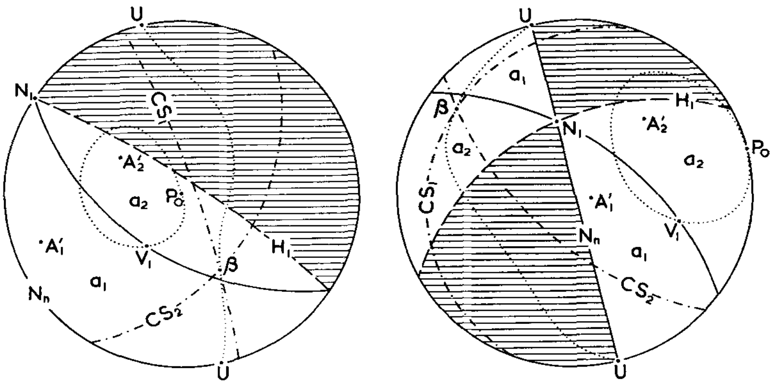
Summary. A new universal stage extinction technique is presented. It is based on the special case of coplanar and conical extinction curves where an optic axis lies on the wave-normal locus. It may be used in favourable circumstances to determine $2V$ when neither optic axis is directly observable.

METHODS for the determination of the optic axes of a biaxial crystal with the universal stage fall into three categories depending upon the number of optic axes that are directly observable: both, one, or neither. When both are directly observable, well-established orthoscopic (Emmons, 1943, pp. 26–28) or conoscopic (Hallimond, 1950) methods or the optical curves of Fedorov (1896) (also summarized by Johannsen, 1918, pp. 494–495) can be used with varying degrees of accuracy (Fairbairn and Podolsky, 1951; Munro, 1963). When only one optic axis is directly observable, this can be determined by any of the above methods and the other located by simple bracketing of a bisectrix or, more accurately, by use of the Biot-Fresnel construction as advocated by Joel and Muir (1958*b*). When neither optic axis is directly observable, use may be made of the ingenious extinction method of Berek (1923) as amplified by Dodge (1934) (this is also set out by Emmons, 1943, pp. 28–39). This method depends, however, upon the use of prepared curves: consequently, absence or non-availability of these curves renders the problem, for practical purposes, insoluble.

The present paper shows that extinction curves may, in addition to their use in the location of principal axes of the indicatrix (Joel and Muir, 1958*a*; Joel and Tocher, 1964), be efficiently employed in the location of both optic axes when neither of these is directly observable. The method proposed has the advantage that it relies on no previously prepared curves: the requisite curves, the extinction curves, are derived and are plotted stereographically as required from simple extinction measurements.

The basis of the method

It has been shown (Tocher, 1964) that, on the basis of any one out of a few plotted extinction measurements, it can be deduced that the optic axes of a biaxial crystal are restricted to certain, generally small, portions of the stereogram. These portions are defined as follows: one of the



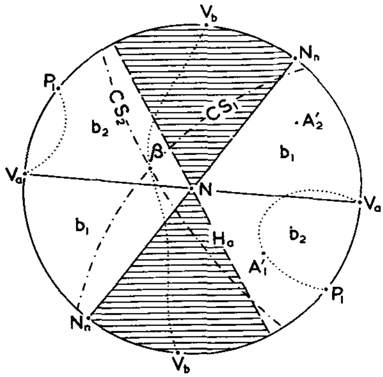
FIGS. 1 and 2: FIG. 1 (left). Stereogram, centre P_0, showing plane of wave-normals, $N_1 N_n$ (primitive circle), and associated equatorial ($U\beta U$) and polar ($P_0 V_1$) extinction curves. Plane $N_1 V_1$ divides the initial search area (blank) into sub-areas a_1, a_2 containing optic axes A'_1, A'_2 respectively. The initial elimination area (ruled) is limited by plane H_1. CS_1, CS_2 are the circular sections. FIG. 2 (right). Fig. 1 replotted with P_0 on the primitive circle after a rotation of 90° about the horizontal axis UU.

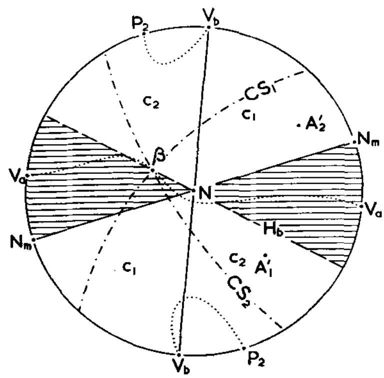
optic axes, A'_1 (figs. 1 and 2), must lie in the acute angle (sub-area a_1) between the plane of wave-normals,[1] $N_1 N_n$, and the plane $N_1 V_1$, where the latter is defined by N_1, the wave-normal, and V_1, the associated vibration on the polar extinction curve; the second optic axis must lie on the other side of plane $N_1 V_1$ within the acute angle (sub-area a_2) subtended at N_1 by planes $N_1 V_1$ and H_1, where plane H_1 is defined by the requirement that the angles $N_n N_1 V_1$ and $H_1 N_1 V_1$ be equal. In the remainder of the stereogram, the initial elimination area (ruled in figs. 1 and 2), no optic axis can lie.

In the case of coplanar extinction curves, i.e. those associated with coplanar wave-normals (Joel and Tocher, 1964), repetition of this process with other selected wave-normals gives rise to the progressive elimination method for location of the optic axes (Tocher, 1964). The

[1] When P_0, the axis of rotation of the crystal, is plotted centrally (fig. 1) plane $N_1 N_n$ is the primitive circle; and when P_0 is plotted on the primitive circle (fig. 2) plane $N_1 N_n$ is the diametral plane normal to P_0.

construction for progressive elimination is very simple with P_0, the axis of rotation of the crystal, plotted centrally, but becomes very laborious and time-consuming with P_0 in any other position.





FIGS. 3 and 4: FIG. 3 (left). Stereogram, centre N, showing polar ($P_1 V_a$) and equatorial ($V_b \beta V_b$) extinction curves derived by rotation about P_1—wave-normal locus is plane NN_n. Elimination procedure applied to wave-normal N and associated polar curve vibration, V_a, produces initial search sub-areas, b_1, b_2, and the ruled initial elimination area. A_1', A_2' are the optic axes, and CS_1, CS_2 the associated circular sections. FIG. 4 (right). Stereogram, centre N, of the same biaxial crystal as in fig. 3 and in the same orientation, showing polar ($P_2 V_b$) and equatorial ($V_a \beta V_a$) extinction curves derived by rotation about P_2—wave-normal locus is plane NN_m. Elimination procedure applied to wave-normal N and associated polar curve vibration, V_b, produces initial search sub-areas, c_1, c_2, and the ruled initial elimination area.

With the universal stage, the available portion of any determinable extinction curve cannot, in general, be guaranteed to cover the requisite range for accurate location of the optic axes by the established progressive elimination method. However, as shown by Joel and Muir (1958*a*), the number of partial extinction curves obtainable with the universal stage is virtually infinite and one has an unlimited choice of axes of rotation, P_1, P_2,..., P_n, in, say, the plane of the outer stage; and these axes of rotation are most conveniently plotted on the primitive circle (*ibid.*). In view of this it has been found convenient to adapt the progressive elimination method for use with several partial extinction curves and with several coplanar axes of rotation, P_1, P_2,..., P_n, plotted on the primitive circle.

Figs. 3 to 5 show the basis of this adaptation as applied to one crystal in a fixed orientation. In fig. 3 the axis of rotation, P_1 ($= A_4$ on the universal stage), is horizontal, and the vertical plane of wave-normals,

NN_n, contains the normal, A_3, to the outer stage—this is wave-normal N. The extinction curve obtainable by 180° rotation about P_1 must then pass through three fixed points: P_1, V_a, and V_b, i.e. the axis of rotation and also the two vibration directions in the primitive circle associated with wave-normal N ($= A_3$) (Joel and Muir, 1958*a*, p. 870).

Of the two vibrations, V_a and V_b, one must, in general, lie in each of the two adjacent dihedrals formed by the circular sections, CS_1, CS_2, of the indicatrix and must bisect these two dihedrals in the plane of the wave-front (the primitive circle in this case). P_1 will, in general, fall in one of the dihedrals. Thus, in general, since the two parts of the extinction curve, the polar and equatorial curves, are each confined to one dihedral and since P_1 must fall on the polar curve (Joel and Garaycochea, 1957, p. 401) it follows that, of the two vibrations, V_a, V_b, one must fall with P_1 on the polar curve and the other on the equatorial curve. In fig. 3, P_1 lies in the same dihedral as and on the polar curve with V_a.

The elimination procedure is now applied to fig. 3 as follows: one optic axis, A'_2, must lie in the acute angle $N_n N V_a$ (sub-area b_1) between the plane of wave-normals, NN_n, and the plane NV_a; the second, A'_1, must lie in the acute angle of the same magnitude (sub-area b_2) between plane NV_a and plane H_a. Neither optic axis can lie in the initial elimination area (ruled) between planes H_a and NN_n.

Fig. 4 shows the case where the axis of rotation, P_2, lies in the same dihedral as the vibration V_b. V_b now lies on the polar extinction curve with P_2 while V_a lies on the equatorial curve in the other dihedral. It can be said here that one optic axis, A'_2, lies in the acute angle $N_m N V_b$ (sub-area c_1) between the plane of wave-normals, NN_m, and the plane NV_b. The other, A'_1, must lie in the acute angle of the same magnitude (sub-area c_2) between plane NV_b and plane H_b. Neither optic axis can lie in the initial elimination area (ruled) between planes H_b and NN_m.

Fig. 5 shows the effect of superimposing figs. 3 and 4: simultaneous consideration of the two sets of initial elimination areas serves to reduce still further the areas in which the optic axes lie. The residual search area now consists of sub-areas d_1 and d_2 in each of which one optic axis may lie. Sub-areas d_1 and d_2 themselves consist of the uneliminated portions of sub-areas b_1, c_1, and b_2, c_2, respectively, of figs. 3 and 4.

At this stage, the axis of rotation, P_1, of fig. 3 may be regarded as variable in position within the primitive circle but confined to the same dihedral as V_a. The points N, V_a, and V_b, and the optic axes, A'_1, A'_2, must, of course, be regarded as fixed. As P_1 migrates anticlockwise towards V_a the plane of wave-normals, NN_n, must likewise migrate anticlockwise with the same angular velocity towards V_b. This anticlockwise movement of plane NN_n with respect to the static plane NV_a serves to increase the acute angle N_nNV_a which defines the angular width of sub-area b_1 as subtended at N. Consequently, plane H_a must migrate

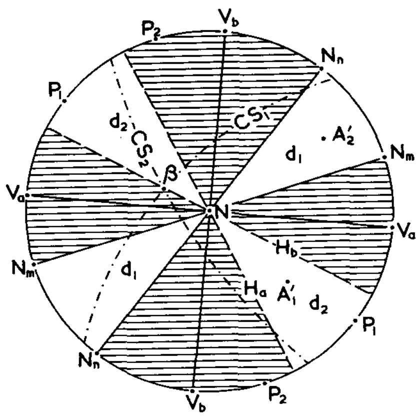


FIG. 5. Composite stereogram, centre N, derived by superposition of figs. 3 and 4, showing much reduced residual search sub-areas, d_1, d_2—these are the uneliminated portions of sub-areas b_1, c_1, and b_2, c_2 respectively, of figs. 3 and 4.

clockwise towards V_b with the same angular velocity in order to maintain sub-area b_2 at the same angular width as sub-area b_1. Clearly, when P_1 reaches V_a, both of the planes NN_n and H_a reach V_b, and the initial elimination area reaches its minimum angular width, zero. It is clear from the above that the angular width at N of the initial elimination area is twice the angle P_1V_a; and that the angular width at N of each of the sub-areas, b_1, b_2, is equal to $90°-P_1V_a$, i.e. equal to P_1V_b.

If P_1 migrates clockwise, away from V_a and towards one of the circular sections defining the dihedral, then planes NN_n and H_a must migrate clockwise and anticlockwise respectively at the same angular velocity until a point is reached where P_1 lies on the circular section: it can migrate no farther and still remain in the same dihedral as V_a. At this limiting point, since P_1 lies on a circular section, the plane of wave-normals, NN_n, must pass through the associated optic axis (A'_2 in fig. 3). It follows from application of the Biot–Fresnel construction to wave-normal N and its associated vibrations, V_a, V_b, that plane H_a in its limiting position must contain the second optic axis (A'_1 in fig. 3).

Similar considerations may be applied to fig. 4, where P_2 may be considered to migrate anticlockwise, away from V_b and towards the same circular section as was dealt with in fig. 3. When P_2 reaches its limiting position, the circular section, the planes NN_m and H_b, in their limiting positions, must each contain one optic axis (A_2' and A_1' respectively in fig. 4). Also, the angular width at N of the initial elimination area is twice the angle $P_2 V_b$; and that of each of the sub-areas, c_1, c_2, is equal to $90^\circ - P_2 V_b$, i.e. equal to $P_2 V_a$.

Extension of the above considerations to fig. 5 shows that the angular width at N of each of the residual search sub-areas, d_1, d_2, is equal to the angle $P_1 P_2$; and that, as P_1 and P_2 migrate clockwise and anticlockwise respectively towards the intervening circular section, the angular width of each of the sub-areas, d_1, d_2, decreases in sympathy with the angle $P_1 P_2$. It is thus clear that the radial direction, with respect to N, of the optic axis in each of the sub-areas, d_1, d_2, is determinable when P_1 and P_2 reach their common limiting position, the intervening circular section: one optic axis (A_2' in fig. 5) lies on the common limiting position of the two planes of wave-normals, NN_n, NN_m; the other (A_1' in fig. 5) on that of the two planes, H_a, H_b.

Practical application

Since this method only permits determination of the radial direction of each optic axis with respect to N ($= A_3$) it is necessary that the optic axial plane and the indicatrix axis β be located first by any of the excellent methods now available: a first approximation obtained by standard orthoscopic techniques may be refined via accurate location of at least two principal axes of the indicatrix using coplanar or conical extinction curves or both (Joel and Muir, 1958*a*; Joel and Tocher, 1964).

The above preliminaries having been completed, the location of the optic axes within the optic axial plane may now proceed. However, the practical application of the method outlined above does not actually necessitate the delineation of initial elimination areas and search sub-areas as in the case where only one axis of rotation is available (Tocher, 1964). In the universal stage application, where an unlimited number of axes of rotation is available, it reduces to a simple search for the special case where the wave-normal plane contains an optic axis, i.e. where, in the case of coplanar extinction curves, the axis of rotation lies on a circular section. A simple method of approach is outlined below.

Start with all axes of the universal stage in their zero positions. Then turn to extinction, either clockwise or anticlockwise, about A_1 and note the reading, α_1.[1] Plotting of α_1, $\alpha_1 \pm 90°$, $\alpha_1 \pm 180°$, and $\alpha_1 \pm 270°$ on the primitive circle of the stereogram serves to delineate the vibration directions, V_a, V_b (fig. 6), that are now parallel to A_4 and A_2. Next, rotate 45° clockwise or anticlockwise about A_1 from the extinction

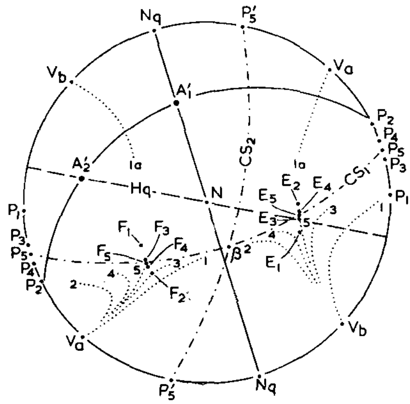


FIG. 6. Stereogram, centre N, showing partial coplanar extinction curves, 1 (and 1a), 2,..., 5 (dotted), derived by rotation about horizontal axes, P_1, P_2,..., P_5, respectively. V_a, V_b are the vibrations associated with wave-normal N. Axis of rotation, P_5, on circular section, CS_1, is associated with plane of wave-normals, NN_q, containing optic axis A'_1. Plane H_q, containing optic axis A'_2, is normal to P'_5, where $P'_5 V_a = V_a P_5$. Points E_1, E_2,..., E_5 and F_1, F_2,..., F_5 bisect acute and obtuse angles respectively between P_1, P_2,..., P_5 respectively and β. Only extinction curves 5 trend towards E_n and F_n points of the corresponding subscript.

position so that A_4 ($= P_1$, fig. 6) bisects the angle $V_a V_b$. Plot P_1 in the correct angle between V_a and V_b. With A_4 ($= P_1$) as axis of rotation, determine and plot both portions of the associated extinction curve to the limit of rotation to both north (curves 1a, fig. 6) and south (curves 1, fig. 6) about A_4. The method of plotting coplanar extinction curves as determined with the universal stage has been fully explained by Joel and Muir (1958a, pp. 871–874) and need not be repeated here. In general, there is little difficulty in distinguishing the polar and equatorial extinction curves: the former, at its farthest from V_a or V_b as the case may be, trends towards P_1; the latter trends towards β. Moreover, if those portions of the extinction curves associated with northwards and

[1] Following the terminology of Joel and Muir (1958a, p. 867) the angles α_1, α_2,..., α_5 are the angular rotations about axes, A_1, A_2,..., A_5 respectively from their zero positions.

southwards rotation about A_4 are considered separately, one set (no. 1, fig. 6) will, in general, show more marked terminal curvature than the other (no. 1*a*, fig. 6); and, in the case of the former set, the terminal curvature will, in general, have the same sense, clockwise or anticlockwise as the case may be, in each member (curves 1, fig. 6, have both clockwise terminal curvature). The set with the more marked terminal curvature clearly occurs within the acute angle between the primitive circle and the plane $P_1\beta$ (no. 1, fig. 6) so that, in fact, this first rotation to the instrumental limit about A_4 need only be to either north or south, as the case may be, in order to produce extinction curves within this acute angle. The other direction of rotation may be ignored. Curves for both directions of rotation about A_4 ($= P_1$) are shown in fig. 6 (curves 1 and 1*a*) in order to illustrate the relationship.

At this stage it is, in general, clear in which dihedral P_1 lies. If, as in fig. 6, it is on the polar curve with and in the same dihedral as V_b then, by suitable rotation about A_1, a new axis of rotation, P_2, must be chosen roughly mid-way between P_1 and V_a and the procedure repeated. However, it will not, in general, be necessary to record the new polar and equatorial curves over the whole tilting range in the appropriate direction about A_4: only those portions of the curves (no. 2, fig. 6) associated with wave-normals within about 30° of the maximum tilt are required in order to determine the direction of terminal curvature—towards P_2 or towards β as the case may be. In fig. 6, both no. 2 curves have anticlockwise terminal curvature, and P_2 is clearly on the same polar curve with and in the same dihedral as V_a. A circular section clearly lies between P_1 and P_2.

By suitable rotation about A_1, a new axis of rotation, P_3, is chosen midway between P_1 and P_2, as in the ranging technique advocated for progressive elimination (Tocher, 1964), and the necessary small portions of extinction curve no. 3 plotted. This ranging process is continued as necessary until a set of curves (no. 5, fig. 6) with terminal curvature simultaneously at a minimum and in opposite senses, one clockwise, one anticlockwise, is plotted. Such a set of curves indicates that the axis of rotation, P_5, contributing thereto lies on a circular section of the indicatrix and that the associated plane of wave-normals, NN_q, includes one optic axis—this optic axis, A_1', is, of course, the pole of the circular section $P_5\beta$.

The plane, H_q, containing the second optic axis, A_2', and the circular section of which A_2' is the pole, are now easily determined as follows. The second circular section meets the primitive circle at a point, P_5', such that V_a (and V_b) bisects the angle $P_5 P_5'$. H_q is the plane of which P_5' is the pole, and the circular section is the plane $P_5' \beta$.

Axial combinations other than A_1, A_4 ($= P_n$), A_5 may, of course, be employed in the determination of the coplanar extinction curves. They are: A_1, A_2 ($= P_n$), A_3; A_1, A_2 ($= P_n$), A_5; A_3, A_4 ($= P_n$), A_5.

It may be noted here that this universal stage technique makes the first deliberate use of a special case of the coplanar extinction curves—a special case hitherto carefully avoided in practice as being of little more than academic interest: the axis of rotation on a circular section and the plane of wave-normals containing an optic axis.

Sensitivity

The sensitivity of this method in terms of the location of a single optic axis, say A_1', without reference to the other is mainly dependent upon the angle NA_1' ($= A_3 A_1'$): other factors being equal, the smaller the value of the angle NA_1' then the greater the sensitivity. However, the sensitivity is not wholly independent of the location of the second optic axis, A_2', with respect to the first, A_1'. If A_1' is always taken as the optic axis nearer to the wave-normal N ($= A_3$), then the location of optic axis A_2' with respect to A_1' can be described in terms of two independent variables: on the one hand, $2V$ and, on the other, the angle $A_2' A_1' N$. In general, the sensitivity increases with the value of $2V$ and reaches its maximum when $2V = 90°$. For any given value of $2V$ the angle $A_2' A_1' N$ ranges in value from a minimum, when $A_1' N = A_2' N$, to a maximum of 180° (provided that both $A_1' N$ and $A_2' N$ exceed 45° and angle $A_2' A_1' N$ is always measured so that $A_2' A_1' \leqslant 90°$), when N lies on the optic axial plane. The sensitivity is inversely related to the value of this angle and reaches its minimum, zero, when the latter reaches 180°. When angle $A_2' A_1' N = 180°$, the optic axes are not determinable individually by the use of coplanar extinction curves: all that is determinable is the plane of wave-normals, NN_n, containing both optic axes—the optic axial plane—which is, in any case in these circumstances, determinable orthoscopically.

Special cases

Special cases are not numerous and, in general, are not troublesome. They may be described in terms of the orientation of N ($= A_3$) with respect to the indicatrix.

N on the $\alpha\beta$ or the $\beta\gamma$ plane. The standard technique is equally sensitive or insensitive as the case may be for both optic axes. Sensitivity varies considerably according to the position of N within these planes:

low for $N = \beta$ ($NA'_1 = NA'_2 = 90°$) (this is also the special case of N on two circular sections); at its minimum, zero, for $N = \alpha$ or $N = \gamma$ (angle $A'_2 A'_1 N = 180°$); and higher for intermediate positions of N.

N on a circular section. Sensitivity differs for the two optic axes: low for A'_2 ($NA'_2 = 90°$) and higher for A'_1, that nearer to N.

N on the optic axial plane. Sensitivity is at its minimum, zero, since angle $A'_2 A'_1 N = 180°$. However, this difficulty can, in some cases, be overcome by the use of additional techniques (see Refinements, below).

Refinements

Refinements may be divided into at least three categories on the following basis: the use of time-saving considerations; the achievement of maximum accuracy with the standard technique as outlined above; and the employment of additional techniques to achieve increased accuracy where possible.

Time-saving considerations. This involves limiting where possible the area over which the ranging technique need be applied in the search for planes NA'_1 and NA'_2 In some cases, particularly where the angle NA_1 or NA'_2 does not greatly exceed the maximum practical tilt about A_4, the axis of rotation of the specimen, this limitation may be achieved as follows. With incident white light and with the specimen between crossed nicols, tilt about A_4 to the practicable limit of observation. Then rotate 360° about A_1, or as far as is necessary to achieve the desired result, noting the variation in relative retardation as displayed in terms of the colours of Newton's scale. While this is being done it will, in general, be necessary to rotate from time to time about A_5 to prevent the specimen passing into extinction. Throughout the operation, the tilt of the inner stage is uniform so that the effective specimen thickness may, in the case of a thin section at least, be considered constant. Thus, for each of the two wave-normals, say B_1 and B_2, of the conical wave-normal locus that are nearest to the two optic axes, A'_1 and A'_2 respectively, the relative retardation will pass through a minimum. These wave-normals, B_1, B_2, serve to define planes NB_1 and NB_2 which will, in turn, be very close to the required planes NA'_1 and NA'_2 respectively. The ranging technique need therefore only be applied in the immediate vicinity of planes NB_1 and NB_2.

There are obvious limitations to the information gained by the use of this refinement. For example, if either of the angles, NA'_1, NA'_2, is close to 90°, the associated minimum will, at best, be very broad—it may even be undetectable. If $2V$ is small, then only one minimum may, in general, be observable. If the angle $A_2' A_1' N$ is near 180° then there are two possible cases: two minima 180° apart, or only one observable minimum.

Maximum accuracy with the standard technique. If the two optic axes differ greatly in angular distance from N ($= A_3$), then the nearer, A_1', can, as a primary objective, be determined with greater accuracy by the use of the above procedure than can the farther, A_2'. Thus, in such a case, maximum accuracy is attainable if the plane of wave-normals, NN_q, containing the nearer optic axis, A_1', is determined as the first objective, and the plane H_q, containing the farther optic axis, A_2', determined by inference.

If the optic axes are approximately equidistant from N ($= A_3$) then each is determinable with about the same accuracy as a primary objective: the planes NN_q and H_q should be determined separately for each optic axis and the results, if not identical, averaged.

Use of additional techniques. These may be used, on the one hand, to confirm or increase the accuracy of determinations made by the standard technique or, on the other, to make determinations that by the standard technique alone would be impossible.

First of all, the accuracy of determinations made by the standard technique may be confirmed or increased without involving other experimental techniques. This is possible by utilizing a knowledge of the geometry of that special case of the coplanar extinction curves for which a search is being made, i.e. where the axis of rotation, P_n, falls on a circular section and where the wave-normal locus contains an optic axis. It can be shown (Tocher, 1965) that, in this special case, the points, E_n, F_n, at which the undifferentiable polar/equatorial curves touch each other and pass into the circular section, are determinable graphically: E_n and F_n lie on the circular section, $P_n\beta$, and bisect the acute and obtuse angles respectively between P_n and β. On the basis that each successive axis of rotation, $P_1, P_2, \ldots, P_n$, may be regarded as lying on a circular section until proved otherwise, a series of points, $E_1, E_2, \ldots, E_n$, and $F_1, F_2, \ldots, F_n$, can be plotted (fig. 6) to bisect the acute and obtuse angles between β and $P_1, P_2, \ldots, P_n$ respectively. Then, only when the axis of rotation, P_n, does lie on a circular section, i.e. only when the associated wave-normal locus contains an optic axis, will the associated extinction curves (no. 5, fig. 6) trend towards the E_n and F_n points of the same subscript.

Alternatively, determinations made by the standard technique may be confirmed or an increase in accuracy may be achieved by the use of conical extinction curves (Joel and Tocher, 1964) of large ψ. As in the standard technique, a deliberate search is made for the special case of the conical extinction curves where the wave-normal locus contains an optic axis (Tocher, 1965). It may be noted that, with conical extinction

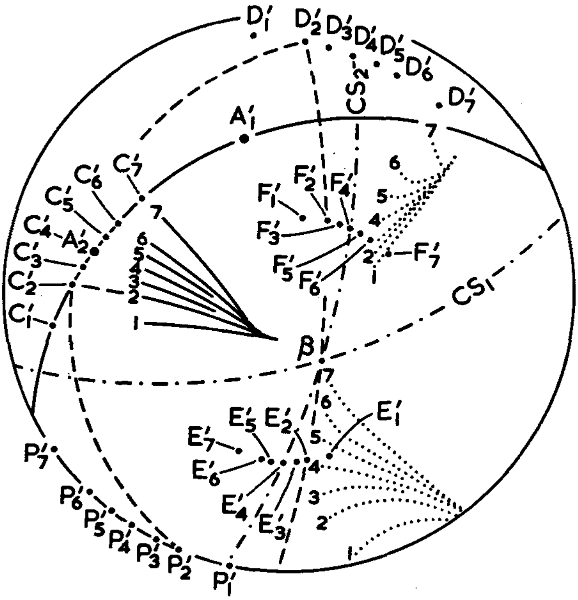


FIG. 7. Representative partial conical extinction curves (dotted) of large ψ. Curves 1, 2,..., 7 are associated respectively with axes of rotation $P'_1, P'_2, \ldots, P'_7$ and with conical wave-normal loci (continuous lines) 1, 2,..., 7 of fixed ψ (70°). Points $C'_1, C'_2, \ldots, C'_7$ are tentative positions of optic axis A'_2 on the optic axial plane. Points $D'_1, D'_2, \ldots, D'_7$ are points of intersection of planes $P'_n C'_n$ with planes polar to points C'_n. Points $E'_1, E'_2, \ldots, E'_7$ and $F'_1, F'_2, \ldots, F'_7$ bisect the acute and obtuse angles respectively between points D'_n and β. The geometrical relationship of points P'_2, C'_2, D'_2, E'_2, F'_2, is depicted. Wave-normal locus 4 contains optic axis A'_2, and only extinction curves 4 trend towards E'_n, F'_n points of the corresponding subscript.

curves, this special case does not, in general, involve the axis of rotation, P'_n, falling on the associated circular section (figs. 7 and 8).

There are two types of approach to the use of the conical extinction curves in this connexion. In one, for a predetermined value of ψ, a search is made for the axis of rotation, P'_n, such that the associated conical wave-normal locus contains an optic axis. In the second, for a preselected position of the axis of rotation, P'_m, a search is made for the value of ψ such that the associated conical wave-normal locus contains an optic axis.

In the former case a simple procedure is as follows. By rotating the tracing paper on the net, a pre-selected small circle, radius ψ, representing the conical wave-normal locus, is made to pass through the provisional position of the optic axis in question. The tentative position of the axis of rotation, say P'_1, associated with this conical wave-normal locus is then the common centre of the small circles on the primitive circle. By suitable rotation about A_1 this direction in the specimen is made to coincide with A_2 $(= P'_1)$, the axis of rotation (Joel and Tocher, 1964). Then, by rotation through the angle $90°-\psi$ to north or south as the case may be about A_4, the universal stage is set for the determination of this conical extinction curve. Only a small part of this need be determined —that part associated with rotation to the instrumental limit about A_2 in the direction necessary to bring the optic axis in question more nearly parallel to A_5, the microscope axis. Plotting is done as described by Joel and Tocher (*ibid.*) for the case where the axis of rotation is on the primitive circle. If the determined portion of the conical extinction curve shows terminal curvature in the same sense in both parts, then the wave-normal locus does not pass through the optic axis. In such a case, suitable minor adjustments, by rotation about A_1, must be made in the location of the axis of rotation, P'_n, until the plotted curves show terminal curvature simultaneously at a minimum and in opposite senses (curves 4, fig. 7). The associated conical wave-normal locus then includes the optic axis. A few representative partial conical extinction curves, nos. 1, 2,..., 7, of fixed ψ and variable axis of rotation, $P'_1, P'_2,..., P'_7$, are shown in fig. 7.

In the second type of approach, the pre-selected position of the axis of rotation, P'_m, is plotted on the primitive circle of the stereogram, and the angle ψ, between it and the supposed position of the relevant optic axis, is measured. The chosen axis of rotation, P'_m, of the specimen is made to coincide with A_2 by suitable rotation about A_1 and then the universal stage is finally set for the determination of the chosen conical extinction curve by rotation through the angle $90°-\psi$ to north or south as the case may be about A_4. If the determined small portion of the resultant conical extinction curve shows terminal curvature in the same sense in both parts, then suitable minor adjustments by rotation about A_4 must be made in the value of ψ until the plotted curves show terminal curvature simultaneously at a minimum and in opposite senses (curves 11, fig. 8). The associated conical wave-normal locus then includes the optic axis. A few representative partial conical extinction curves, nos. 8, 9,..., 14, with fixed axis of rotation, P'_m, and variable ψ are shown in fig. 8.

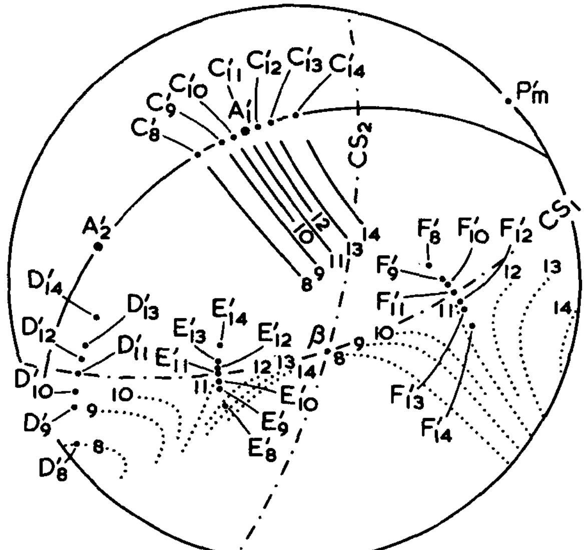


FIG. 8. Representative partial conical extinction curves 8, 9,..., 14 (dotted) associated respectively with conical wave-normal loci (continuous lines) 8, 9,..., 14 of variable large ψ (86° to 54° respectively) and fixed axis of rotation, P'_m. Points C'_n, D'_n, E'_n, F'_n, have same connotation as in fig. 7. Wave-normal locus 11 contains optic axis A'_1 and only curves 11 trend towards E'_n, F'_n points of the corresponding subscript.

As in the case of the standard technique based on coplanar extinction curves, these additional techniques are also capable of refinement on the basis of the geometry of the special case of the conical extinction curves where the wave-normal locus contains an optic axis. It can be shown (Tocher, 1965) that, in this special case also, the points, E'_n, F'_n, at which the two portions of the conical extinction curve touch each other and pass into the circular section are determinable graphically. The points E'_n and F'_n lie on the appropriate circular section (that whose pole is the optic axis contained by the wave-normal locus) such that they bisect the acute and obtuse angles respectively between β and the point D'_n. D'_n is determinable as the point of intersection of the circular section, on the one hand, and the plane through the axis of rotation and the relevant optic axis, on the other (figs. 7 and 8). This is, of course, a generalization of the construction given above for the graphical determination of points E'_n and F'_n in the case of coplanar extinction curves, where $\psi = 90°$. As before, on the basis that each successive conical wave-normal locus, 1, 2,..., n, may be regarded as containing an optic axis until proved otherwise, a series of points, E'_1, E_2,..., E'_n, and F'_1, F'_2,..., F'_n, can be plotted to bisect the acute and obtuse angles, $D'_1\,\beta$, $D'_2\,\beta$,..., $D'_n\,\beta$ (figs. 7 and 8). The successive tentative positions, D'_1, D'_2,..., D'_n, are determinable as the points of intersection of, on the one hand, the supposed circular sections polar to points C'_1, C'_2,..., C'_n and, on the other, the planes through points C'_1, C'_2,..., C'_n and the associated axes of rotation. The points C'_1, C'_2,..., C'_n, the tentative positions of the optic axis in question, are on the known optic axial plane where it is intersected by the successive conical wave-normal loci. The relation of one such set of points, P'_2, C'_2, D'_2, E'_2, F'_2, is depicted in fig. 7. As in the case of the coplanar extinction curves, only when the conical wave-normal locus contains an optic axis will the associated conical extinction curve (no. 4, fig. 7; no. 11, fig. 8) trend towards the E'_n and F'_n points of the same subscript.

It is clear, of course, that either of the two additional techniques based on large ψ conical extinction curves can be used alone for the initial location of an optic axis in cases where the standard technique is, for some reason, inapplicable. However, although both optic axes can be determined simultaneously by the standard technique, provided that the optic axial plane is known, it must be noted that each optic axis must, in general, be the subject of a separate determination when using conical extinction curves. Alternatively, if the nearer optic axis, A'_1, is determinable in this way, the farther, A'_2, may be located with respect to A'_1 by means of the Biot–Fresnel construction as advocated by Joel and Muir (1958*b*). In fact, this is potentially the more accurate method of locating A'_2 when the angle NA'_2 has a high value, approaching 90°.

One special case is incapable of solution by the simple standard technique—the case where N ($= A_3$) falls on or near the optic axial plane, i.e. where the angle $A'_2 A'_1 N \approx 180°$. However, depending upon the values of the angles NA'_1 and NA'_2 and upon whether the upper hemisphere projections of A'_1 and A'_2 are on the same or opposite sides of N ($= A_3$), a solution, partial or complete, may be possible by the use of additional techniques, either alone or in conjunction with the standard technique. For example, the optic axes may, in favourable circumstances, be determined separately by the use, in each case, of one or more large ψ conical extinction curves in conjunction with the determinable optic axial plane. Alternatively, a solution, partial or complete depending upon circumstances, may be possible by the use of the standard technique if a preliminary setting of the inner stage is made by suitable rotations about A_1 and A_2: before employing the standard technique, the optic axial plane is set parallel to A_2 by rotation about A_1 and then tilted to the right or left by rotation about A_2 in order to increase its angular distance from N ($= A_3$). With the 4-axis universal stage, this now limits the axial combination for coplanar extinction curves to A_3, A_4 ($= P_n$), A_5. In such a case, plotting may proceed in the normal fashion and the optic axis or axes so determined can eventually be rotated to the proper orientation with respect to the plane and zero point of the inner stage (Joel and Muir, 1958*a*, pp. 873–874).

Finally, it is clear that, should the optic axial plane be, for any reason, indeterminable or determinable with insufficient accuracy, it may still be possible to use a combination of the standard and additional experimental techniques to locate the optic axes. The problem reduces then to that of finding, in the case of each optic axis, a series of wave-normal loci, both planar and conical, each of which contains the optic axis concerned. The optic axis then lies at the common intersection of the several wave-normal loci. A final check on the accuracy can, of course, be made by confirming that the several special extinction curves do trend towards the relevant E_n, F_n and E'_n, F'_n points. In this case, these points will, of course, have to be plotted in terms of the determined position of each optic axis and of the position of β deduced therefrom.

Other uses

Although this technique was originally designed to meet the case where both optic axes are beyond the normal tilting range of the universal stage, it is clear that it may be used with profit in other circumstances. For example, there occur cases where at least one optic axis may be within the normal tilting range of the universal stage but is nevertheless inaccessible due to some inhibiting factor, optical or physical, peculiar to the specimen, e.g. neighbouring grains or lamellae overlapping with that under investigation. It may be possible to locate that optic axis with a fair degree of accuracy by the use of the standard technique or the additional techniques outlined above. Among the additional techniques may now be numbered, for this purpose, small ψ conical extinction curves (Joel and Tocher, 1964). These may be determined and plotted with a fixed axis of rotation, P'_n ($= A_1$ or A_3), and with variable ψ. Alternatively, ψ may be fixed in value and the axis of rotation, P'_n ($= A_3$), treated as the variable—this can be achieved by varying the preliminary setting of the inner stage by suitable rotations about A_1 or A_2 or both while A_3 remains the axis of rotation.

Further, this extinction curve technique may be used with profit to confirm or refine determinations made by orthoscopic or conoscopic methods, especially in those cases where an optic axis, although directly observable, is near the limit of tilt of the universal stage.

Uniaxial case

There are cases where the orientation of the optic axis of a uniaxial crystal is determinable with only marginal accuracy by orthoscopic or conoscopic methods or by the use of general conical extinction curves (Joel and Tocher, 1964), e.g. where the optic axis and circular section are near the limit of tilt of the universal stage. In such cases, the new technique, using the special case of coplanar or conical extinction curves, may be used to locate the single optic axis.

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