

*A test of the discriminant function in the amphibolite problem*

By D. M. SHAW and A. M. KUDO<sup>1</sup>

McMaster University, Hamilton, Ontario

---

NUMEROUS authors have attempted to use chemical composition as a means of distinguishing amphibolites derived by isochemical metamorphism of basic igneous intrusives or flows from those of sedimentary parentage. In recent years contributions have come from Engel and Engel (1951, 1953, 1962), Lapadu-Hargues (1953, 1958), Poldervaart (1953), Eckelmann and Poldervaart (1957), Wilcox and Poldervaart (1958), Evans and Leake (1960), and Walker, Joplin, Lovering, and Green (1960).

Evidence has also been presented that some amphibolites have not retained the composition of their parent rocks, but are of metasomatic origin. Notable contributions have come from Adams and Barlow (1910), Hewitt and James (1955), and Tilley (1957) in addition to the authors mentioned previously.

It is, however, generally admitted that regional metamorphism is not uncommonly isochemical, at least to a first approximation, excepting mobile components such as H<sub>2</sub>O and CO<sub>2</sub>, and excluding migmatite zones. This has been demonstrated in a few cases (e.g. Shaw, 1954*a, b*, 1956; Schwarcz, 1960). In some cases, therefore, it may be expected that ortho- and para-amphibolites will inherit chemical differences of diagnostic value from their parent rocks. The universality of any proposed diagnostic tests of this kind might, however, be vitiated by regional differences in composition of rocks of a particular type, as has been demonstrated in the case of basalts for Zr (Chao and Fleischer, 1960) and Sr (Turekian and Kulp, 1956). In addition, the lack of natural compositional boundaries between rock types may be expected to lead to overlap and blurring of any chemical differences, which would therefore be more difficult to detect.

In making comparisons of chemical composition, petrologists are

<sup>1</sup> Present address: University of California, San Diego.

accustomed to treating one, two, or at the most three elements at once, using qualitative graphical methods or simple averages. If  $n$  elements have been determined on each sample, a comparison will be much more powerful if it can (a) take into account the ranges of values encountered, and (b) use all the elements at once. In other words, it is desirable to search for the separation between clusters of points in  $n$ -dimensional space.

The methods of multivariate statistical analysis are suitable here, and the discriminant function has been developed specifically for this purpose (see, e.g., Rao, 1952). Details of the technique will not be given here, other than to state that the method gives the coefficients of the best linear combinations of the  $n$  variables to discriminate between the two (or more if necessary) clusters of points, together with a measure of their degree of overlap. Recent geological articles which discuss the procedure more fully include Middleton (1962) and Potter, Shimp, and Witters (1963).

As a prelude to a more extended study of amphibolites it was decided to try the value of this approach in a limited area where undoubted ortho- and para-amphibolites occur. Chandos township in the Grenville province of south-eastern Ontario was suitable and had been mapped and sampled by one of us (Shaw, 1962). Analyses and computations were carried out by the second author and have been recorded in a thesis (Kudo, 1962).

We are glad to acknowledge much advice and assistance on statistical matters from G. V. Middleton, McMaster University, whose programmes were used on a Bendix G-15 computer. It is a pleasure and a privilege to dedicate this article to Professor C. E. Tilley.

### *Samples*

Twenty-nine rocks (ortho-amphibolites) were collected from bodies recognized in the field as having intrusive igneous origins, including the Duck Lake metagabbro, the Tallan Lake metagabbro, the Glen Alda metagabbro, the Loon Lake complex, and various smaller dykes and sills. Most of these were massive or faintly foliated rocks, mapped as metagabbro, diorite, hornfels, or amphibolite. All except one contain hornblende and plagioclase as essential constituents, other major minerals including biotite, quartz, and garnet, and less commonly augite or hypersthene. Accessory constituents usually include magnetite, apatite, sphene, calcite, and pyrite. The one rock containing no hornblende was a noritic hornfels, an inclusion in the Loon Lake granite.

One para-amphibolite (69-28-5) was later found to have been erroneously included in this group.

Twenty-one rocks (para-amphibolites) were collected from strata intercalated in layered meta-sedimentary sequences correlative to the Mayo Group and the Hermon Formation. Fourteen were feather amphibolites from the Dungannon Formation, the remainder miscellaneous banded amphibolites and calc-silicate rocks; all were clearly of non-intrusive origin. Hornblende and plagioclase were essential constituents of all; quartz, biotite, and calcite were common major minerals, while microcline, augite, scapolite, clinozoisite, and chlorite were major constituents of some. Accessory minerals usually included sphene, magnetite, and one or more of the minerals included among major constituents: occasional apatite, zircon, and tourmaline were noted. Whether or not these rocks represent tuffaceous sediments is not apparent.

A further 26 amphibolites were collected and analysed. Although for some of these the field relationships indicated clearly either an intrusive or a metasedimentary origin, the majority could not be classed in either category. For purposes of testing, all the individuals in this group were treated as having uncertain origins. In mineral composition they resembled the previous groups.

Sample numbers, rock names, and generalized localities are shown for the first two groups of rocks in tables I and II. Detailed localities are listed by Kudo (*op. cit.*).

#### *Analysis*

All the samples were analysed spectrographically in triplicate for Cr, V, Ni, Co, Sc, Sr, Ba, Zr, using a Stallwood jet technique described elsewhere (Shaw, 1960). Precision was estimated from a single-factor analysis of variance which showed that the variation between samples was much greater than the analytical errors of replication. The estimates were mostly satisfactory, except perhaps for Zr, as presented by the per cent. relative deviation (*C*) in table III.

Accuracy was judged by quadruplicate analysis of the standard diabase W-1 and by triplicate analysis of four analysed amphibolite powders kindly supplied by A. E. J. Engel (table III). Results for W-1 are generally satisfactory with respect to the recommended values, except for Ni and Sc which are low.

This same relation holds for our results on Engel's samples: with other elements there is general conformity of results for V, Co and Zr, but Engel's results are higher for Cr, Sr, and Ba.

TABLE I. Ortho-amphibolite samples

Number	Rock name	Locality*
68-75-6	Garnet metagabbro	T.L.
68-76-3	Metagabbro	T.L.
69-28-2	"	D.L.
69-28-5†	Augite feather amphibolite	(D.L.)
69-29-5	Garnet amphibolite gneiss	D.L.
69-30-2	" " "	D.L.
69-30-6	Augite amphibolite	D.L.
69-31-1	Garnet biotite amphibolite	D.L.
69-31-2	Garnet amphibolite	D.L.
69-34-3	Biotite hornblende diorite	G.A.
69-34-7	Diorite	G.A.
69-35-7	Hypersthene hornblende gabbro	G.A.
70-143-5e	Hypersthene hornblende hornfels	L.L.
70-148-1	Amphibolite gneiss	L.L.
70-148-3	Diorite	L.L.
71-46-9	Biotite garnet amphibolite	D.1
71-47-2	Metadiabase	D.1
71/2-178-1	Amphibolite	D.1
71/2-178-7	Metagabbro	D.1
71/2-180-1	Amphibolite	D.2
71/2-181-3	Diorite	D.2
71/2-184-3	Hypersthene augite hornfels	L.L.
74-228-2	Amphibolite	D.2
74-228-4	Metadiabase	D.2
TL-61-2	Metagabbro	T.L.
TL-61-2	"	T.L.
TL-61-3	"	T.L.
TL-61-4	"	T.L.
TL-61-5	"	T.L.
TL-61-6	"	T.L.
TL-61-7	"	T.L.

T.L. Tallan Lake metagabbro.

D.L. Duck Lake metagabbro.

G.A. Glen Alda metagabbro.

L.L. Loon Lake pluton

D.1. Metadiabase area at west end Con VII.

D.2. Other miscellaneous metadiabase intrusions.

\* See Shaw (1962).

† Amphibolite xenolith in Duck Lake metagabbro, included erroneously with ortho-amphibolites.

The validity of the later discussion is not influenced by considerations of accuracy. For any comparisons with data from other laboratories it will be necessary to correct our Ni and Sc values by a factor of 1.5.

Eleven of the rocks in each group were also analysed for major constituents by rapid methods and the results are presented in table IV. Average trace element values for 20 amphibolites in each group are in

TABLE II. Para-amphibolite samples

Number	Rock name	Unit*
67-171-1	Feather amphibolite	D.F.
67-74-16	Augite feather amphibolite	H.F.
68-77-1	Leucocratic feather amphibolite	D.F.
68-78-1	Feather amphibolite	D.F.
68-81-3	Chlorite feather amphibolite	L.G.A.
68-81-10	Biotite feather amphibolite	L.G.A.
68-81-11a	Garnet biotite feather amphibolite	L.G.A.
68-81-11b	Hornblende plagioclase marble	L.G.A.
69-27-2	Biotite feather amphibolite	D.F.
69-32-1	Feather amphibolite	L.G.A.
70-139-1	" "	D.F.
70-143-2	Biotite hornblende gneiss	D.F.
70-143-8	Scapolite epidote augite hornblende gneiss	U.G.A.
70-145-9	Scapolite epidote amphibolite	U.G.A.
70-146-6	Biotite scapolite augite amphibolite	U.G.A.
71-46-7	Calcite hornblende gneiss	W.H.
71/2-178-5	Feather amphibolite	W.H.
71/2-178-8	Epidote feather amphibolite	W.H.
72-194-3	Biotite feather amphibolite	W.H.
73-110-1	Biotite augite hornblende gneiss	L.A.
FA-102-61	Feather amphibolite	D.F.
	D.F. Dungannon formation	
	H.F. Hermon formation	
	U.G.A. Upper Glen Alda amphibolite	
	L.G.A. Lower Glen Alda amphibolite	
	W.H. Wolfe Hill amphibolite	
	L.A. Lasswade amphibolite	

\* See Shaw, 1962, p. 21.

TABLE III. Precision and accuracy of amphibolite spectrographic analyses

	Precision C %	A 8		A 16		A 17		AE 326		W 1	
		ppm		ppm		ppm		ppm		ppm	
		A	E	A	E	A	E	A	E	A	B
Cr	7.9	118	350	155	470	52	128	49	150	114	120
V	7.1	897	530	378	350	647	505	402	430	217	240
Ni	6.5	53	52	46	64	35	57	37	46/60	53	80
Co	4.1	63	46	41	40	50	95	44	46	35	38/51
Sc	11.7	20	67	18	44	20	58	20	58	21	33
Sr	11.3	141	165	166	400	125	180	111	240	194	220/175
Ba	9.8	117	150	154	640	152	700	67	140	202	225
Zr	14.3	181	140	91	51	136	114	79	150	92	100

C. Ratio of standard deviation to mean in per cent., converted from logarithms.

A. This study.

B. Fleischer and Stevens (1962).

E. Engel and Engel (1962).

TABLE IV. Major element analysis of twenty-two Chondros amphibolites

	SiO <sub>2</sub>	TiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>2</sub> O <sub>3</sub>	FeO	MnO	MgO	CaO	Na <sub>2</sub> O	K <sub>2</sub> O	P <sub>2</sub> O <sub>5</sub>	H <sub>2</sub> O	CO <sub>2</sub>	Ign	Sum
Ortho-amphibolites															
69-28-2	50.5	3.03	14.5	3.0	13.85	0.32	2.6	6.3	4.58	0.44	1.31	0.88	0.06	1.63	101.4
69-28-5	48.5	0.94	13.2	0.72	4.83	0.07	3.8	16.8	2.75	1.33	0.34	0.73	6.95	7.74	101.0
69-29-5	42.0	4.88	13.0	3.5	16.54	0.38	4.8	8.6	2.71	0.52	2.43	1.53	0.96	2.64	101.9
69-30-2	53.3	1.92	13.8	3.0	12.10	0.29	3.6	6.6	3.44	0.40	0.43	0.97	0.74	2.16	100.7
69-30-6	48.7	1.47	15.7	1.8	9.53	0.18	6.8	10.4	2.69	0.32	0.09	1.19	0.49	1.84	99.5
69-34-7	50.0	2.15	15.7	5.5	5.23	0.11	3.6	7.0	3.67	3.25	1.48	0.82	0.33	1.47	98.8
69-35-7	48.5	1.86	15.1	2.2	10.46	0.19	7.1	9.5	2.84	0.20	0.16	1.19	0.67	1.72	100.0
TL-61-3S	47.1	2.75	13.0	3.8	16.18	0.37	3.3	6.4	2.92	1.43	1.52	1.36	0.00	2.13	100.1
TL-61-4S	46.0	4.36	13.0	3.9	14.15	0.32	4.3	8.6	3.10	0.50	2.33	1.20	0.04	1.60	101.8
TL-61-6S	47.5	5.04	13.6	3.5	13.68	0.29	5.2	7.8	2.82	0.71	0.54	1.19	0.75	1.61	102.6
TL-61-7S	46.1	5.21	14.2	3.8	13.88	0.32	4.8	7.9	2.96	0.51	0.39	0.70	0.00	1.58	100.8
Para-amphibolites															
67-171-1	61.5	0.27	4.2	0.17	1.67	0.25	17.4	9.8	1.11	1.61	0.06	1.41	0.02	0.53	99.5
68-74-16	48.7	1.60	15.6	3.3	8.74	0.20	5.4	10.3	3.76	0.26	0.15	1.19	0.67	2.01	99.9
68-77-1	42.7	0.53	10.9	0.57	3.48	0.06	2.8	22.8	2.64	0.68	0.13	12.98	0.57	13.40	100.8
68-78-1	60.2	0.80	14.9	0.76	5.72	0.02	4.6	5.4	4.29	0.91	0.13	0.34	1.02	1.60	99.1
68-81-10	62.7	0.91	15.4	2.1	4.58	0.10	2.6	4.4	4.19	1.98	0.26	0.05	0.78	0.91	100.1
68-81-11b	39.8	0.32	7.9	0.66	2.64	0.08	1.9	24.9	1.64	0.41	0.05	18.70	0.45	18.69	99.5
69-27-2	58.8	0.76	15.6	0.88	5.69	0.04	4.5	5.8	5.31	0.45	0.19	1.22	0.94	2.23	100.2
69-32-1	56.5	0.93	16.4	2.9	4.30	0.15	2.9	7.5	4.88	0.89	0.27	1.10	0.69	1.73	99.4
70-139-1	57.3	0.78	13.5	0.75	3.43	0.05	3.4	13.1	2.64	0.33	0.21	0.69	3.84	4.65	100.0
70-146-6	55.5	0.76	15.8	2.1	4.55	0.07	4.3	10.1	2.04	2.24	0.22	0.63	1.09	1.77	99.4
71/72-178-8	61.4	0.78	15.8	1.6	4.27	0.13	2.4	8.2	3.58	0.53	0.16	0.68	0.38	1.77	99.9

Analyst: J. Muysson, Department of Geology, McMaster University.

tables V and VI. Where values of 'tr' or '\*' appear it was necessary to assign arbitrary numbers for purposes of calculation: the numbers chosen were usually one-half and one-tenth respectively of the appropriate analytical sensitivity limit (see Kudo, *op. cit.*, pp. 48-50).

### *Discriminant function analysis*

A comparison of the arithmetic means of most of the elements determined for each group (table VII) suggests a marked difference in average composition, in the cases of V, Ni, Co, Sc, Ba, Zr, TiO<sub>2</sub>, FeO, P<sub>2</sub>O<sub>5</sub>, and CO<sub>2</sub>. The higher Ti value in ortho-amphibolites is in accord with the conclusions of Lapadu-Hargues (1958). Inspection of tables IV, V and VI shows, however, that there is considerable overlap in values for most elements, and single-variate comparisons would be more efficiently made using the Student *t*-function. These were not made, since the multi-variate procedure is more powerful and would in any case include single-variate analysis as a limiting case. The multi-variate analogue (analysis of dispersion) was in fact carried out on the data for 8 trace elements and showed a highly significant difference between the variate means, indicating that the two point-clusters in 8-space were distinct and that discriminant function analysis would consequently be profitable.

Separate computations were made with the trace element data and the major element data. In each case analyses were first converted to logarithms (base 10).

The discriminant function  $X_1$  calculated for 8 trace elements is as follows (values in ppm):

$$X_1 = -2.69 \log \text{Cr} - 3.18 \log \text{V} - 1.25 \log \text{Ni} + 10.57 \log \text{Co} + \\ + 7.73 \log \text{Sc} + 7.54 \log \text{Sr} - 1.95 \log \text{Ba} - 1.99 \log \text{Zr} - 19.58. \quad (1)$$

For an amphibolite of unknown affinities, an analysis will permit the calculation of  $X_1$ : this quantity will be positive for an ortho-amphibolite and negative for a para-amphibolite. It is possible to demonstrate from the data used that the probability of incorrect classification is 5.4 %, which is reasonably small. It is also possible to ascertain which elements contribute most effectively towards discrimination, and in the present case Co and Sc are clearly the most important. A second discriminant function ( $X_2$ ), based on these elements alone follows:

$$X_2 = 3.89 \log \text{Co} + 3.99 \log \text{Sc} - 8.63. \quad (2)$$

TABLE V. Trace element content of 20 ortho-amphibolites (ppm)

Sample number	Cr	V	Ni	Co	Sc	Sr	Ba	Zr
68-76-3	17	tr.	4	11	13	402	302	317
69-28-2	tr.	20	10	26	19	319	146	420
69-29-5	23	376	tr.	10	66	426	276	142
69-30-2	23	143	54	24	49	633	197	575
69-30-6	446	263	39	38	50	189	91	151
69-31-1	tr.	292	18	35	21	185	66	502
69-31-2	tr.	275	34	32	30	182	83	453
69-34-3	tr.	349	47	37	*	2400	5420	326
69-34-7	tr.	207	53	32	*	3500	6500	193
69-35-7	242	362	159	44	55	264	73	190
70-148-1	tr.	237	20	27	*	455	231	262
71/72-178-7	72	257	48	37	tr.	277	98	61
71/72-181-3	*	721	25	50	42	610	513	459
74-228-2	233	335	49	45	56	240	66	190
TL-61-2S	16	tr.	7	18	25	206	576	1130
TL-61-3S	tr.	52	90	56	26	82	220	264
TL-61-4S	tr.	287	53	66	43	182	127	340
TL-61-5S	tr.	24	40	19	44	263	320	1070
TL-61-6S	tr.	346	56	81	43	162	270	255
TL-61-7S	tr.	472	81	75	42	115	171	416

\* Not found.

TABLE VI. Trace element content of 20 para-amphibolites (ppm)

Sample number	Cr	V	Ni	Co	Sc	Sr	Ba	Zr
67-171-1	21	41	8	8	*	1240	300	103
68-74-16	150	433	46	41	26	240	46	222
68-77-1	44	72	26	11	tr.	584	165	128
68-81-3	*	265	12	25	*	274	247	200
68-81-10	37	101	21	13	*	569	431	166
68-81-11a	*	320	38	22	*	208	105	103
68-81-11b	tr.	48	52	10	*	201	97	84
69-27-2	115	128	60	15	19	373	260	183
69-32-1	37	101	14	16	16	440	446	256
70-139-1	*	136	11	10	*	192	161	23
70-143-2	tr.	113	13	15	*	493	205	161
70-143-8	44	118	23	23	*	244	tr.	229
70-145-9	34	124	27	23	*	325	255	186
70-146-6	tr.	133	23	3	tr.	1250	345	186
71-46-7	*	28	7	*	*	154	tr.	180
71/72-178-5	*	57	7	8	*	358	152	214
71/72-178-8	30	94	9	10	tr.	676	258	10
72-194-3	*	23	6	*	*	620	524	211
73-110-1	20	108	12	14	*	273	364	228
FA-102-61	34	123	14	14	tr.	237	220	254

\* Not found.

The probability of misclassification is now 11.9 %, so it is preferable to use equation (1) which included all 8 elements.

To test equation (1) a number of amphibolites and related rocks of known origins were classified. Table VIII includes 2 known para-amphibolites and 8 known ortho-amphibolites (taken from tables I and II, but not used to calculate the d.f.): all were assigned correctly by the

TABLE VII. Average composition of each amphibolite group

	8 trace elements (ppm)		9 oxides (wt. %)		
	Ortho	Para		Ortho	Para
Cr	55	33	TiO <sub>2</sub>	3.06	0.77
V	246	135	Al <sub>2</sub> O <sub>3</sub>	14.07	13.27
Ni	43	21	Fe <sub>2</sub> O <sub>3</sub>	3.16	1.44
Co	37	14	FeO	11.86	4.46
Sc	30	5	MnO	0.26	0.10
Sr	595	449	MgO	4.54	4.75
Ba	785	323	CaO	8.72	11.12
Zr	376	181	P <sub>2</sub> O <sub>5</sub>	1.00	0.17
			CO <sub>2</sub>	1.00	3.67
<i>n</i>	21*	21†		11	11

\* Including sample 69-28-5, see table VIII.

† Including sample 68-78-1, see table VIII.

discriminant function. Also included were an analysis of diabase W-1 carried out in this laboratory, and analyses of two other rocks from the literature. In these the figures for Ni and Sc were divided by 1.5 to adjust for our systematic error: in addition, the average carbonate rock data were multiplied by the factor of 1.67, assuming that about 40 % carbon dioxide would be lost during metamorphism of such a rock. Otherwise it is assumed that the values are meaningful in relation to our own, which is not by any means certain in the case of trace element studies.

In spite of these uncertainties, the discriminant function  $X_1$  assigns these three rocks correctly. In addition to the rocks reported in table VIII values of  $X_1$  were also calculated for our analyses of A. E. J. Engel's rocks (table III), giving the following results:

A 8:	-0.51 para
A 16:	-0.42 para
A 17:	+0.26 ortho
AE 326:	+1.15 ortho

These values of  $X_1$  are so close to zero that there is little confidence in the discrimination.

In the case of major element data the following discriminant function ( $X_3$ ) was obtained (wt. %):

$$X_3 = 7.07 \log \text{TiO}_2 + 1.91 \log \text{Al}_2\text{O}_3 - 3.29 \log \text{Fe}_2\text{O}_3 + 8.48 \log \text{FeO} + \\ + 2.97 \log \text{MnO} + 4.81 \log \text{MgO} + 7.80 \log \text{CaO} + \\ + 3.92 \log \text{P}_2\text{O}_5 + 0.15 \log \text{CO}_2 - 15.08. \quad (3)$$

TABLE VIII. Classification of several rocks by means of the discriminant function  $X_1$

Sample	Cr	V	Ni	Co	Se	Sr	Ba	Zr	G	Classification	
										$X_1$	
68-78-1	124	267	17	16	tr.	468	2160	140	Para	-6.97	Para
69-28-5	75	150	17	16	tr.	1420	754	174	„	-1.25	„
70-143-5e	142	599	51	50	39	115	n.d.	238	Ortho	3.80	Ortho
70-148-3	354	163	54	30	28	354	98	65	„	2.89	„
71-46-9	71	287	30	29	24	291	95	203	„	2.03	„
71-47-2	125	440	28	48	37	151	n.d.	193	„	5.41	„
71/72-178-1	102	267	100	43	28	84	tr.	97	„	1.52	„
71/72-180-1	tr.	622	18	44	11	139	21	118	„	4.37	„
71/72-184-3	211	394	40	58	36	369	n.d.	235	„	8.29	„
74-228-4	106	341	58	52	28	79	n.d.	163	„	3.01	„
Standard dia- base W-1*	114	217	53	35	21	194	202	92	—	6.89	Ortho
Average On- tario diabase†	220	220	110	39	37	300	280	100	—	2.98	„
Average On- tario diabase (corrected)	220	220	73	39	25	300	280	100	—	1.89	„
Average car- bonate rock‡	9	15	12	4.3	0.3	475	150	17	—	-11.09	Para
Average car- bonate rock (corrected)	15	25	13	7.2	0.3	793	251	28	—	-9.26	„

\* This study.

† Fairbairn, Ahrens, and Gorfinkle, 1953, table 3.

‡ Graf, 1960, table 38.

G. Geological evidence.

The probability of incorrect classification is 5.7 %. The most effective oxides for discrimination are  $\text{TiO}_2$ ,  $\text{FeO}$ , and  $\text{P}_2\text{O}_5$ : it was not expected that  $\text{CaO}$  and  $\text{CO}_2$  would turn out to be of negligible importance, yet this is apparently the case.

To test the function  $X_3$ , a number of analyses from the literature have been used (table IX), in the hope that major element analyses are nowadays comparable.

*Discussion*

In assessing the value of discriminant function analysis for resolving the problem of amphibolite classification, several points should be mentioned.

TABLE IX. Classification of several rocks by means of the discriminant function  $X_3$ 

Sample	Classification		
	$G$	$X_3$	
W-1	Ign	0.85	Ortho
Average basic rock*	Ign	3.05	Ortho
Average of three Roan amphibolites, North Carolina†	Para	-0.48	Para
Connemara amphibolites‡			
BL 2518	Ign	5.76	Ortho
BL 763A	"	1.19	"
BL 2404	"	5.97	"
BL 1092	"	4.75	"
BL 2494	"	1.74	"
BL 1310	"	-0.78	Para
BL 165	"	0.45	Ortho
BL 229	"	-0.66	Para
BL 2506	"	-1.28	"
BL 2503	"	-6.72	"
Adirondack amphibolites§			
A 8	Ign ?	3.83	Ortho
AE 326	Ign ?	2.47	"

\* Nockolds, 1954, p. 1032, no. V.

† Wilcox and Poldervaart, 1958, p. 1351.

‡ Evans and Leake, 1960, p. 346.

§ Engel and Engel, 1962, pp. 47 and 58.

$G$ . Geological evidence.

Certain assumptions are necessary to the statistical procedure, which restrict its applicability:

- (a) the populations are multivariate normal;
- (b) the populations have identical dispersion matrices;
- (c) the discriminant functions are normally distributed with equal variances in each group;
- (d) the *a priori* probability of an individual belonging to any group must be known;
- (e) every individual must belong to one of the stipulated groups.

The first three assumptions are similar to many statistical procedures where normality is assumed: natural distributions commonly deviate

from normality, but in many cases the logarithms of abundance approximate more closely to this requirement. With respect to (d), it has been assumed that the probabilities are equal that a given amphibolite might belong to the ortho or the para groups: this does not necessarily conform to reality. If, for example, basic igneous flows and intrusions were known to be ten times more common than sediments which can metamorphose to similar assemblages, then this would necessitate a significant modification to the arithmetic.

Perhaps the most serious restriction is (e), since the present analysis has only considered two alternatives. Any analysed rock, for example a granite, would have to be classified in either the ortho or the para group. Obviously one would not attempt such a meaningless assignment, but in practice quite arbitrary decisions must be made concerning how much hornblende is necessary to make a rock worth classifying. Future development should allow for a third group, to accommodate rocks which are not clearly assignable to either of the two major categories.

Some points should also be considered on geological grounds. First of all there is the question of the meaning of 'para-amphibolite'. The term is commonly equated to sedimentary rocks, yet Engel has pointed out on several occasions (e.g. 1962, pp. 75-78) the difficulty of finding unmetamorphosed rocks of non-igneous origin which chemically resemble basalts. It might well be that para-amphibolites can only form from tuffaceous sediments which have undergone considerable chemical re-working. If this is the case then such tuffs might be unlikely to constitute a sharply defined compositional group, rather being transitional to unaltered basalts, or ortho-amphibolites.

Secondly, the empirical success of the present study cannot safely be extrapolated to amphibolites in other areas and of other ages. Mention has already been made of the existence of regional variations in basalt composition.

Thirdly, the possibility of metasomatic amphibolites has here been ignored, although such rocks are widespread within the Grenville province itself. There seems no *a priori* reason why metasomatic rocks should match in composition either of the two groups used.

Notwithstanding these and other criticisms, we feel that the potentialities of discriminant function analysis are particularly suited for the type of problem presented by the amphibolites. The present article is a preliminary test of the method and the results are sufficiently encouraging to merit a more thorough study.

*References*

- ADAMS (F. D.) and BARLOW (A. E.), 1910. Geol. Surv. Canada, Mem. 5.
- CHAO (E. C. T. R.) and FLEISCHER (M.), 1960. Int. Geol. Congr. XXI, Proc. Part 1, p. 107.
- ECKELMANN (F. D.) and POLDERVAART (A.), 1957. Bull. Geol. Soc. Amer., vol. 68, p. 1225.
- ENGEL (A. E. J.) and ENGEL (C. G.), 1951. *Ibid.*, vol. 62, p. 1435.
- — 1953. *Ibid.*, vol. 64, p. 1013.
- — 1962. *Ibid.*, A. F. Buddington vol., p. 37.
- EVANS (B. W.) and LEAKE (B. E.), 1960. Jour. Pet., vol. 1, p. 337.
- FAIRBAIRN (H. W.), AHRENS (L. H.), and GORFINKLE (L. G.), 1953. Geochim. et Cosmochim. Acta, vol. 3, p. 34.
- FLEISCHER (M.) and STEVENS (R. E.), 1962. *Ibid.*, vol. 26, p. 525.
- GRAF (D. L.), 1960. Illinois State Geol. Surv., circular 301, p. 5.
- HEWITT (D. F.) and JAMES (W.), 1955. Ontario Dept. Mines, Ann. Rept., vol. 64, pt. 8, p. 1.
- KUDO (A. M.), 1962. M.Sc. thesis (unpublished), McMaster University.
- LAPADU-HARGUES (P.), 1953. Bull. Soc. Géol. France, ser. 6, vol. 3, p. 153.
- 1958. C.R. Somm. Soc. Géol. France, vol. 8, p. 132.
- MIDDLETON (G. V.), 1962. Proc. Roy. Soc. Canada, vol. 56, sect. iii, p. 119.
- NOCKOLDS (S. R.), 1954. Bull. Geol. Soc. Amer., vol. 65, p. 1007.
- POLDERVAART (A.), 1953. *Ibid.*, vol. 64, p. 259.
- POTTER (P. E.), SHIMP (N. F.), and WITTERS (J.), 1963. Geochim. et Cosmochim. Acta, vol. 27, p. 669.
- RAO (C. R.), 1952. Advanced statistical methods in biometric research. New York: John Wiley & Sons, Inc.
- SCHWARCZ (H. P.), 1960. Unpublished Ph.D. Thesis, California Institute of Technology.
- SHAW (D. M.), 1954 a. Bull. Geol. Soc. Amer., vol. 65, p. 1151.
- 1954 b. *Ibid.*, vol. 65, p. 1167.
- 1956. *Ibid.*, vol. 67, p. 919.
- 1960. Canadian Min., vol. 6, p. 467.
- 1962. Ontario Department of Mines, Geol. Rep. No. 11.
- TILLEY (C. E.), 1957. Quart. Journ. Geol. Soc., vol. 113.
- TUREKIAN (K. K.) and KULP (J. L.), 1956. Geochim. et Cosmochim. Acta, vol. 10, p. 245.
- WALKER (K. R.), JOPLIN (G. A.), LOVERING (J. F.), and GREEN (R.), 1960. Jour. Geol. Soc. Australia, vol. 6, p. 149.
- WILCOX (R. E.) and POLDERVAART (A.), 1958. Bull. Geol. Soc. Amer., vol. 69, p. 1323.
-