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Types of distribution of the minerals of the Meldon Aplite, Devonshire

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SYNOPSIS

A STATISTICAL study on the modal analysis data of eight minerals of the Meldon aplite is made with a view to determining their distribution patterns in the rock. Various parameters and statistics have been calculated and the results are tabulated.

Topaz and elbaite are considered as censored samples; 'normalization' in the case of these two minerals is obtained by double probit transforms. Findings of the statistical study are correlated with the modes of formation of the minerals.

TYPES OF DISTRIBUTION OF THE MINERALS OF THE MELDON APLITE, DEVONSHIRE, ENGLAND

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THE Meldon aplite is a sodium and lithium-rich aplite dyke about 20 to 25 m thick and three kilometres long which outcrops in Devonshire about a kilometre to the northwest of the Dartmoor granite. A summary of the petrology of the dyke is given by Worth (1920) and aspects of the mineralogy have been described by Chaudhry and Howie (1970, 1973) and Chaudhry (1971). The intrusion is heterogeneous in colour, grain-size and mineralogy and there are pegmatitic patches and veins. However, the intrusion is essentially an aplite and the variability in its composition and grain-size is the result of secondary processes such as recrystallization, autometasomatism and greisenization which have played their roles to various degrees. About 92% of the intrusion is aplite, 6% pegmatite, 1.5% greisen and about 0.5% xenolithic material. The aplites contain albite, quartz, orthoclase, and lepidolite as essential minerals, while elbaite, topaz, fluorite, apatite, and petalite are accessories. The pegmatite veins are between 2 and 7 cm thick. There are two kinds, one made of orthoclase, quartz, lepidolite, albite, elbaite, and the other of orthoclase, quartz, lepidolite, albite, topaz, and petalite.

The chilled marginal facies of the intrusion is flow textured and blue in colour. It is porphyritic with large grains of albite, orthoclase, and quartz. These blue rocks grade into white and then brown aplites by increase in grain-size and increase in quartz, orthoclase, and lepidolite content, and decrease in the amount of albite. Further alteration results in the formation of aplites characterised by abundant topaz, elbaite, or petalite, or combinations of these minerals, at the expense of albite, lepidolite and quartz. Elbaite, topaz, petalite, apatite, lepidolite, are regarded as of metasomatic origin. Their occurrence is extremely irregular though they tend to be most abundant in the pegmatite-rich areas. Both the metasomatism and the formation of the pegmatites are attributed to the influence of the volatile-rich residuum of the crystallizing aplite.

Statistical distributions of the minerals. The statistical distributions of all the minerals mentioned above with the exception of apatite have been determined for 86 specimens. The modes were made by point counting and from 4 500 to 6 000 points were counted for each specimen over a typical area of 4 by 2 centimetres (Table 1). The sill was sampled randomly as far as was possible given limited exposure and difficulty in obtaining samples. Also, as far as we are aware the numbers of samples obtained corresponds with the abundances of the various rock types we have been able to recognise. The same sampling procedures and criteria were employed for collecting all the 86 specimens and similar measures were taken in making sections in order to avoid sampling and experimental bias in the data.

Histograms of the distributions are given in Figure 1 and various statistics are given in Tables II and III. Elbaite, topaz, fluorite and petalite show extreme positive skewness and positive kurtosis. Orthoclase also shows extreme positive skewness and albite is less positively skewed. Lepidolite and quartz are normal, (Fig. 2) but quartz shows slight negative skewness.

Albite appears to be mainly of magmatic origin, for it displaces the flow lineation of the chilled rock. It is most abundant in the blue and white aplites. These two types of aplite were first to form and their predominant mineral is albite. In later aplites albite crystallized less abundantly and its premier place was taken by quartz and orthoclase. The amount of albite was also reduced by its metasomatic replacement with other phases. The hypothesis that the albite is of mainly magmatic origin may be tested against the statistical distribution. Significantly strong skewness of the distribution curve of albite appears to have been generated by its abundance in the early blue and white aplites which cover only a small area of the intrusion and for that matter represent only a small portion of the statistical sample. A relatively low coefficient of variation of albite is a good indication that the mineral was derived from a melt, which was more or less uniform in composition.

Orthoclase occurs in all rocks but is most abundant in mica-poor white aplites and porphyritic brown aplites. Its primary magmatic generation has been succeeded by a partial replacement by lepidolite, elbaite, topaz, petalite, quartz and other phases and by the superimposition of a much less general metasomatic growth which is quantitatively more important than the magmatic generation. This metasomatic orthoclase has also been altered with the formation of lepidolite, quartz, elbaite and topaz.

Quartz and lepidolite occur in all rocks and both increase rapidly from the blue to the white aplites and are even more abundant in the brown rocks. As for orthoclase, an early magmatic generation of these two minerals has been added to by a metasomatic generation. The distribution curves for quartz and lepidolite are however normal while that for orthoclase is significantly skewed. This is because orthoclase is present as post-solidification segregations whereas the other two minerals are uniformly spread throughout the intrusion. Orthoclase is especially abundant in the mica-poor varieties of white aplite and this occurrence largely accounts for the skewness of its distribution.

The distribution curves of albite and orthoclase are brought to a normal form by logarithmic transformation of the variate. This lognormality of the distributions was checked by such statistics as skewness and kurtosis as well as the use of probability graphs (Fig. 3). To obtain the most efficient, unbiased estimates of the population parameters from which these lognormally distributed samples were drawn, we have followed the method proposed by Koch and Link (1970) and Koch (1971). The results of these calculations are as follows: albite, $m = 43.81$, $V^2 = 72.99$; orthoclase, $m = 4.25$, $V^2 = 18.25$, where m and V^2 are respectively the unbiased, efficient estimates of the population mean and population variance, Koch (1971).

Elbaite, topaz, fluorite and petalite. These four minerals all occur as replacement phases but their statistical distributions show that the effects of the processes involved in their formation were not alike. Elbaite is low in the blue and white aplites but increases slightly in the brown and markedly in the much altered rocks. Topaz is absent from the blue aplite but otherwise broadly conforms with the occurrence of elbaite, though it does not always reach high concentration in the same samples as elbaite. All four minerals show extremely strong positive kurtosis and strong positive skewness. Their distributions reflect the fact that most values for these minerals are 0.00% or nearly so, i.e. they may be present but in such small amounts that they are not detected in the counting routine. Elbaite and topaz are therefore treated as censored samples and the point of censoring Y_o is put equal to the upper class limit of the first grouping interval. When the individuals at one end of a sample are unmeasured but assumed to be larger or smaller than those which are measured the sample is said to be censored and according to Bliss (1967) two types of censoring may occur. In type 1 the point of censoring Y_o is fixed by the experimenter while sampling. In a grouped sample Y_o is the outer class limit of the last measured interval, whether it is the lower or upper limit. In type 2 the number of censored observations f_o is fixed experimentally or by the natural process involved, as for example when some samples fall above or below the range of the measuring instrument, or when some samples have failed to respond to the natural phenomena. If the point of censoring Y_o is put equal to the smallest or largest completely measured variate Y, estimates for a type 2 censored sample are identical with type 1. Several methods are available for calculating mean and standard deviation from censored distributions (e.g. Bliss 1967 and Miesch 1967).

Before calculating mean and standard deviation for the two censored distributions, their normalisation was achieved by a double probit transform. The equation describing the curve of the normal distribution

$Z = (1/\sigma\sqrt{2\Pi})^{e-\frac{1}{2}((y-\mu)/\sigma)^2}$ may be reduced to a standard measure by putting

$\sigma = 1$ so that $Z = (1/\sqrt{2\Pi})^{e-X^2/2}$ where $X = (y-\mu)/\sigma$. When $y = \mu$, X is zero and the density Z is a maximum. X is called the standardised normal deviate and $X+5$ is the probit. When these standardised normal deviates or probits are plotted against cummulative percentage areas under a perfect bell-shaped curve of normal distribution the result is a symmetrical sigmoidal curve. Tables are available (Bliss, 1967) from which the probit or X values for cummulative percentage frequencies can be read directly. In this way the cummulative percentage frequencies may be converted to a scale equivalent to that of the variate. In the case of a grouped sample, when the upper class limits of the observed variates are plotted against the probits a straight line results, provided that the underlying distribution is normal.

In the case of the double probit transform we convert into probits both the cummulative percentage frequencies as well as the upper class limit of each percentage grouping interval. If the distribution is normal, the plots of one set of probit values against the other should fall on a straight line. Bliss (1967) states that "When the variate in a frequency distribution is a graded response measured in percentage, the frequency histograms may assume a wide variety of shapes with no apparent common element. The fact that the variates are percentages

Table 1

MODAL COMPOSITION OF BLUE APLITE

Specimen No.	Albite	Orthoclase	Quartz	Lepidolite	Topaz	Elbaite	Fluorite	Apatite	Petalite
B/M15	85.25	1.40	11.40	0.15	0.00	0.20	0.10	1.50	0.00
B/MN75N	81.03	2.56	12.31	2.03	0.00	0.04	0.15	1.87	0.00
B/MN92A	79.27	1.30	13.77	2.25	0.00	2.80	0.00	0.60	0.00
B/M158	77.31	1.10	16.28	1.50	0.00	1.81	0.00	0.00	0.00
B/M15C	76.15	1.44	18.91	1.43	0.00	2.01	0.00	0.00	0.00
B/M15D	72.60	2.15	18.92	1.10	0.00	4.11	0.00	0.00	0.00
W/MN11N	66.40	0.50	15.00	10.00	4.00	2.00	0.00	1.50	0.50
W/MN41B	60.81	0.58	24.78	10.25	0.69	2.54	0.00	0.00	0.35
W/MN10	58.20	4.40	18.10	11.90	2.00	4.00	1.40	0.00	0.00
W/RA4A	54.20	5.79	25.05	12.53	2.00	0.00	0.43	0.00	0.00
W/M99	54.02	2.13	27.28	11.41	3.34	1.82	0.00	0.00	0.00
W/M79	53.14	2.08	27.66	10.51	3.72	2.06	0.44	0.41	0.00
W/H21	53.06	3.71	25.75	13.82	0.65	1.18	0.00	0.32	1.51
W/MN71	51.74	2.78	31.40	11.85	0.00	1.51	0.00	0.72	0.00
W/MN307	50.49	0.33	34.17	9.80	4.96	0.19	0.00	0.00	0.06
W/MN1	48.91	1.79	33.31	9.61	1.66	4.78	0.00	0.00	0.00
W/H218	48.17	4.08	32.81	11.95	2.17	0.62	0.00	0.00	0.00
W/MN116A	54.50	3.50	15.50	18.50	5.60	2.40	0.00	0.00	0.00
W/MN11N	50.50	1.55	21.93	17.62	5.42	0.86	1.72	0.00	0.00
Br/H2	47.21	7.06	33.90	11.11	0.00	0.72	0.00	0.00	0.00
Br/MN79	49.79	1.57	29.80	16.98	1.57	0.00	0.00	0.00	0.29
Br/MN35	49.30	2.20	23.00	23.00	2.00	0.00	0.50	0.00	0.00
Br/MN575	48.87	0.35	25.65	21.64	2.79	0.70	0.00	0.00	0.00
Br/H2A	48.84	3.00	24.69	14.49	0.36	2.33	0.00	0.00	6.26
Br/MN325	47.11	1.11	27.63	20.09	2.37	0.00	0.50	0.00	1.19
Br/MN636	46.81	0.60	24.90	26.89	0.00	0.80	0.00	0.00	0.00
Br/MN63	46.80	0.81	31.06	18.49	2.84	0.00	0.00	0.00	0.00
Br/MN718	46.00	5.00	27.50	17.00	1.40	1.60	0.20	1.30	0.00
Br/M38NA	45.75	3.39	25.05	22.60	2.07	1.13	0.00	0.00	0.00
Br/M1	85.14	1.12	34.12	17.78	0.78	0.45	0.61	0.00	0.00
Specimen No.	Albite	Orthoclase	Quartz	Lepidolite	Topaz	Elbaite	Fluorite	Apatite	Petalite
Br/MN75	40.88	1.93	34.08	15.09	5.66	1.80	0.00	0.00	0.56
Br/MN206	40.48	1.86	36.52	17.32	2.42	1.40	0.00	0.00	0.00
Br/M126	38.71	4.22	37.66	14.33	5.08	0.00	0.00	0.00	0.00
Br/MN4(5)	37.85	1.18	35.35	22.86	0.00	0.26	0.00	0.00	2.50
Br/MN327	37.80	4.67	36.55	19.92	2.45	0.00	0.00	0.00	0.61
Br/M.O.2	37.60	8.30	33.00	13.10	7.00	1.00	0.00	0.00	0.00
Br/MN5(A)	36.80	7.80	28.10	22.80	3.00	0.00	1.50	0.00	0.00
Br/M75	36.53	3.06	34.00	17.18	5.47	3.75	0.00	0.00	0.00
Br/MN2861	36.36	8.91	32.91	18.55	2.91	0.36	0.00	0.00	0.00
Br/M57A	35.65	9.74	35.48	17.68	0.14	0.88	0.00	0.00	0.43
Br/MN85	35.40	6.00	28.00	21.00	1.21	2.80	4.19	1.40	0.00
Br/MN635A	34.93	4.57	29.11	25.16	2.49	3.74	0.00	0.00	0.00
Br/MNA'N	33.16	0.26	30.77	30.24	2.30	0.80	2.39	0.00	0.00
Br/M1111G	32.86	0.20	32.14	33.73	0.00	0.00	1.07	0.00	0.00
Br/MN22	32.40	4.10	27.50	32.40	0.00	0.00	3.50	0.00	0.00
Br/MN3	31.40	1.50	41.01	24.35	0.00	0.12	0.00	0.60	0.00
Br/MN9	31.36	4.47	37.30	21.43	2.92	0.92	0.97	0.63	0.00
Br/MN83	30.82	7.85	32.55	22.70	0.00	5.36	0.00	0.72	0.00
Br/MN83A	31.28	9.00	25.28	19.97	3.87	0.00	0.80	0.00	0.00
Br/MN9(5)A	29.93	3.90	25.38	34.48	3.69	1.08	0.00	0.00	1.52
Br/M22	23.25	2.87	30.69	37.90	0.00	0.00	3.18	1.91	0.00
Br/MK	22.89	9.52	24.89	34.55	0.64	5.15	0.00	0.00	2.36
M.P.W.A/MN70	67.23	4.08	22.68	3.22	1.07	1.72	0.00	0.00	0.00
M.P.W.A/MN371	62.95	1.30	25.13	3.63	4.40	2.59	0.00	0.00	0.00
M.P.W.A/MN77A	61.83	2.45	27.10	6.15	0.41	2.05	0.00	0.00	0.00
M.P.W.A/MN309	59.91	0.30	30.07	3.86	2.03	1.83	0.00	0.00	0.00
Por.M.P./MN308	56.95	27.46	13.54	1.11	0.00	0.91	0.00	0.00	0.00
Por.M.P./MN48	67.00	19.78	11.20	0.20	0.00	1.42	0.00	0.40	0.00
Por.M.P./MN310	45.79	23.23	24.75	0.34	2.70	3.19	0.00	0.00	0.00
Por.M.P./MN318A	49.20	21.70	18.48	1.00	8.50	1.11	0.00	0.00	0.00
Topazified Aplite/M.O.3	36.00	13.30	24.20	16.60	10.00	0.00	0.00	0.00	0.00
Petalitized Aplite/MN304	47.67	1.06	26.27	12.29	0.00	0.21	0.00	0.00	12.50
" " /MN301	31.93	11.60	31.17	9.79	3.46	1.81	0.00	0.00	10.24
" " /MN300	26.24	7.10	34.62	9.89	1.15	2.51	0.00	0.00	10.19
Por.Br.A/MK27	36.99	18.37	21.94	14.80	1.78	6.12	0.00	0.00	0.00
Por.Br.A/MN33A	34.49	16.54	30.24	16.22	0.94	0.00	1.57	0.00	0.00
Por.Br.A/MN5	32.53	13.70	33.26	11.86	1.12	7.53	0.00	0.00	0.00
Por.Br.A/MH2S	30.43	13.03	34.96	14.55	3.18	3.85	0.00	0.00	0.00
Topazified Aplite/MN16	45.31	10.19	26.54	7.33	10.03	0.25	0.35	0.00	0.00
Topazified Aplite/MN18	39.03	8.72	29.74	11.51	11.00	0.00	0.00	0.00	0.00
Topazified Aplite/MN206	37.88	7.13	28.92	15.89	9.98	0.00	0.00	0.00	0.00
R.Mixed/N.S10(S)	34.53	1.47	35.07	18.74	8.63	0.30	1.26	0.00	0.00
Topazified Aplite/MN206B	30.75	0.50	39.10	12.33	16.32	1.00	0.00	0.00	0.00
Topazified Aplite/M.O.1	30.40	11.50	35.00	10.00	13.10	0.00	0.00	0.00	0.00
R.Mixed/MN71B	44.14	7.56	23.52	13.06	3.26	8.46	0.00	0.00	0.00
Tourmalinised Aplite/M76	34.36	4.41	25.99	19.38	0.00	12.78	0.00	0.00	3.08
Tourmalinised Aplite/M91	45.37	0.40	26.15	16.36	1.23	10.49	0.00	0.00	0.00
Tourmalinised Aplite/M91B	30.58	6.47	20.72	25.40	0.65	12.46	0.00	0.00	0.00
Tourmalinised Aplite/MK7	30.38	5.27	23.09	23.54	2.82	14.79	0.00	0.00	3.72
R.Mixed/MN72	32.38	8.90	30.08	8.90	11.35	2.20	0.00	0.00	0.11
R.Mixed/MN309	31.22	8.79	32.97	8.46	0.00	6.57	0.00	0.00	6.19
R.Mixed/M9	29.76	11.00	33.56	15.22	4.21	6.25	0.00	0.66	11.33
R.Mixed/MK552	27.61	5.52	32.82	16.56	5.52	4.91	0.00	0.00	0.00
R.Mixed/MN301	23.50	8.14	39.39	10.68	5.21	2.67	0.00	0.00	7.06
R.Mixed/M68	23.21	7.38	26.58	25.11	5.79	7.59	0.00	0.00	10.41
R.Mixed/MK9	22.75	9.72	35.93	20.75	0.00	8.75	0.00	0.00	4.43

B : Blue aplite, W : White aplite, Br : Brown aplite

M.P.W.A. : Mica-poor white aplite)
Por.M.P. : Por hyritic mica-poor) strongly metasomatised aplites
R.Mixed : Reconstituted mixed)
Por.Br.A : Porphyritic brown aplite)

restricts them to the range 0 to 100, although the condition underlying a given response may represent a graded array of internal states". Wright (in Bliss, op. cit.) has proposed that these internal states are themselves distributed normally on a scale in which the same elementary factor has a uniform effect. According to him, by transforming the percentage variate to the deviates of a normal curve or probit the distribution should assume a normal form. This method of double probit transform for the normalisation of distributions should prove useful for geochemical or point counter data which are always expressed in percentages.

The point of censoring Y_o in the case of elbaite is 2.20 (volume percentage in probit) and in the case of topaz is 2.42 (volume percentage in probit). These termini are the upper class limits of the first intervals with $y = \alpha$ at 100%. Agreement with the hypothesis of censored normal samples by double probit transform is tested numerically by comparing the expected and observed frequencies and by X^2 test (Table III). The graphic proof is given in Fig. 4.

Both elbaite and topaz are metasomatic in the rocks described here. It is possible that the censored portions in the case of these two minerals correspond to the conditions of latent metasomatism (as regards the intensity of metasomatism) below such a degree as was critical for the formation of the minerals in amounts higher than 0.00 or values close to 0.00.

Fluorite and petalite are highly inconsistent in their occurrence. Their distribution curves could not be transformed to the normal form by the various methods tried. It appears that in terms of these two variates the 86 individuals do not represent a homogeneous population, as may also be tested by their plots on probability graphs. The petrological implications of the statistical distributions of these two minerals may be that the metasomatic processes responsible for their formation were operative only at certain points in the intrusion, causing strong local heterogeneities in the rocks. On the other hand, the metasomatic processes which gave rise to elbaite and topaz are thought to be of general occurrence, affecting the whole rock body.

Conclusions. The study of the frequency distributions for the minerals of the Meldon aplite has provided a means for testing the various models which have been proposed already for their formation. It has also brought out some aspects of the formation of the minerals that were not appreciated before. A significantly skewed distribution curve and a relatively low coefficient of variation for albite conform with the predominantly magmatic origin postulated for that mineral. For orthoclase, quartz, and lepidolite, it has been shown that the distribution patterns for the minerals, which owe their formation to the superimposition of processes, may be considerably variable. For the metasomatic minerals of the aplite this study has revealed that the effects of the metasomatic processes involved in the formation of elbaite and topaz on the one hand and petalite and fluorite on the other were not essentially similar. The metasomatising agents appear to have permeated the intrusion as a whole in the case of elbaite and topaz, whereas in the case of petalite and fluorite they seem to have been operative only at restricted localities in the intrusion. Further studies may provide geological evidence for the conclusions drawn from statistical considerations. The study has stressed the need that, in order to obtain the most efficient and unbiased estimates of the parameters of a parent population from which the samples for statistical study are drawn, the distribution curves must be brought to the normal form. Since the point counter and geochemical data are expressed in percentages, a probit or standardised normal deviate transform of the variate expressed in percentage and of the cummulative percentages may prove a convenient method of achieving the desired normalisation.

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Table 3

Mineral		Censored Distributions				
			$\hat{\mu}$			
N = 86	N'	Y_o	in probits.	in percent	$\hat{\sigma}$	X^2
Elbaite	64	2.20	2.66	1.00	0.64	7.08 n = 5
Topaz	62	2.42	2.82	1.3	0.62	6.120 n = 5

Note: For explanation of symbols see the text.

Table 2

Mineral	Normal Distributions				
N = 86	$\bar{X}$	s	V%	SK	K
Lepidolite	15.04	8.25	54.87	0.28	-0.22
Quartz	28.15	6.79	24.13	-0.55	-0.24
	Lognormal Distributions				
N = 86	$\check{X}$	slgX	V%	SK	K
Albite	41.69	0.14	8.47	0.29	-0.64
Orthoclase	3.23	0.50	98.18	-0.51	-0.52
5% level				0.55	0.75
1% level				0.65	1.06

Distribution is considered normal if $SK \leq SK_{5\%}$ and $K \leq K_{5\%}$ and not normal if $SK \geq SK_{1\%}$ and $K \geq K_{1\%}$.

$\bar{X}$ - arithmatic mean of mineral content (in volume percent)
S - standard deviation
slgX - standard deviation of logarithms of mineral content
$\check{X}$ - geometric mean of mineral content
V - coefficient of variation
SK - skewness of the distribution curve
K - kurtosis of the distribution curve

APPENDIX

Estimates of the population parameters, mean $\hat{\mu}$ and variance $\hat{\sigma}^2$, of the censored samples (elbaite and topaz) are calculated from the following formula (see also Bliss, 1967):

$$\hat{\mu} = \bar{y} - \hat{\theta}^*(\bar{y} - Y_o)$$

$$\hat{\sigma}^2 = m_2 + \hat{\theta}^*(\bar{y} - Y_o)$$

$\hat{\theta}^*$ is a factor which depends upon H' and P_o

$$P_o = \frac{f_o}{N}$$

f_o is the number of "unmeasured samples" or those falling in the censored portion, and N is the total number of samples;

$$H' = \frac{m_2}{(\bar{y} - Y_o)^2}$$

m_2 is the second moment and is calculated from the formula:

$$m_2 = [y^2]/N' \quad \text{where} \quad [y^2] = \Sigma(fy^2) - \frac{\Sigma^2(fy)}{N}$$

and $N' = N - f_o$, f being the frequency and y the mid-point of the grouping interval on the scale on which the variate is normally distributed. $\bar{y}$ which is employed along with factor $\hat{\theta}^*$ in the calculation of population parameters is computed from the following formula:

$$\bar{y} = \frac{\Sigma fy}{N}$$