62 M. V. von Lang on a Horizontal Goniometer.

The excess of lime in the analysis is probably traceable to an augitic mineral, a diopside, probably present as an ingredient, although not yet recognized by the microscope in the sections made from the South-African rocks. In fact the microscope, so invaluable as an instrument for pioneering in the realm of petrology, is frequently an untrustworthy guide when too much relied on—that is to say, when the results obtained by it are not checked and confirmed, and in fact supplemented by the more tedious methods of investigation pursued in the laboratory.

XIV. On a Horizontal Goniometer. By VICTOR VON LANG. [Plate IV. figs. 7 & 8.]

The instrument represented in fig. 8 owes its existence to the necessity of being able to measure refractive indices at different angles of incidence. This is of great importance when one wants, like Professor Stokes, to verify the theoretical formulæ of double refraction by experiment, or when one tries, on the contrary, to determine with the aid of these formulæ the constants of double refraction from observations on a prism of arbitrary position.

The measurements I made in order to determine the figure of the wave-surface in quartz near its axis belong to the first kind of researches; whereas the determination of the refractive indices of gypsum I have just finished gives an example of the second kind of researches, as for that determination one single prism was made use of.

The chief requisites of such an instrument are two concentric axes—the inner one carrying the vernier and the prism-table, the outer movement carrying the telescope and the graduated circle. A collimator with slit is, of course, fixed to the tripod of the instrument.

The two axes are not put into one another, as is done in geodetical instruments, for the sake of repeating the angles; but here the tripod (G) is terminated in a strong cone that forms the axis on which the circle (C) turns, whereas a hole in the centre of the cone supports the axis (A) of the vernier (n). Fig. 7 shows this arrangement in a sectional drawing.

Both axes may be clamped by the screws L L', and micro-

metrically moved by the screws M M'. Suppose we clamp the inner axis with the prism, of which the angles of deviation (D) and of incidence (i) are to be measured. Let S be the reading of the vernier by direct observation of the slit, R the reading by observation of the slit after reflection from the first plane of the prism, and T the reading by observation of the spectrum. Then we have

$$D=T-S, i=90^{\circ}-\frac{1}{2}(S-R);$$

and the index of refraction in the direction given by the angle r is found from the known formulæ

$$r = \frac{\sin i}{\sin r}, \quad \tan\left(r - \frac{A}{2}\right) = \tan\frac{A}{2}\tan\left(i - \frac{A+D}{2}\right)\cot\frac{A+D}{2},$$

A being the angle of the prism.

As long as the relative position of the prism and the vernier is not altered, we have

$$S + R = C = constant.$$

For if we turn the prism (with the vernier of course) through an angle $+\chi$, all readings will increase in the same ratio; but the reflected image of the slit turns at the same time through an angle -2χ , so that the sum S + R will indeed remain constant. We find by the quantity C,

$$i = 90^{\circ} + \frac{C}{2} - S;$$

and this formula allows us to find the angle i from the reading S. In this way it is possible to determine the angle of incidence even when it is very small; in which case it cannot be observed directly, as telescope and collimator cannot be brought so near to each other.

If we put the telescope to the slit and then turn the inner axis till the vernier gives $90^{\circ} + \frac{\text{C}}{2}$, then the light will fall perpendicularly on the first plane of the prism, the angle *i* being then evidently zero.

Of course the angle of the prism can also be determined with this instrument; and it is not necessary to alter any thing in its arrangement or to move the prism. We have only to clamp the telescope in a position of about 90° to the collimator, and then to observe the reflected image alternately on both faces of the prism.

In order to be able to use the instrument for other purposes, both telescope and collimator can be shifted parallel to their axes and be fixed by the screws OO'. If the instrument is to be used as a spectroscope, one may put conveniently before the objectives of telescope and collimator prisms with direct vision.

XV. Note on the Analysis of the Rhombohedral System. By W. J. LEWIS, M.A., Fellow of Oriel College, Oxford*.

THE methods followed by Professor Miller and most writers for obtaining the formulæ employed in determining the indices of a form in the rhombohedral system from the measured angles, or, conversely, the angles from the given indices, are, though elegant, difficult and perplexing. It occurred to me that they might be easily obtained by means of the anharmonic ratio of four poles in a zone applied to three known poles in one of the planes of symmetry, and a fourth pole whose position and indices can be directly connected with the poles of the form to be determined. This method brings out in a prominent manner the relation (2); a relation to be found in all the books, but so disguised and so little noticed as easily to be passed over, whereas from its simplicity, and from the fact that the angle involved in it is the first deduced from the measured angles of a scalenohedron, it contains a smaller error than any other equation.

The figure represents the stereographic projection of some of the principal poles and planes of a rhombohedral crystal, together with the poles P of a form $\{h \ k \ l\}$ to be determined. The poles r are $\{1 \ 0 \ 0\}$, $o(1 \ 1 \ 1)$; therefore the poles b and a are $\{2 \ \overline{1} \ \overline{1}\}$ and $\{0 \ 1 \ \overline{1}\}$ respectively. Let P be $(h \ k \ l)$, P₁, P₁₁ the corresponding faces repeated over O b and O b₁₁. Then P₁ is $(h \ l \ k)$, and



* Read November 24, 1878.

