A NEW METHOD OF INTERPRETATION OF PETROFABRIC DIAGRAMS[1]

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ABSTRACT

According to the present technique of petrofabric analysis a large personal factor enters into the interpretation of the diagrams expressing the data. Two statistical methods are described which are easily and conveniently applied to petrofabric diagrams. The results of these tests can be quickly interpreted by means of graphs accompanying the discussion, or by means of published tables. These tests tend to eliminate this personal factor.

PART I. GENERAL DISCUSSION

The purpose of this discussion is to call attention to a weakness in the present method of interpretation of the aspect of petrofabric diagrams. Briefly, this weakness is the personal factor introduced in determining the pattern of the contoured high spots. The problem is to determine the likelihood that a given diagram would be duplicated by another, the first showing significant concentrations, the second, random distribution.

Sander's technique consists of plotting the orientations of the crystal units as points on an equal area projection—a diagram which somewhat resembles a stereographic projection (8: 118–135).[2] He then contours the density of the points to bring out groupings which indicate parallel orientations. An equal area projection, like a stereographic net, represents a sphere of reference.[3] Points on it indicate critical directions such as crystallographic, tectonic, or geographic directions. A diagram containing a number of such points may be called a statistical scatter diagram. A second diagram is constructed from the scatter diagram, and shows by contours the relative density of concentration of the points.[4]

[1] A petrofabric diagram consists of points within a circle, contoured to bring out concentrations. It represents the orientation data of crystal units in relation to the structural setup.

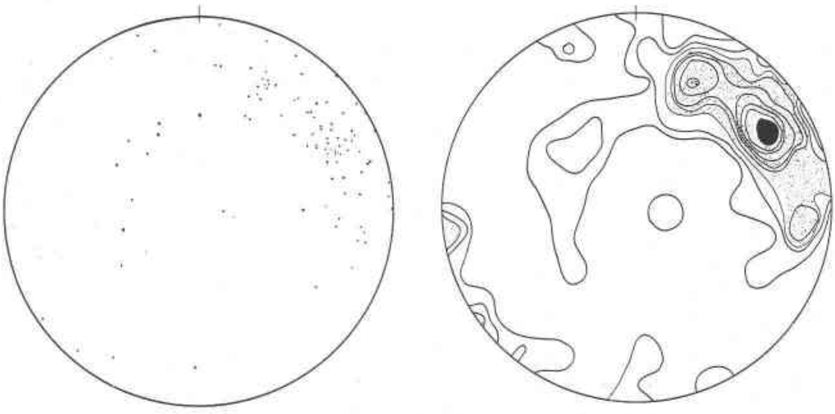
[2] First numbers in parenthesis refer to numbered references in bibliography; numbers following the colon (:) refer to pages.

[3] Description and tables for the construction of the Lambert equal area projection are given in (1:71–76).

[4] This diagram is customarily contoured in the following manner (8: 118–135, 2: 22–24). Place the diagram over a sheet of coördinate paper and cover it with a sheet of plain paper. This may be done to advantage on a light table. Considering the total area of the diagram as 100 units, describe small circles of either ½, 1, 2, or 4 units, at selected intersections of the lines of the coördinate paper. Count the number of points within each small circle and convert this to per cent per unit area; record on the topsheet at the center of the small circle. Contour lines are then drawn on this sheet, connecting points of equal density or

It is not always certain from a study of the density diagrams that the concentrations are of enough importance to be significant, as the following diagrams will illustrate.[5]

Fig. 1A is a scatter diagram and Fig. 1B the density diagram showing by contours the concentrations of the points. This diagram represents the orientations of the quartz grains in a sample of quartz schist. The specimen was chosen because of the high orientational concentration, obvious both in the scatter diagram and in the density diagram.



A B

FIG. 1A. Scatter diagram. FIG. 1B. Density diagram.

FIG. 1. High concentration. Quartz schist. Petrofabric diagram of a specimen of quartz schist from Freiburg, Saxony, with 93 quartz axes. The contour intervals are 0, 1, 2, 3, 4, 5, 6 and 7, 8 and 9, 10 and 11, 12 and 13, 14% and up; the maximum is 17%. Statistical analysis gives the following results: general test $P = .000013$; zone test $P' = .000008$ (—),* .0000005 (130), .0003 (40), .048 (20). These analyses show what a remarkable concentration exists simply by the fact that the highest value of P' obtained with the zone test is .048. Slide E-10–6, University of Wisconsin petrographic collection.

* In the descriptions of this and the following figures, notations in parentheses after values of P and of P' indicate the orientation of the zone test diagram as follows: (—), as in Fig. *9a*; (20), as in Fig. *9b*, rotated 20 degrees clockwise; (*a.l.*), oriented about the axial line of the fold; (*a.p.*), oriented about the normal to the axial plane; (60° *a.p.*), oriented about a direction 60 degrees from the normal to the axial plane, and perpendicular to the axial line. The fold referred to is in every case the regional structure in the pre-Cambrian rocks of the Baraboo region, Sauk Co., Wis. The axial line is horizontal, striking east-west; the axial plane dips about 60 degrees north. For all diagrams of known orientation, representing specimens from the oriented Baraboo suite, the axial line is represented by the center of the diagram, and the axial plane, by the diametral line. In all the diagrams, the dotted areas represent regions of concentration of 3% or more.

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concentration. The heaviest concentration is generally shaded to call attention to its location and distribution. The equal area projection is of course necessary because of the method of contouring used.

[5] There is necessarily a personal factor involved also in deciding the contour interval safely to be used in making density diagrams of this type. A contour interval of .2% would accentuate the maxima and bring out smaller ones, and one of 2% would perhaps conceal some important concentrations entirely. Statistical analysis of the scatter diagrams eliminates this personal factor.

Fig. 2 contains some points of doubtful concentration. It represents the orientations of some quartz axes in a specimen of deformed quartzite. In comparison with Fig. 1, this diagram seems to have no concentration at all; it remains to decide if there is any notable concentration that would be unlikely to occur in a random sample. This diagram well illustrates the difficulty of determining the correct concentration pattern without the aid of statistics.

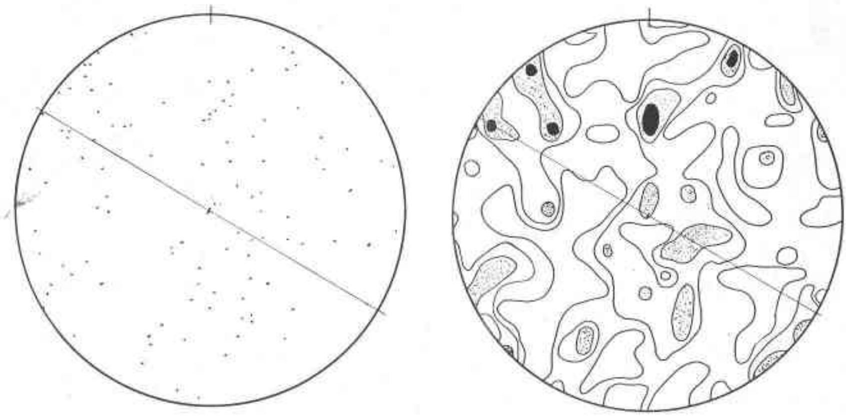


FIG. 2. Questionable concentration. Quartzite. Petrofabric diagram of a specimen of Baraboo quartzite from a fracture cleavage zone near the north end of Devil's Lake, Sauk County, Wisconsin, with 110 quartz axes. The contour intervals are 0, 1, 2, 3, 4%. Statistical analysis gives following results: general test $P=.16$; zone test $P=.18$ (*a.l.*),* .67 (*a.p.*), .21 (60° *a.p.*). As a deformed rock, this specimen might be expected to have a tectonite pattern, but no orientation of any kind is evident from the data. Results of the statistical analysis of the scatter diagram show the axes to be oriented probably at random. Specimen 20, University of Wisconsin oriented Baraboo suite, collected for the purposes of this discussion.

* See note to Fig. 1.

Figs. 3 and 4 further illustrate questionable patterns of concentration. Fig. 5 seems to exhibit at least some tendency toward a concentration in the center.

Thus it is readily seen that the present method of interpretation of petrofabric diagrams sometimes may be fully adequate; at other times it is at best only an approximation depending largely upon a personal factor; occasionally it is little better than guesswork. Statistical analysis attempts to reduce this personal factor and eliminate guesswork. This is done by means of a statistical comparison between a theoretical random distribution and the observed distribution.

The chance already mentioned that a random sample would have

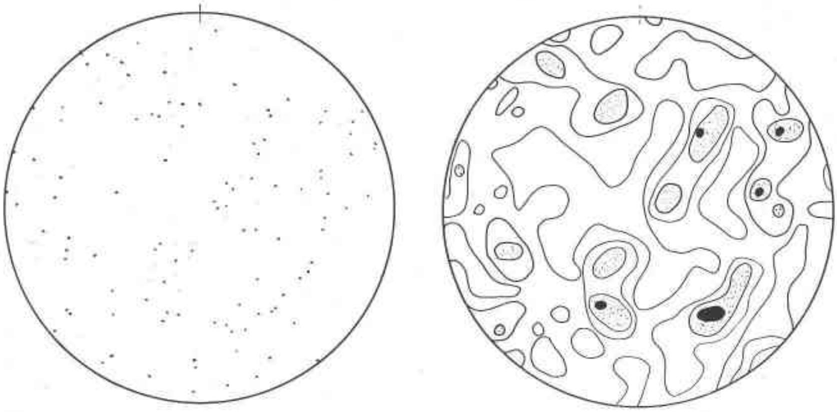


FIG. 3. Random orientation. Sandstone. Petrofabric diagram of a specimen of sandstone from the St. Peter formation (Ordovician) in a quarry in Lafayette Co., Wis., with 100 quartz axes. The contour intervals are 0, 1, 2, 3, 4%. Statistical analysis gives the following results: general test $P = .24$; zone test $P' = .83$ (−),* .91 (40). This formation and those above and below it show practically no indication of deformation of any kind. The sand grains in the St. Peter are generally rounded and frosted, and almost spherical, giving apparently ideal conditions for the production of a purely random orientation. The statistical analysis affirms this conclusion. Slide D–17–11, University of Wisconsin petrographic collection.

* See note to Fig. 1.

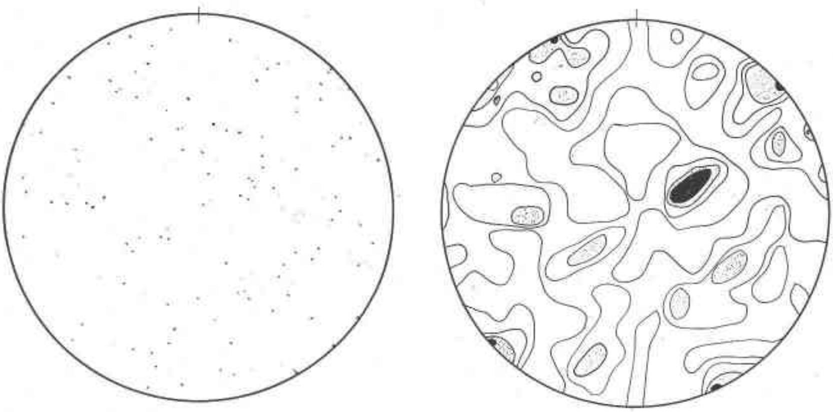


FIG. 4. Quartzite. Petrofabric diagram representing 102 quartz axes in a quartzite pebble from a conglomerate in a siliceous slate formation in the Wasatch Mountains. The contour intervals are 0, 1, 2, 3, 4, and 5%. Statistical analysis gives the following results: general test $P = .34$; zone test $P' = .86$ (−),* .66 (45). There is no indication in this diagram of any significant concentration. Specimen 15759A, slide 8151, U.S.G.S. collection at the University of Wisconsin.

* See note to Fig. 1.

concentration equal to, or greater than that of the sample in hand is, therefore, a prime concern of the structural petrologist, and it is the subject of this study. A scale for measuring this chance will be set up, the object being to remove the personal factor from the conclusions reached. This scale will be a measure of the probability that a random sample would have a divergence from a predetermined standard random, as great as, or greater than that of the sample studied. The fundamental problems to be solved are therefore first, the setting up of this theoretical random, and second, the comparison of this random with the distributions studied. The relationship between these two distributions will be expressed in terms of the probability defined above.

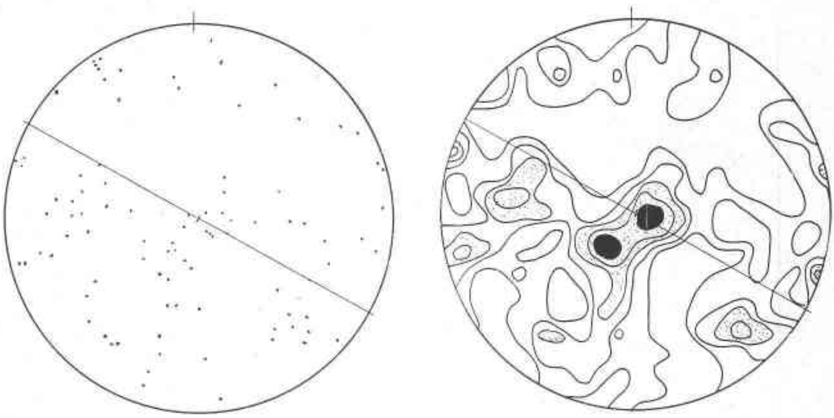


FIG. 5. Rhyolite porphyry. Petrofabric diagram of a specimen of rhyolite porphyry from the formation underlying the Baraboo quartzite, Lower Narrows, Baraboo district, Sauk Co., Wis., with 93 quartz axes. The contour intervals are 0, 1, 2, 3, 4, 5%. Statistical analysis gives the following results: general test $P=.003$; zone test $P=.055$ (*a.l.*).* 48 (*a.p.*), .37 (60° *a.l.*). This is a fairly ideal case where the general test leads to the conclusion that there is some sort of concentration, and the zone test shows the direction of the concentration. Note that only one position of the zone test axis gives a distinctly lower and more significant value of P than the other two perpendicular to it. Specimen 13, University of Wisconsin oriented Baraboo suite.

* See note to Fig. 1.

Leaving the details of the procedure to part II, some results of the various applications of the tests may be cited:

Figure 1 is shown to have a distribution of points in the scatter diagram that would happen without a controlling force only about 13 times in 1,000,000 trials—good evidence that concentration exists. This diagram, furthermore, has a zonal concentration in one direction (about a point on the periphery of the circle, 40° east of the arbitrary north marked on the diagram) such as would not be likely to occur even once in 1,000,000 trials. For comparison, the zonal concentration about another point on the periphery of the circle, 90° from the first, was tested, and it is found that the probability is less than 1 chance in 20 ($\frac{1}{20}$) of obtaining equal or greater deviation from uniformity in that direction. Ordinarily, it would be thought that the value of 1/20 might be significant, but here attention must be given to the comparison between 1/20 and 1/1,000,000.

As already remarked, Fig. 2 contains some points of doubtful concentration, a conclusion based on the usual method of interpretation. It is found, however, that about 1 time in 6 ($\frac{1}{6}$), a random sample would have just as much or more deviation from the theoretical random. This means that the chances are 1 in 6 that this rock has a purely random orientation of the quartz axes—not very convincing evidence for a significant concentration.

Figure 3 is a diagram showing the orientation of some quartz axes in a specimen of sandstone. In the region from which this sample was collected, this sandstone formation, and the formations next above and below it show no indications whatever of any deformation. The sand grains in this formation are rounded frosted quartz grains which should be ideally adapted for yielding a random orientation. The concentrations shown by the grain orientations of this specimen would be duplicated or surpassed in about 1 out of every 4 ($\frac{1}{4}$) trials.

Figure 4 represents some quartz axes in a quartzite pebble from a conglomerate in a siliceous slate formation. The statistical analysis shows that in the sample the deviation from the theoretical random would be duplicated or surpassed in about 1 out or every 3 ($\frac{1}{3}$) trials by random sampling.

Figure 5 shows the orientations of some quartz axes in a specimen of rhyolite from a zone immediately below a quartzite formation. The diagram is so oriented that the axial line of the major fold of the quartzite is at the center of the diagram, and the diametral line represents the axial plane. The concentration in this case would be duplicated or surpassed only 3 times in 1,000 (3/1000) random trials. The zonal concentration about the axial line gives a probability of only about 1 in 18 (1/18), while in two directions perpendicular to the axial line, the probability is such as to show a very uniform distribution of points with respect to these directions—the chances are about 1 in 3 to 1 in 2 ($\frac{1}{3}$ to $\frac{1}{2}$) of duplicating or surpassing this sample's deviation from uniformity in these directions.

Table 3, part II of this paper, shows these results and the results obtained on other diagrams (Figs. 10–15 inclusive). It is seen on comparing the diagrams with the results shown, that low values of the quantity P

given in the third column are usually obtained when there is any significant concentration in the diagram. The values in columns four to seven are the results of the application of a test more descriptive than the simple comparison of the observed distribution with a theoretical random distribution. Part II, containing the detailed descriptions of these tests, should be consulted for a more exact definition of the meanings of the numbers given in the table.

Part II. Details of a Statistical Method for the Interpretation of Petrofabric Diagrams

Attention has been called to a weakness in the present technique of interpretation of petrofabric diagrams. Several instances were cited in

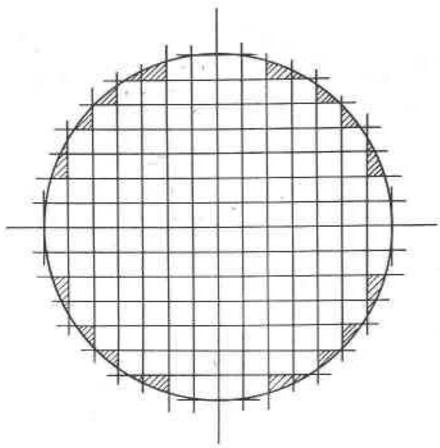


Fig. 6. Circle divided into 148 equal squares for the "general test."

which this weakness is apparent. The results of a statistical study were used in part for comparison with the doubtful results obtained by the standard procedure.

It is the purpose in the second part of this discussion to describe the essential details of two statistical methods for the testing of petrofabric diagrams. In the first of these methods a theoretical random distribution of points in the petrofabric diagram is set up and compared with the distribution of points obtained by the actual study of rock thin sections. The degree of departure from random is thereby measured, the results giving a clue to the degree of concentration. The second test is a test of the departure toward a specific type of concentration located in an arbitrarily chosen direction.

For the purposes of the test to be described, the scatter diagrams (e.g. Fig. 1*A*) have been found more useful than the density diagrams (e.g. Fig. 1*B*). The diagram to be tested is divided arbitrarily into a large number of equal areas in some such manner as shown in Fig. 6. A copy of this net on transparent paper was found very useful in this procedure. This figure has 148 squares wholly or almost wholly within the circle. Superimpose it upon the scatter diagram and count the num-

TABLE 1

CALCULATION OF χ^2 FOR GENERAL TEST AS APPLIED TO FIG. 1

$$\text{Formula: } \chi^2=\sum_{r=0}^{r=n}\frac{(m_r-m_r')^2}{m_r}$$

Column	1	2	3	4	5	6	7
Symbol	r	m_r'	rm_r'	m_r	m_r-m_r'	$(m_r-m_r')^2$	$\frac{(m_r-m_r')^2}{m_r}$
	0	106	0	81	25	625	7.7
	1	25	25	49	24	576	11.6
	2	11	22	18	1	1	.1
	3	2	6	0*			$\chi^2=19.4$
	4	0	0	0			
	5	0	0	0			
	6	0	0	0		$P=.005$ or less as shown by the graph.	
	7	1	7	0			
	8	1	8	0			
	9	1	9	0		$P=.000013$ according to tables.	
	10	0	0	0			
	11	0	0	0			
	12	1	12	0			
		148	89	148			

* This entry could be made 3 and the one above it 15, but so small a cell frequency is not desirable in this column; much more reliable results are obtained by grouping the small numbers.

ber of squares containing 0 points, the number containing 1 point, the number containing 2 points, etc., until all the squares have been counted. When a point falls on the dividing line between two squares, it must be assigned to one of them by some arbitrary rule. The rule used is commonly of the following type. Points on a north-south line are assigned to the square to the east; points on an east-west line are assigned to the square to the north. The procedure has been applied to Fig. 1 and recorded in Table 1.

The first column shows the number of points per square; the second column shows the number of squares containing the given number of points (0, 1, 2, 3, etc.), and should of course total 148. The third column shows the number of points counted, and must be used to obtain column 4 from the graph shown in Fig. 7. Column 4 shows the theoretically most probable number of squares containing 0, 1, 2, etc., points and represents the theoretical random distribution with which the actual distribution shown in column 2 is to be compared. It is obtained from Fig. 7 as follows.[6] On the upper abscissa scale of Fig. 7, find n, the total number of points counted (sum of column 3); above this point read on the ordinate scale 148 $P_n(r)$ the vertical distance to each of the curves marked $x=0$, $x=1$, $x=2$, etc. and record the results in column 4 opposite the appropriate numbers in column 1. Curves are missing where the expectation is less than 4 squares; write 0 for each entry in column 4 corresponding to the missing portions of the curves, and increase the last number not 0 enough to make the column total 148.[7] Group together and add all the numbers in column 2 corresponding to 0's and to the last number not 0 in column 4; this sum will be compared with the

[6] The expected frequency of squares containing 0, 1, 2, etc., points is given by the following theorem (3: 420, 9: 455).

Let p denote the probability of the event in one trial, and q the probability of its failure. Then $p+q=1$. The probability $P_n(r)$ that the event will happen exactly r times in n trials is then

$$\frac{n!\, p^r q^{n-r}}{r!\,(n-r)!} \qquad (1)$$

This is at once seen to be the formula for the rth term of the binomial expansion of $(p+q)^n$. The most probable number of squares in Fig. 6 containing r points is therefore 148 $P_n(r)$, if (as in the figure) the total number of squares is 148, and the total number of points is n. Since it is obviously impractical to calculate the value of $P_n(r)$ by this formula, a function known as Poisson's exponential is used for the purpose. Poisson's exponential function is an approximate expression for the value $P_n(r)$ in terms of e (2.718——), r, and $m(=np)$, if p is small and n large so that m is of moderate size. The expression giving $P_n(r)$ is as follows:

$$P_n(r)=\frac{e^{-m}m^r}{r!} \qquad (2)$$

Tables accurate to .000001 for various values of m and r have been prepared and should be used to obtain maximum accuracy; one was used in the construction of Fig. 7 (6: table LI). For the benefit of those who wish to divide the large circle into a different number of equal areas than 148, the two auxiliary scales m and $P_n(r)$ were included. The principal scales 148 m and 148 $P_n(r)$ can be used of course only when the test diagram has exactly 148 equal areas.

[7] "In applying the χ^2 test to such a series it is desirable that the number expected should in no group be less than 5, since the calculated distribution of χ^2 is not very closely realized for very small classes" (4: 81).

last number not 0 in column 4 in place of the single number that would otherwise be used. Find the difference between corresponding entries in columns 2 and 4 and record in column 5. Square each entry in column 5 and record in column 6. Divide each entry in column 6 by the corresponding number in column 4 and record in column 7. The sum of the numbers in column 7 is "chi-squared" (χ^2).[8] If a slide rule is used, columns 5 and 6 may be omitted.

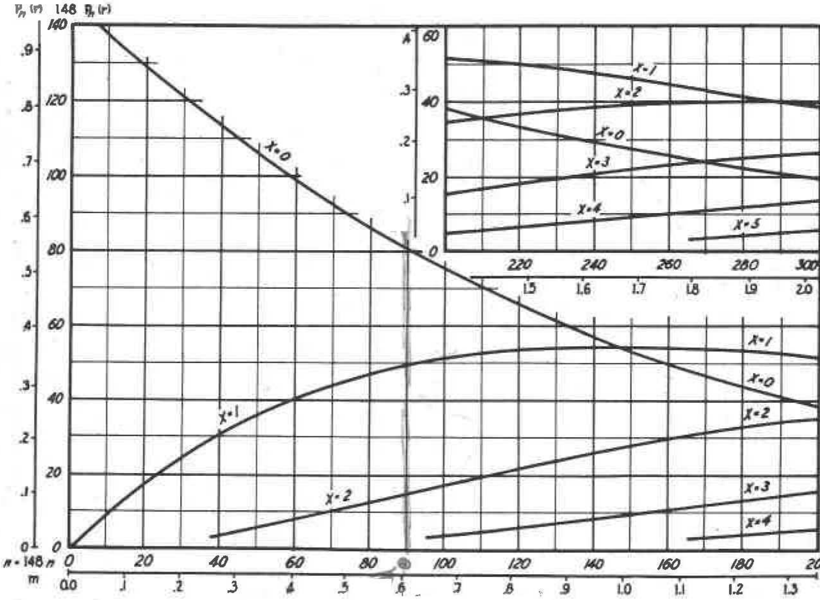


FIG. 7. Graph showing the most probable number of squares in Fig. 6 that will contain 0, 1, 2, etc., points, for any given total number of points up to 300. This graph is based on Poisson's exponential function, $P_n(r)=e^{-m}m^r/r!$. Table LI (6) was used in the construction of the graph.

[8] Pearson's χ^2 test for goodness of fit is based upon the equation (4, 7)

$$\chi^2=\sum_{i=1}^{i=n}\frac{(m_i-m_i')^2}{m_i}. \tag{3}$$

The value χ^2 may be converted into a probability P that a random sample would have as great or greater deviation from theory as the sample studied by means of the following formulas: (4, 6, 7)

$$P=\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty}e^{-\chi^{1/2}}d\chi+\sqrt{\frac{2}{\pi}}\,e^{-\chi^{2/2}}\left(\frac{\chi}{1}+\frac{\chi^3}{3}+\frac{\chi^{n-3}}{1\cdot 3\cdot\cdots\cdot(n-3)}\right) \tag{4}$$

for n even, and

Each entry in column 7, together with all the corresponding entries used in deriving it, constitutes one "cell" according to statistical definition. For this test, the number of cells is decreased by 2 to determine the value of n to be used in Fig. 8. Consult Fig. 8 which is a graph relating χ^2, n, and a value P, the index sought, expressing the probability that a random sample would have as great (or greater) deviation from theory as the one studied. χ^2 is located on the abscissa scale, and the vertical distance to the curve corresponding to the proper n is found on the ordinate scale P. When greater accuracy is desired, and when the graph is inadequate, tables may be used instead of the graph (4, 6, 7).

The probability index, P, may have any value between 0 and 1. It may be multiplied by 100 and expressed as per cent if desired. "If P is between .1 and .9, there is certainly no reason to suspect the hypothesis [of random distribution] tested. If it is below .02 it is strongly indicated that the ~~hypothesis~~ random dist. fails to account for the whole of the facts. We shall not often be led astray if we draw a conventional line at .05 and consider that higher values of χ^2 [i.e., lower values of P than .05] indicate a real discrepancy" (4: 77). In other words, if P is found to be less than .05, it is reasonably safe to conclude that there is a significant deviation from the theoretical random distribution postulated. No definite limit of this nature can logically be set, but for the purposes here discussed a limit may be chosen arbitrarily, based upon the experience of the operator and the suggestion of the statistician.

This method of determining the degree of scatter was applied to a number of petrofabric scatter diagrams; the results obtained are, as already stated, in agreement with the interpretations which would ordinarily be made concerning these diagrams, but they give a much more definite index of the degree of scatter. Figs. 1 to 5 have already been discussed. Fig. 11 gives a value of $P=.00027$, showing definite evidence of

$$P=e^{-\chi^2/2}\left(1+\frac{\chi^2}{2}+\frac{\chi^4}{4}+\cdots+\frac{\chi^{n-3}}{2\cdot 4\cdot\ \cdots\ (n-3)}\right) \tag{4}$$

for n odd. n is the number of degrees of freedom in determining cell frequencies; for the zone test described later it is 9, for 9 of the cells can be filled arbitrarily and the last one is then the remainder necessary to make up the correct total number of points. n is 2 less than the number of cells in the general test described, because in addition to this first limitation, that the total number of points must fulfil a condition, the number of squares counted must also fulfil the condition of totalling 148, making two limitations instead of one. (4) Tables giving the values of P for various values of n and χ^2 have been prepared (4, 6: table XII, 7). In using such tables, it should be remembered that where some authors (e.g. 4) give n, the number of degrees of freedom, others (e.g. 6) give a value n' which is equal to $n+1$. In most cases, as in the zone test about to be described, the value n' is the number of cells.

orientation; other figures give values which are recorded in column 3 of table 3. The values given in this table suggest that every diagram studied probably has significant orientation except Figs. 2, 3, 4, 10, 12, and 13.

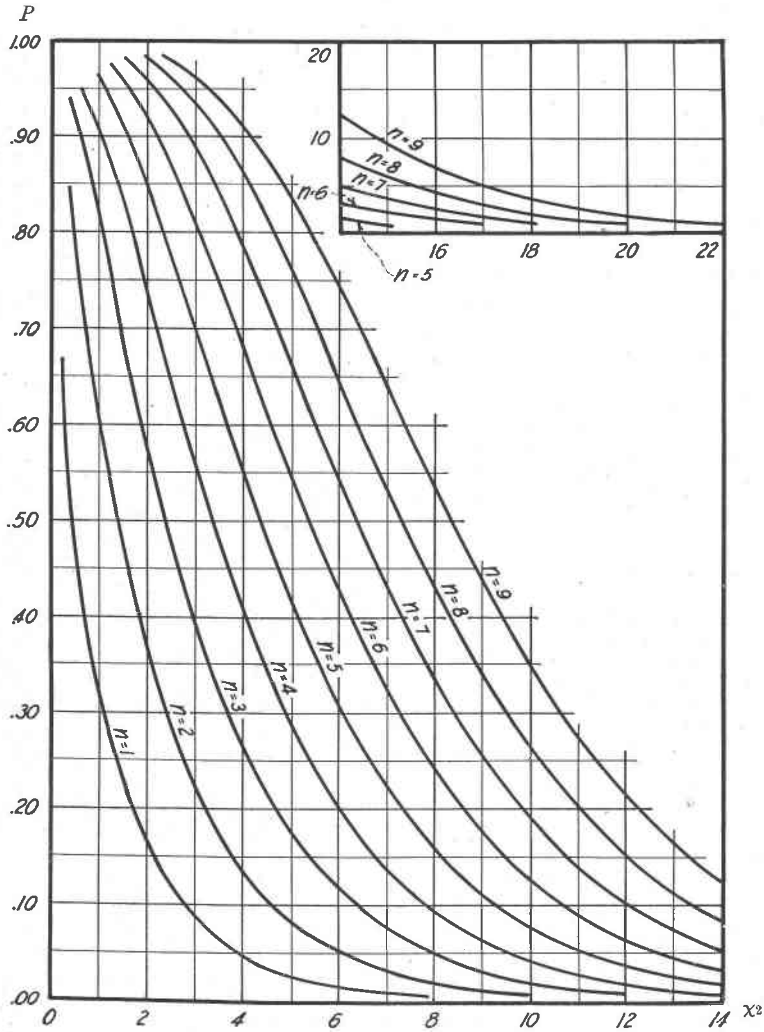


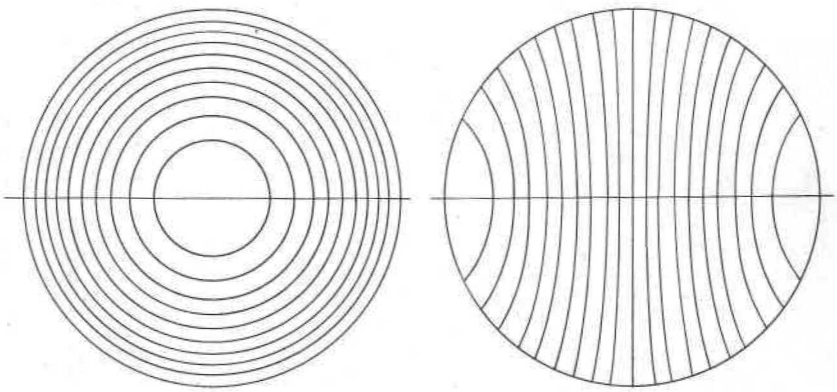
FIG. 8. Graph showing the probability that a random sample would deviate as much or more from the theoretical sample as the sample studied. This graph is based on equations (4), and was constructed from Table XII (6). Equivalent tables are given by Fisher (4), and are referred to by Rietz (7) as follows:

Elderton, W. P., *Biometrika*, 1: 155–63.

Elderton, W. P., *Tables for Biometricians and Statisticians*, pp. 26–29.

An unusually high value of P should often be considered a hint to lead to further investigation, though less definite in its meaning than a low value would be. If P is as high as .9, some significant force may often be suspected, acting to produce a distribution more nearly in agreement with the theoretical distribution than is likely to occur in random sampling. A value somewhere in the vicinity of .5 is the strongest indication of random, meaningless distribution of the points on the scatter diagram.

Since the general test just described reveals only the likelihood that the orientation is that of a random sample, and since it does not give positive descriptive results, it is in some respects inadequate. Another test was therefore devised which is believed useful in determining the



a *b*

FIG. 9*a*. Circle divided into 10 rings of equal area for the "zone test."
FIG. 9*b*. Same, rotated 90 degrees about a north-south axis.

nature of the orientation with respect to field relations, but necessitates certain limitations in its use. For this purpose, a circle of the size of the standard diagram was divided into ten concentric rings of equal area, as in Fig. 9*a*. These rings represent zones of equal area on the surface of the sphere of reference. This figure is superposed upon the scatter diagram in which concentration is to be tested in any zone parallel to the periphery of the diagram or around the center of the diagram,—that is, in any one or more of the delimited areas.

Let Fig. 5 be chosen for testing by this method. The areas are numbered from 1 to 10 starting at the center. A line is drawn diametrically across the test diagram in any arbitrary position, as shown in the figure; points on one side of this line falling on circular dividing lines are assigned arbitrarily to the higher numbered zone, and those falling on circular dividing lines on the other side of the diameter are assigned to the lower numbered zone. Table 2 is a convenient form for the tabulation of the following procedure. Column 1 shows the zone numbers—1, 2, . . . , 10. Column 2 shows the number of points counted within each zone. This column is summed and the result divided by 10 to obtain the average number of points per zone; the decimal should be retained. The third column contains the differences between the entries of column 2 and the average of column 2. Column 4 is obtained by squaring the numbers of column 3. The sum of column 4 is divided by the average of column 2 to obtain χ^2; n is 9, and a value P is obtained from Fig. 8.

TABLE 2

CALCULATION OF χ^2 FOR ZONE TEST AS APPLIED TO FIG. 5

$$\text{Formula: } \chi^2=\sum_{r=1}^{r=n}\frac{(m_r-m_r')^2}{m_r}$$

Column	1	2	3	4	
Symbol	r	m_r'	(m_r-m_r')	$(m_r-m_r')^2$	
	1	19	9.7	94.0	
	2	6	3.3	10.9	$93/10=9.3=m_r$
	3	6	3.3	10.9	
	4	7	2.3	5.3	$\chi^2=154.1/9.3$
	5	10	.7	.5	$=16.6$
	6	10	.7	.5	$n=9$
	7	11	1.7	2.9	$P=0.055$
	8	6	3.3	10.9	
	9	6	3.3	10.9	
	10	12	2.7	7.3	

If P is to have a meaning comparable to that ascribed to it in the previous test—namely the probability that a random sample would have as great (or greater) deviation from the theoretical distribution postulated, as the sample studied—then the test circles must be oriented on a basis of field evidence.[9] The P determined by orienting the zone test diagram on the basis of evidence shown by the scatter diagram or by the density diagram has not the same meaning as that determined when the zones are located by the use of *a priori* evidence, such as field evidence. The meaning of the P determined with the zones oriented on the basis of evidence from the diagram is not known.

For instance, in Fig. 5, the field evidence upon which the orientation of the test diagram is based is the fact that the center of the diagram

[9] In spite of considerable time and energy spent upon the problem of determining the probability that a random sample would lead to as great (or greater) concentration for at least one circular region of given radius or for at least one location of a zone of fixed dimensions, as a given sample, Professor M. H. Ingraham of the Department of Mathematics of the University of Wisconsin was unable to find the solution. He consented to the use of his name as authority here.

represents the direction of the axial line of the fold affecting the quartzite formation overlying the rhyolite from which the sample was taken. Thus it is tested whether there is any concentration, not in the center of the diagram, but parallel to the axial line of the fold (or in a zone around it). The resulting $P=.055$ shows that in about 1 out of every 18 trials, a random sample would have equal or greater concentration of this type.

TABLE 3

RESULTS OF APPLICATION OF THE TESTS TO SOME PETROFABRIC DIAGRAMS

1	2	3	4	5	6	7
		P—general test	P—zone test	P—zone test	P—zone test	P′—zone test
Fig. 1	93	.000013				.000008 (−)
						.000000+ 130
						.0003 40
						.048 20
2	110	.16	.18	.67	.21	
3	100	.24				.83 (−)
						.91 40
4	102	.34				.86 (−)
						.66 145
5	93	.003	.055	.48	.37	
10	105	.31	.00034	.18	.60	
11	104	.00027				.014 (−)
						.123 65
12	100	.10				.94 (−)
						.12 145
						.92 55
13	57	.35				.82 (−)
						.65 80
						.77 170
14	100	.01	.98	.42	.0035	
15	127	.001	.63	.58	.03	

In column 7, (−) after the value of P' indicates that the zones were oriented as in Fig. 9*a;* numbers after the values of P' indicate the number of degrees rotation clockwise, from the position of Fig. 9*b*.

If the concentration to be tested is not centered in the diagram, then either the test diagram or the scatter diagram must be rotated into a more favorable position. Such a rotation of the test diagram has been carried out as shown in Fig. 9*b*.

This method of testing has been applied to the diagrams already studied by means of the general test first described, and the results tabulated with those of that test. It is to be emphasized again that the best use of this test is to study concentrations in relation to field data, that is, to test the zonal concentration about predetermined directions such as a specific tectonic axis, or a specific geographic direction.

Table 3 contains the results of the application of these two tests to the diagrams shown in Figs. 1–5 and 10–15 inclusive. Column 1 shows the diagram number; column 2, the total number of points on the scatter diagram (number of quartz axes measured), and column 3, the result of the application of the general test. The remaining columns show results of the application of the zone test as follows. Column 4, the result of testing with the zones oriented about the axial line of the fold in the Baraboo quartzite formation from which or near which the

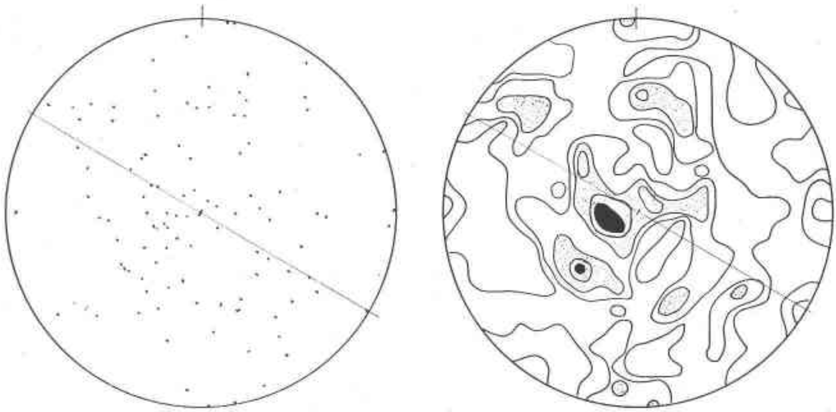


FIG. 10. Quartzite. Petrofabric diagram of a specimen of quartzite from the Baraboo formation, Devil's Lake, Sauk Co., Wis., with 105 quartz axes. This specimen was taken from a massive bed of quartzite below and adjacent to the fracture cleavage zone from which the specimen of Fig. 2 was obtained. The contour intervals are 0, 1, 2, 3, 4, 5%. Statistical analysis gives the following results: general test $P=.31$; zone test $P=.00034$ (*a.l.*),* .18 (*a.p.*), .60 (60° *a.p.*). Specimen 21, University of Wisconsin oriented Baraboo suite.

* See note to Fig. 1.

specimens were taken; column 5, with the zones oriented about the direction normal to the axial plane, and column 6, with the zones oriented about a direction perpendicular to the axial line and 60° from the normal to the axial plane. Column 7 shows values designated P' to distinguish them from P with which they are not comparable, as explained above. These values were obtained by means of the same procedure, but without the use of field evidence to orient the zone test diagram.

As already stated, every diagram except Figs. 2, 3, 4, 10, 12, and 13 has a value of P in column 3 (general test) which suggests orientation of some sort. The actual orientation is shown to a considerable extent in columns 4 to 7. An example will illustrate.

The general test on Fig. 14, for instance, gives a value of $P=.01$ which is significant.[10] Concentration here is shown by the zone test centered about a direction 60° from the normal to the axial plane and perpendicular to the axial line of the fold in the quartzite associated with the rhyolite porphyry from which the specimen was taken. The two other values obtained with the zone test, .98 and .42, show that there is an unusually uniform distribution of points with respect to the axial line orientation of the diagram, and approximately normal random distribution with respect to the orientation normal to the axial plane.

Figure 10 is an example of very nice Poisson distribution ($P=.31$, column 3), but a considerable concentration is shown about the axial

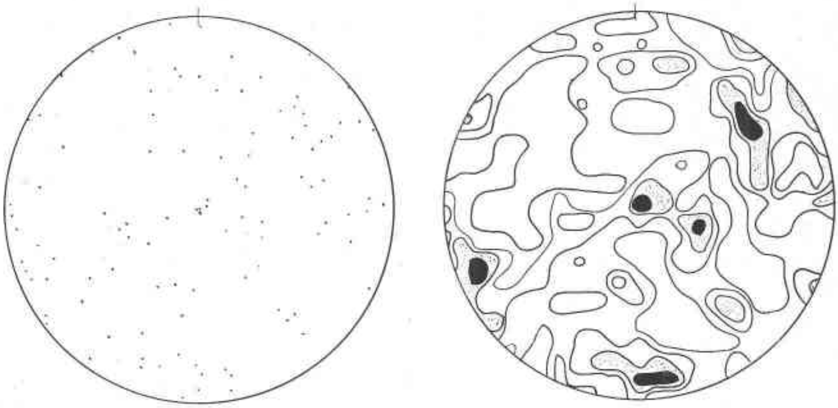


FIG. 11. Quartzite. Petrofabric diagram of a specimen of quartzite showing enlargement of the grains, taken from the vicinity of the Chapin Mine, Iron Mountain, Mich., with 104 quartz axes. The contour intervals are 0, 1, 2, 3, 4%. Statistical analysis gives the following results: general test $P=.00027$; zone test $P'=.014$ (−),* .12 (65). The general test proves considerable concentration somewhere, and the zone test shows the direction of the orientation. It is not unlikely that a different orientation of the zones, intermediate between the positions of Figs. 9*a* and 9*b*, would give even more striking values of P' than the position indicated. Specimen E-10, University of Wisconsin metamorphic collection.

* See note to Fig. 1.

line of the fold ($P=.0003$, col. 4), and rather even distribution perpendicular to this ($P=.18$, a.p., $P=.60$, a.p.).[11] The seemingly anomalous results of the application of the two tests to Fig. 10 do not invalidate either of them. The zone test shows with considerable assurance that there is concentration about the axial line. Although the general test fails to show evidence of this concentration, because it does not take into account the position or distribution of the squares, but only the numbers of

[10] Fisher believes that values less than .02 are surely significant, and values less than 0.5 are usually significant (4: 77).

[11] See note to Fig. 1.

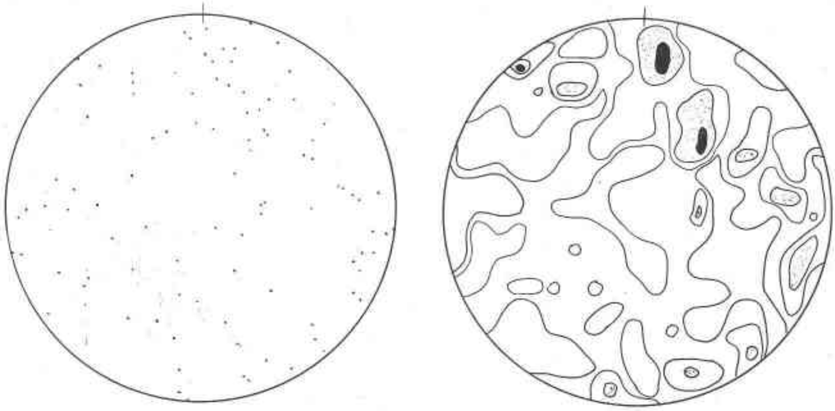


FIG. 12. Marble. Petrofabric diagram of a specimen of marble from Ashley Falls, Mass., with 100 calcite axes. The contour intervals are 0, 1, 2, 3, 4%. Statistical analysis gives the following results: general test $P=.10$; zone test $P'=.94$ (—),* .12 (145), .92 (55). The zone test shows that with respect to two positions, the distribution is very uniform, and with respect to the third at right angles to both of these two, the distribution is not as uniform. A more detailed study, involving several times as many points, might possibly bring out a concentration which would be significant. Specimen L–15, University of Wisconsin metamorphic collection.

* See note to Fig. 1.

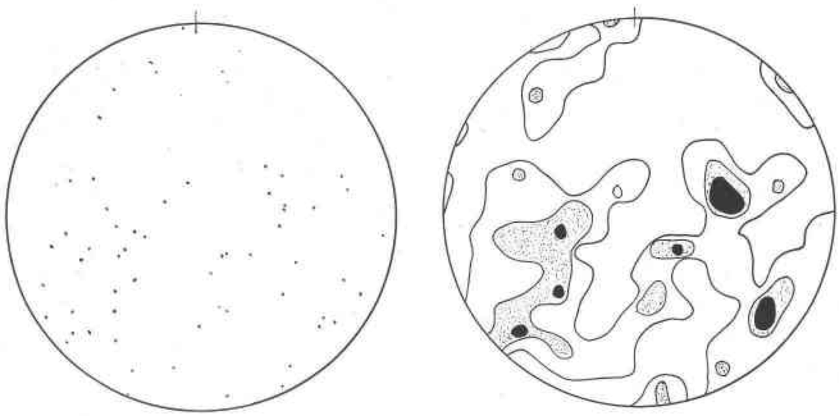


FIG. 13. Quartzose slate. Petrofabric diagram of a quartzone phase in some slates near Deadwood, S. D., with 57 quartz axes. The contour intervals are 0, 1.8, 3.5, 5.3%. Statistical analysis: general test $P=.35$; zone test $P'=.82$ (—),* .65 (80, .77 (170). The general test and the zone test in all three positions all indicate random distribution of orientation. The results of the zone test show that the distribution is very uniform in every respect. In view of the statistical data, it is doubtful whether a greater number of grains measured would discover any significant orientation. Specimen 14849, slide 7610, U.S. Geol. Survey collection.

* See note to Fig. 1.

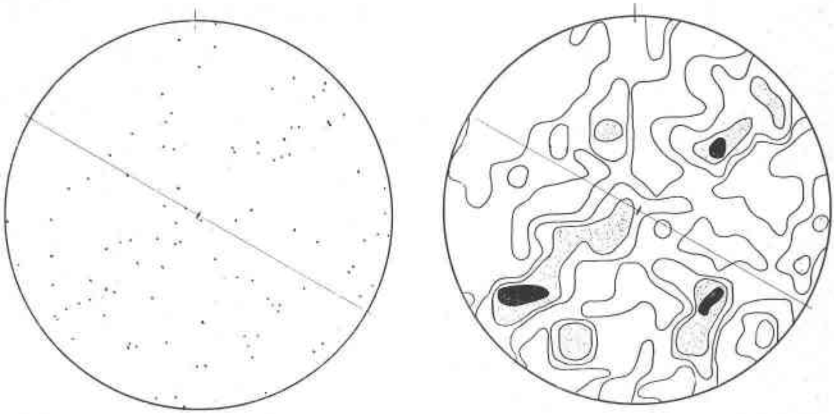


FIG. 14. Rhyolite porphyry. A petrofabric study of a specimen of rhyolite porphyry from the formation underlying the Baraboo quartzite, Lower Narrows, Sauk Co., Wis., with 100 quartz axes. The contour intervals are 0, 1, 2, 3, 4, and 5%. The statistical analysis gives the following results: general test $P=.01^-$; zone test, $P=.98$ (*a.l.*),* .42 (*a.p.*). .0035 (60° *a.p.*). The tests show clearly that concentration exists for the third orientation of the zone test. The density diagram helps determine which zone or zones are important regions of concentration. Specimen 14, oriented Baraboo suite.

* See note to Fig. 1.

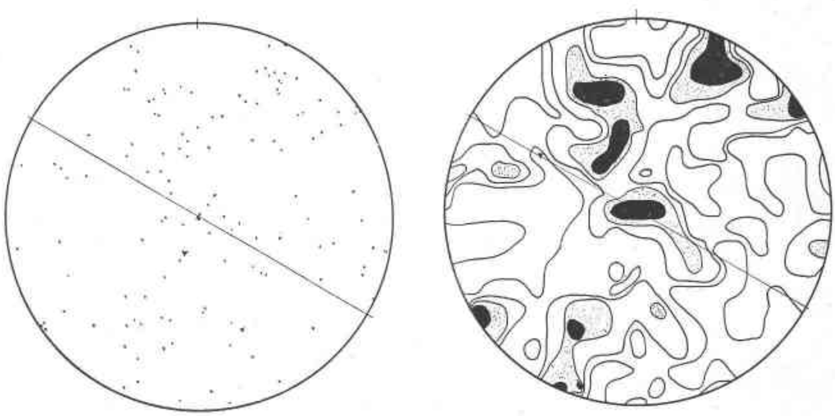


FIG. 15. Quartzite. A petrofabric study of a sample of quartzite from the Baraboo formation, Devil's Lake, Sauk Co., Wis., with 127 quartz axes. The contour intervals are 0, 0.8, 1.6, 2.3, 3.1 and 3.9%. The results of the statistical analysis are as follows: general test $P=.001$; zone test $P=.63$ (*a.l.*),* .58 (*a.p.*), .03 (60° *a.p.*). An interesting comparison can be made of the results of the statistical tests as applied to this figure, and to figures 2 and 10; these three samples represent samples of adjacent beds. The fracture cleavage zone in the middle shows no concentration; the massive bed below it shows concentration in a different direction. Specimen 19, University of Wisconsin oriented Baraboo suite.

* See note to Fig. 1.

them, containing 0, 1, 2, etc., points, this test does give definite evidence useful in describing the kind or type of concentration further. For the concentration is now describable, not only in its relation to the axial line of the fold (or to the areas of the zone test), but also in its distribution type, as shown by the general test. The general test has shown that concentration to be not remotely different from the Poisson distribution used as a basis of comparison. This is an example of one of the inadequacies of the general test; the writer has no knowledge or experience to show whether it is to be considered a frequent occurrence, but it is the only such example found in a series of 12 petrofabric studies made in connection with the development of these tests.

It is not to be overlooked that this zone test is equally applicable to diagrams having either point, or girdle concentration. Thus it can be used with *b*-tectonites as well as with *s*-tectonites.

Acknowledgment

The writer is under special obligation to Professor R. C. Emmons of the Geology Department of the University of Wisconsin for his critical reading of the paper, and for his aid and suggestions as to presentation, and to Professor M. H. Ingraham, Chairman of the Mathematics Department of the University of Wisconsin, who spent much time in the consideration of the statistical problems involved in this study. The writer expresses thanks also to his father, Professor A. N. Winchell, of the Geology Department, and to other members of the faculty of that department, for their suggestions and interest.

Summary

Petrofabric analysis as discussed in this paper is the detailed study of the orientation of selected minerals in a rock section, leading to conclusions concerning parallelism or tendencies toward parallelism of orientation of these grains. The orientations of individual grains are indicated by points on an equal area projection, and the diagram is contoured to show the density of concentration of these points. This contouring method of interpreting the data introduces a personal factor which can be largely eliminated by the use of the proper statistical tests. Such a test has been devised and its application to a number of petrofabric diagrams has been described. The test measures the probability that a random sample would have concentrations equivalent to, or greater than those of the sample studied.

The results of applying the tests to a series of diagrams prepared from oriented and non-oriented slides cut from specimens of tectonites and non-tectonites are tabulated, showing that the tests are capable not

only of determining simply the degree of departure from random, but also to a certain extent at least, of describing the concentration with respect to its direction and degree.

APPENDIX. *Descriptions of samples collected for the purpose of this paper, from the vicinity of Baraboo, Sauk Co., Wis.*[12]

To record the orientation of specimens, two arrows were made on each with colored pencils, after first knocking the sample free, trimming slightly, and then fitting back into its original position. A red arrow was in every case made horizontal, and its compass direction recorded. A blue arrow pointing upward was made perpendicular to the red one, and its dip recorded.

Each thin section was cut with the intention of orienting it so as to lie in a vertical plane striking north, with the arrow marked on it in such a position as to be in a horizontal plane and pointing north. Since the rocks of the Baraboo district form an asymmetric fold with a horizontal east-west axial line and with its axial plane dipping about 60 degrees north, the resulting thin sections are approximately normal to the axial line, and the normal to the axial plane lies 30 degrees from the arrow. The average error of orientation of the thin sections is probably not more than 10 degrees in any direction. The position of the arrow, i.e., on the east or the west side of the thin section is recorded.

Specimens 1 to 17 inclusive are from the vicinity of the Lower Narrows, Baraboo River, near the town of Baraboo, Sauk Co., Wisconsin. Specimens were taken at varying elevations, mostly in the middle one-third of the hill, on the west nose. They were located by their approximate distance in meters north or south from the center of the depression which represents the contact between the Baraboo quartzite formation and the rhyolite porphyry underlying it. Those specimens marked with an asterisk (*) are of questionable orientation. The first number refers in each case to the specimen number, the second to the distance in meters north (N) or south (S) from the contact; the third notation is a brief description of the nature of the rock; following this are the notations describing the position of the red (horizontal) and blue (in vertical plane normal to red) arrows marked on the specimen. The last notation refers to the east (E) or west (W) side of the thin section, and indicates the side on which the arrow is marked. This notation is also carried out in the description of specimens 18 to 21, except that the location is completely described.

[12] Designated *U. W. oriented Baraboo suite.*

1	21	S	massive quartzite: red N 8 W, blue 83 W: E
2	15	S	massive quartzite: red N 73 E, blue 77 M: E
3	20	S	quartzite: in fracture cleavage zone; fracture cleavage dip 60 N strike N 85 W, bedding dip *ca.* 90 strike N 75 E: red N 23 W, blue 81 W: E
*4	3	S	vein quartz, sheared quartzite: red N 3 W, blue 77 E: W
*5	11	S	quartzite: red N 75 W, blue 87 S (X on top side): W
6	8	S	quartzite: red S 38 W, blue 59 W: ?
*7	5	S	quartzite: red N 10 W, blue 87 E: E
8	3	S	quartzite: red N 13 W, blue 87 E: W
9	2	S	quartzite: red N 45 E, blue 60 W: W
10	2	S	quartzite: red N 52 W, blue 48 E: W
11	2	N	rhyolite: schistosity N 75 W dip 45 N, red N 78 W, blue 73 S: W
12	5	N	rhyolite etc: red N 65 E, blue 90: fragment chipped, W or E not certain; if W, arrow ok, if E, arrow reversed; arrowhead not certain.
13	10	N	rhyolite, etc: red N 25 W, blue 74 N: W
14	20	N	rhyolite porphyry: red N 78 E on bottom of sample; blue 39 N: W
15	30	N	rhyolite etc: red N 10 E, blue 87 E: W
16	50	N	estimated, rhyolite etc: red N 15 E, blue 45 E: E
17	85	N	estimated, rhyolite etc: red N 44 W, blue 90: E
18	Located about 2 miles W of Lower Narrows on N side of N range; sample shows various joint system directions; joint systems very well developed; major joint or bedding plane N 5 E dip 26 W; striations on this plane, direction N 30 E to N 5 E: red N 35 E, blue 62 N: line is in N–S direction, but arrowhead is not known and line may be on E or W side of thin section.		
19–21	inclusive: Located at small zone of fracture cleavage near base of E Bluff, behind the cottages at the N end of Devil's Lake.		
19	upper massive quartzite bed, contiguous to bed showing fracture cleavage: red N 35 W, blue vertical: W		
20	fracture cleavage zone: red N 35 W, blue vertical: W		
21	lower massive quartzite bed, contiguous to beds howing fracture cleavage: red N 25 E, blue vertical: W		

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