APPLICATION OF THE RULE OF GLADSTONE AND DALE TO MINERALS*

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ABSTRACT

The relation, $(n-1)/d=K$, the rule of Gladstone and Dale, holds very well for crystalline substances based upon data for 121 minerals selected from modern literature. The specific refractive energy values of the constituents of minerals, determined by Larsen, need very little revision. New values are given for Ce_2O_3, La_2O_3, Nd_2O_3, Pr_2O_3, Sm_2O_3, Sc_2O_3, MoO_3, Y_2O_3, and V_2O_5 in minerals.

To emphasize the relation between index of refraction, density, and chemical composition, Larsen (1921) suggested that the rule of Gladstone and Dale be applied to crystalline substances. Gladstone and Dale (1864) showed that $(n\text{-}1)/d=K$, and $K=(k_1)(p_1/100)+(k_2)(p_2/100)$ +etc., where K is the specific refractive energy of a liquid, k_1, k_2 are the specific refractive energies of the components of the liquid, and p_1, p_2 are the weight percentages of the components. A table of specific refractive energies (k) of the constituents of minerals was published by Larsen (1921 and 1934). Larsen's specific refractive energies are average values calculated from the most reliable mineralogical data available more than 30 years ago. Recently, Prof. Larsen suggested that the author publish a revised set of k values based upon more recent optical, chemical, and density data. With few exceptions, it was found that Larsen's data did not need revision. The few suggested changes are as given at the top of the next page.

When the rule, $(n-1)/d=K$, is applied to minerals, the arithmetical mean index of refraction, $(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$, is used and the relations are stated to hold approximately (Larsen and Berman, 1934).

Mineralogists have made relatively little use of the rule of Gladstone and Dale in the belief that the relations would not hold sufficiently well because of major differences in the manner of combination of the constituents of minerals. Where the optical, chemical, and density data are accurate, the Gladstone and Dale relations hold very well for most minerals, with only slight modifications due to the manner of combination of the constituents. Mineralogists would do well to reexamine their data before deciding that a mineral is an exception to the rule $[(n-1)/d=K]$. This paper was written on the advice of Prof. Larsen and several of the author's colleagues who did not expect that the rule would hold so well for most minerals.

All of the data are presented as a comparison of the calculated mean index of refraction, $dk+1$, with the arithmetical mean index of refraction, $(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$, determined experimentally.

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	Larsen and Berman (1934)	Jaffe (1955)
k Ce_2O_3	.160	.149 (*a*)
k La_2O_3	.149	.142 (*a*)
k Nd_2O_3	—	.138 (*a*)
k Pr_2O_3	—	.140 (*a*)
k Sm_2O_3	—	.141 (*a*)
k (Ce, La, Nd, Pr, Sm)$_2O_3$	—	.144 (*b*)
k Y_2O_3	.144	.170 (*c*)
k Fe_2O_3	.308	.290 (*d*)
		.310 (*e*)
		.404 (*f*)
k V_2O_5	.430	.340 (*g*)
k Sc_2O_3	—	.248 (*h*)
k MoO_3	—	.234 (*i*)

(*a*) derived from artificial inorganic compounds and minerals.
(*b*) derived from numerous minerals.
(*c*) derived from $Y_2(SO_4)_3 \cdot 8H_2O$, xenotime and yttrogarnet.
(*d*) derived from silicates.
(*e*) derived from hydrated ferric sulfates.
(*f*) derived from the oxide.
(*g*) derived from rossite and metatyuyamunite.
(*h*) derived from thortveitite.
(*i*) derived from powellite.

In Table 1, the calculated and experimental mean indices of refraction are compared for minerals representative of different silicate structure types.

Regardless of the different linkages of SiO_4 tetrahedra, the silicates in Table 1 show excellent agreement between the calculated and experi-

TABLE 1. COMPARISON OF CALCULATED AND EXPERIMENTAL MEAN INDICES OF REFRACTION FOR MINERALS OF DIFFERENT SILICATE-STRUCTURE TYPES

Mineral	O:Si Ratio	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
Forsterite Mg_2SiO_4	4:1	1.652	1.649	−0.003
Pyrope $Mg_3Al_2(SiO_4)_3$	4:1	1.705	1.705	0.000
Akermanite $Ca_2MgSi_2O_7$	7:2	1.634	1.628	−0.006
Benitoite $BaTiSi_3O_9$	3:1	1.773	1.779	+0.006
Enstatite $MgSiO_3$	3:1	1.654	1.649	−0.005
Talc $Mg_3(OH)_2Si_4O_{10}$	5:2	1.572	1.570	−0.002
Quartz SiO_2	2:1	1.547	1.551	+0.004

mental mean indices of refraction with a maximum deviation of only 0.006. The relations can be shown to hold very well for minerals of extremely complex chemical compositions. This may be demonstrated by a calculation of the mean index of refraction ($dk+1$) of schroeckingerite, $Ca_3Na[UO_2(CO_3)_3(SO_4)F]\cdot 10H_2O$.

Oxide	$p/100$		k		$(k)(p/100)$	
CaO	18.93	×	.225	=	.04259	
Na_2O	3.49	×	.181	=	.00632	
UO_3	32.19	×	.134	=	.04313	
CO_2	14.86	×	.217	=	.03225	
SO_3	9.01	×	.177	=	.01595	
H_2O	20.28	×	.340	=	.06895	
F	2.14	×	.043	=	.00092	
	100.90				.21011	
$-O=F$	.90	×	.203	=	.00183	
	100.00		K	=	.20828	$\alpha=1.489$
			d	=	2.51	$\beta=1.542$
			dK	=	.52278	$\gamma=1.542$

Mean index, $dk+1=1.523$ (n calculated).
Mean index, $(\alpha+\beta+\gamma)/3=1.524$ (n experimental).

Agreement between the calculated and experimental mean index of refraction is not always as good as $\pm 0.00X$. Some of the calculated and measured values for various minerals show much greater deviations. In some minerals it is thought that the chemical composition or density values are slightly in error inasmuch as the equation $(n-1)/d=K$ is very sensitive to small differences in density.

For example, schroeckingerite with $d=2.51$ and $K=.20828$ gives $dK+1=1.523$ and $(\alpha+\beta+\gamma)/3=1.524$. The calculated mean index of refraction would vary as follows with errors in density:

if $d=2.41$, $dK+1=1.502$ (-0.022)
if $d=2.61$, $dK+1=1.544$ $(+0.020)$.

Similarly small errors in chemical analysis may cause differences of as much as ± 0.02 in the calculated and measured mean indices of refraction. On occasion, however, small chemical errors may balance each other so that they may not be apparent if the constituents involved have similar specific refractive energies. For example, $k_{N_2O}=.181$ and $k_{K_2O}=.189$ and small errors in their determination may be hidden.

Differences in the manner of combination may be expected to modify the relations (Barth, 1930) but the effects are not as pronounced as might

be expected. For example, the relations in calcite and aragonite are illustrative, as follows:

calcite $dK+1=1.602$ $(2\omega+\epsilon)/3=1.601$ $(+0.001)$
aragonite $dK+1=1.652$ $(\alpha+\beta+\gamma)/3=1.632$ $(+0.020)$.

Assuming that the density and optical data are correct for aragonite, the deviation of 0.020 for the calculated mean index of aragonite must be a function of the different manner of combination of the constituents. In general, it can be expected that, regardless of the manner of combination of the mineral constituents, a deviation of not more than ± 0.020 in the mean index of refraction calculation can be obtained for most minerals from accurate chemical and density data. In most minerals studied by the author the calculated and experimental mean indices of refraction agree within ± 0.009 where the data are known to be reliable. If the relation $[(n-1)/d=K]$ holds so well for pure minerals, what would be the effect of isomorphism in a complex silicate series? There are too few complex mineral groups for which density and index of refraction

TABLE 2. COMPARISON OF THE CALCULATED AND EXPERIMENTAL MEAN INDICES OF REFRACTION FOR 7 MEMBERS OF THE ANTHOPHYLLITE SERIES

	1	8	9	14	17	29	30
SiO_2	42.90	45.47	46.06	48.38	50.32	57.16	57.05
TiO_2	0.49	0.44	0.53	0.41	0.43	None	None
Al_2O_3	17.82	15.86	14.95	13.23	8.05	1.40	1.94
Fe_2O_3	1.03	2.94	0.62	1.28	2.18	None	None
FeO	18.36	15.34	17.45	14.56	18.34	8.73	11.10
MnO	0.14	0.07	0.04	None	None	0.09	0.11
MgO	15.58	17.62	18.30	20.51	17.55	28.88	26.78
CaO	None	0.14	0.07	0.04	0.74	1.48	0.64
Na_2O	1.52	0.28	0.47	0.11	0.70	0.66	0.27
K_2O	0.03	None	None	None	None	None	0.06
F	0.31	None	None	None	None	None	None
H_2O+	1.95	1.84	1.51	1.48	1.69	1.60	2.05
	100.13						
$-O=F$	0.13						
	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Density	3.277	3.261	3.245	3.259	3.279	3.106	3.102
$(\alpha+\beta+\gamma)/3$	1.667	1.663	1.661	1.655	1.660	1.630	1.628
$dK+1$	1.666	1.672	1.660	1.667	1.670	1.637	1.633
Difference	−0.001	+0.009	−0.001	+0.012	+0.010	+0.007	+0.005

Chemical analyses by F. A. Gonyer (Rabbitt, 1948) after recalculation to 100.00 per cent.

Density and optical data determined by J. C. Rabbitt (Rabbitt, 1948).

measurements were made on analyzed material. Usually the density or indices of refraction are given as a range covering several chemical analyses. One good example of complete data is given by Rabbitt (1948) for a series of seven anthophyllites from Montana. Calculation of the mean index of refraction ($dK+1$) for each anthophyllite is compared with the experimental mean index of refraction, $[(\alpha+\beta+\gamma)/3]$ in Table 2. In calculating the mean index of refraction, all of the minor constituents were included and the analyses were recalculated to 100 per cent. Inasmuch as the analytical totals range from 99.77 to 100.23, the effects of the recalculations to 100.00 are negligible. The recalculation must be made because the relation, $[(n-1)/d=K]$, obviously requires that the specific refractive energy of the compound, K, be based upon 100 per cent. The agreement between the calculated and experimental mean n for each anthophyllite is excellent although the analyses show major variations in silica, alumina, and iron. Thus, if the chemical, optical, and density data are reliable, large-scale isomorphism will not seriously affect the relation $[(n-1)/d=K]$.

In Table 3, the calculated and experimental mean indices of refraction are compared for 121 minerals of widely different chemical composition for which the data seemed most reliable. Several minerals were rejected on the basis of (1) large differences in the calculated and measured density, (2) large amounts of material designated "remainder" in chemical analyses, and (3) poor analytical summations. For example, the measured density of cordylite and that calculated from x-ray data are given as 4.31 and 5.61, respectively (Palache, Berman, and Frondel, 1951). Obviously one of the values is in error for material of a given composition. Application of the rule of Gladstone and Dale suggests the density of cordylite should be near 4.76, assuming that the indices of refraction and chemical composition are correct. Table 3 includes the chemical composition, density, and indices of refraction for each of 121 minerals from which the n values ($dK+1$) were calculated. All of the analyses in Table 3 were recalculated to 100.00 per cent as required by the law of Gladstone and Dale.

Differences of the calculated from the experimental mean indices of refraction are less than ± 0.020 for 92.6 per cent of the entries (Table 3) of which 64.5 per cent are less than ± 0.009. The differences of $dK+1$ from $(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$ are graphically represented in Fig. 1. Some of the 7.4 per cent (9 in 121) of the entries for which $dk+1$ deviates by greater than ± 0.020 obviously result from poor data; others apparently reflect failure of the rule, $[(n-1)/d=K]$, to hold because of marked differences in polarizabilities of the constituents. For example, the data for 16 amphiboles in Table 3 show good agreement between the

(*Continued on page 775*)

TABLE 3. COMPARISON OF EXPERIMENTAL AND CALCULATED MEAN INDICES OF REFRACTION, $[(\alpha+\beta+\omega)]$ OR $[(2\omega+\epsilon)/3]$ WITH $dk+1$ FOR 121 MINERALS

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
Oxides			
1. Periclase, MgO MgO 94.03, FeO 5.97 *d* 5.72, *n* 1.745	1.745	1.732	−0.013
2. Bromellite, BeO BeO 97.89, CaO 1.03, BaO 0.55, MgO 0.07, Sb_2O_3 0.29, Al_2O_3 0.17 d 3.017, ω 1.719, ϵ 1.733	1.724	1.715	−0.009
3. Gahnite, $ZnAl_2O_4$ FeO 1.70, MnO 0.50, ZnO 41.31, Al_2O_3 53.28, Fe_2O_3 2. 51, SiO_2 0.70 *d* 4.57, *n* 1.818	1.818	1.816	−0.002
4. Zincite, ZnO ZnO 99.64, MnO 0.27, FeO 0.01, SiO_2 0.08 *d* 5.66, ω 2.013, ϵ 2.029	2.018	2.036	+0.018
5. Sassolite, $B(OH)_3$ B_2O_3 56.39, H_2O 43.61 *d* 1.48, α 1.340, β 1.456, γ 1.459	1.418	1.403	−0.015
6. Corundum, Al_2O_3 Al_2O_3 99.09, Fe_2O_3 0.91 *d* 4.0, ω 1.769, ϵ 1.760	1.766	1.775	+0.009
7. Baddelyite, ZrO_2 ZrO_2 98.93, Fe_2O_3 0.82, CaO 0.06, SiO_2 0.19 *d* 5.72, α 2.13, β 2.19, γ 2.20	2.173	2.154	−0.019
8. Quartz, SiO_2 SiO_2 100.00 *d* 2.66, ω 1.544, ϵ 1.553	1.547	1.551	+0.004
9. Tridymite, SiO_2 SiO_2 100.00 *d* 2.30, α 1.469, β 1.470, γ 1.473	1.471	1.476	+0.005
10. Cristobalite, SiO_2 SiO_2 100.00 *d* 2.3, *n* 1.486	1.486	1.476	−0.010
11. Rutile, TiO_2 TiO_2 100.00 *d* 4.23, ω 2.612, ϵ 2.899	2.708	2.691	+0.017
12. Anatase, TiO_2 TiO_2 100.00 *d* 3.90, ω 2.561, ϵ 2.488	2.537	2.548	+0.011
13. Brookite, TiO_2 TiO_2 100.00 *d* 4.14, α 2.583, β 2.584, γ 2.700	2.622	2.643	+0.021

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
14. Portlandite, $Ca(OH)_2$			
CaO 75.64, H_2O 24.31			
d 2.230, ω 1.574, ϵ 1.547	1.565	1.564	−0.001
Carbonates			
15. Calcite, $CaCO_3$			
CaO 56.03, CO_2 43.97			
d 2.710, ω 1.658, ϵ 1.486	1.601	1.602	+0.001
16. Aragonite, $CaCO_3$			
CaO 56.03, CO_2 43.97			
d 2.947, α 1.531, β 1.681, γ 1.685	1.632	1.652	+0.020
17. Magnesite, $MgCO_3$			
MgO 47.81, CO_2 52.19			
d 3.00, ω 1.700, ϵ 1.509	1.636	1.627	−0.009
18. Siderite, $FeCO_3$			
FeO 62.01, CO_2 37.99			
d 3.96, ω 1.875, ϵ 1.633	1.794	1.786	−0.008
19. Rhodochrosite, $MnCO_3$			
MnO 61.71, CO_2 38.29			
d 3.70, ω 1.816, ϵ 1.597	1.744	1.743	−0.001
20. Smithsonite, $ZnCO_3$			
ZnO 64.90, CO_2 35.10			
d 4.43, ω 1.848, ϵ 1.621	1.772	1.777	+0.005
21. Strontianite, $SrCO_3$			
SrO 62.55, CaO 6.10, CO_2 31.35			
d 3.628, α 1.520, β 1.667, γ 1.669	1.618	1.621	+0.003
22. Alstonite, $CaBa(CO_3)_2$			
CaO 17.64, BaO 48.64, SrO 4.25, CO_2 29.47			
d 3.707, α 1.526, β 1.671, γ 1.672	1.623	1.636	+0.013
23. Barytocalcite, $CaBa(CO_3)_2$			
BaO 51.56, CaO 18.85, CO_2 29.59			
d 3.66, α 1.525, β 1.684, γ 1.696	1.632	1.630	−0.002
24. Shortite, $Na_2Ca_2(CO_3)_2$			
Na_2O 20.25, CaO 36.63, CO_2 43.12			
d 2.605, α 1.531, β 1.555, γ 1.570	1.552	1.554	+0.002
25. Thermonatrite, $Na_2CO_3 \cdot H_2O$			
Na_2O 50.03, CO_2 35.45, H_2O 14.52			
d 2.255, α 1.420, β 1.509, γ 1.525	1.485	1.489	+0.004
26. Nesquehonite, $MgCO_3 \cdot 3H_2O$			
MgO 29.11, CO_2 31.65, H_2O 39.24			
d 1.842, α 1.417, β 1.503, γ 1.527	1.482	1.480	−0.002
27. Lansfordite, $MgCO_3 \cdot 5H_2O$			
MgO 23.25, CO_2 25.06, H_2O 51.69			
d 1.694, α 1.456, β 1.469, γ 1.508	1.478	1.469	−0.009

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
28. Natron, $Na_2CO_3 \cdot 10H_2O$			
Na_2O 21.66, CO_2 15.38, H_2O 62.96			
d 1.478, α 1.405, β 1.425, γ 1.440	1.423	1.424	+0.001
29. Pirssonite, $Na_2Ca(CO_3)_2 \cdot 2H_2O$			
Na_2O 25.73, CaO 23.41, CO_2 36.11, H_2O 14.75			
d 2.352, α 1.504, β 1.509, γ 1.575	1.529	1.536	+0.007
30. Gaylussite, $Na_2Ca(CO_3)_2 \cdot 5H_2O$			
Na_2O 20.42, CaO 19.03, CO_2 30.05, H_2O 30.50			
d 1.991, α 1.443, β 1.516, γ 1.523	1.494	1.495	+0.001
31. Lanthanite, $(Ce, La)_2(CO_3)_3 \cdot 8H_2O$			
$(Ce, La)_2O_3$ 55.03, CO_2 21.95, H_2O 24.21			
d 2.84, α 1.520, β 1.587, γ 1.613	1.573	1.587	+0.014
32. Artinite, $Mg_2(CO_3)(OH)_2 \cdot 3H_2O$			
MgO 40.99, CO_2 22.37, H_2O 36.64			
d 2.02, α 1.488, β 1.534, γ 1.556	1.526	1.515	−0.011
33. Hydrocerussite, $Pb_3(CO_3)_2(OH)_2$			
PbO 86.33, CO_2 11.35, H_2O 2.32			
d 6.94, ω 2.09, ϵ 1.94	2.040	2.046	+0.006
34. Dawsonite, $NaAl(CO_3)(OH)_2$			
Na_2O 21.81, Al_2O_3 36.01, CO_2 30.51, H_2O 11.61			
d 2.44, α 1.466, β 1.542, γ 1.596	1.535	1.524	−0.011
35. Bastnaesite, $(Ce, La)(CO_3)F$			
Ce_2O_3 38.23, La_2O_3 36.79, CO_2 20.30, F 7.94, H_2O 0.08			
d 5.12, ω 1.717, ϵ 1.818	1.751	1.763	+0.012
36. Ancylite, $(Ce, La)_4(Sr, Ca)_3(CO_3)_7(OH)_4 \cdot 3H_2O$			
CaO 1.54, SrO 21.25, FeO 0.35, Ce_2O_3 22.45, La_2O_3 24.29, CO_2 23.53, H_2O 6.59			
d 3.95, α 1.625, β 1.700, γ 1.735	1.687	1.692	+0.005
37. Tychite, $Na_6Mg_2(SO_4)(CO_3)_4$			
Na_2O 35.58, MgO 15.42, SO_3 15.32, CO_2 33.68			
d 2.588, $n=1.510$	1.510	1.506	−0.004
38. Roentgenite, $Ca_2(Ce, La)_3(CO_3)_5F_3$			
CaO 13.11, $(Ce, La)_2O_3$ 57.32, CO_2 25.71, F 6.66			
d 4.19 (calc.), ω 1.662, ϵ 1.756	1.693	1.694	+0.001
39. Parisite, $(Ce, La)_2 Ca(CO_3)_3F_2$			
CaO 10.10, BaO 0.33, $(Ce, La)_2O_3$ 61.57, CO_2 23.95, F 7.00			
d 4.32, ω 1.667, ϵ 1.760	1.698	1.695	−0.003
40. Sahamalite, $(Ce, La)_2(Mg, Fe)(CO_3)_4$			
$(Ce, La)_2O_3$ 59.8, MgO 6.2, FeO 2.0, CO_2 32.0			
d 4.30, α 1.679, β 1.776, γ 1.807	1.754	1.738	−0.016
41. Andersonite, $Na_2Ca(UO_2)(CO_3)_3 \cdot 6H_2O$			
MgO 0.5, CaO 8.1, Na_2O 9.6, UO_3 44.9, CO_2 20.3, H_2O 16.0			
d 2.8, ω 1.520, ϵ 1.540	1.527	1.552	+0.025

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
42. Bayleyite, $Mg_2(UO_2)(CO_3)_3 \cdot 18H_2O$			
MgO 9.76, Na_2O 0.21, K_2O 0.10, UO_3 35.29, CO_2 16.72, H_2O 37.92			
d 2.05, α 1.455, β 1.490, γ 1.500	1.482	1.477	−0.005
43. Swartzite, $CaMg(UO_2)(CO_3)_3 \cdot 12H_2O$			
CaO 7.32, MgO 5.47, Na_2O 0.26, K_2O 0.49, UO_3 38.85, CO_2 17.92, H_2O 29.69			
d 2.3, α 1.465, β 1.51, γ 1.540	1.505	1.508	+0.003
44. Leadhillite, $Pb_4(SO_4)(CO_3)_2(OH)_2$			
PbO 82.78, SO_3 7.36, CO_2 8.17, H_2O 1.690			
d 6.55, α 1.87, β 2.00, γ 2.01	1.960	1.982	+0.022
Nitrates			
45. Darapskite, $Na_3(NO_3)(SO_4) \cdot H_2O$			
Na_2O 38.00, N_2O_5 22.10, SO_3 32.65, H_2O 7.25			
d 2.20, α 1.391, β 1.481, γ 1.486	1.453	1.449	−0.004
46. Gerhardtite, $Cu_2(NO_3)(OH)_3$			
CuO 66.12, N_2O_5 22.67, H_2O 11.21			
d 3.43, α 1.703, β 1.713, γ 1.722	1.713	1.750	+0.037
47. Nitrobarite, $Ba(NO_3)_2$			
BaO 58.67, N_2O_5 41.33			
d 3.250, *n* 1.571	1.571	1.564	−0.007
48. Niter, KNO_3			
K_2O 46.58, N_2O_5 53.42			
d 2.109, α 1.332, β 1.504, γ 1.504	1.447	1.456	+0.009
49. Soda-Niter, $NaNO_3$			
Na_2O 36.46, N_2O_5 63.54			
d 2.266, ω 1.587, ϵ 1.336	1.503	1.495	−0.008
50. Nitrocalcite, $Ca(NO_3)_2 \cdot 4H_2O$			
CaO 23.75, N_2O_5 45.75, H_2O 30.50			
d 1.90, α 1.465, β 1.498, γ 1.504	1.489	1.507	+0.018
Iodate			
51. Lautarite, $Ca(IO_3)_2$			
CaO 14.95, I_2O_5 85.04			
d 4.59, α 1.792, β 1.840, γ 1.888	1.840	1.845	+0.005
Borates			
52. Kernite, $Na_2B_4O_7 \cdot 2H_2O$			
Na_2O 22.65, B_2O_3 50.80, H_2O 26.55			
d 1.93, α 1.455, β 1.472, γ 1.487	1.471	1.469	−0.002
53. Probertite, $NaCaB_5O_9 \cdot 5H_2O$			
Na_2O 8.53, CaO 15.45, B_2O_3 50.44, H_2O 25.58			
d 2.141, α 1.514, β 1.524, γ 1.543	1.527	1.531	+0.004

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
54. Colemanite, $Ca_2B_6O_{11}\cdot 5H_2O$			
CaO 27.84, B_2O_3 49.61, H_2O 22.55			
d 2.423, α 1.568, β 1.592, γ 1.614	1.591	1.602	+0.011
55. Tincalconite, $Na_2B_4O_7\cdot 5H_2O$			
Na_2O 21.47, B_2O_3 47.42, H_2O 31.11			
d 1.880, ω 1.461, ϵ 1.474	1.465	1.468	+0.003
56. Borax, $Na_2B_4O_7\cdot 10H_2O$			
Na_2O 16.26, B_2O_3 36.51, H_2O 47.23			
d 1.715, α 1.447, β 1.469, γ 1.472	1.463	1.463	0.000
57. Inyoite, $Ca_2B_6O_{11}\cdot 13H_2O$			
CaO 20.5, B_2O_3 37.2, H_2O +26.1, H_2O −16.2			
d 1.875, α 1.495, β 1.510, γ 1.520	1.508	1.510	+0.002
Sulfates			
58. Barite, $BaSO_4$			
BaO 65.70, SO_3 34.30			
d 4.50, α 1.636, β 1.637, γ 1.648	1.641	1.649	+0.008
59. Celestite, $SrSO_4$			
SrO 56.42, SO_3 43.58			
d 3.97, α 1.621, β 1.624, γ 1.631	1.625	1.625	0.000
60. Anhydrite, $CaSO_4$			
CaO 41.19, SO_3 58.81			
d 2.98, α 1.570, β 1.575, γ 1.614	1.586	1.586	0.000
61. Alunite, $KAl_3(SO_4)_2(OH)_6$			
K_2O 10.02, Al_2O_3 39.65, SO_3 35.50, H_2O 14.83			
d 2.752, ω 1.572, ϵ 1.592	1.579	1.574	−0.005
62. Boussingualtite, $(NH_4)_2Mg(SO_4)_2\cdot 6H_2O$			
$(NH_4)_2O$ 14.44, MgO 11.18, SO_3 44.40, H_2O 29.98			
d 1.722, α 1.472, β 1.473, γ 1.479	1.475	1.474	−0.001
63. Ilesite, $MnSO_4\cdot 4H_2O$			
MnO 31.82, SO_3 35.88, H_2O 32.30			
d 2.25, α 1.511, β 1.519, γ 1.521	1.517	1.527	+0.010
Phosphates, Arsenates, Vanadates, Tungstates			
64. Moraesite, $Be_2PO_4(OH)\cdot 4H_2O$			
BeO 25.32, P_2O_5 34.82, H_2O 39.86			
d 1.805, α 1.462, β 1.482, γ 1.490	1.478	1.473	−0.005
65. Svanbergite, $SrAl_3(PO_4)(SO_4)(OH)_6$			
Al_2O_3 36.98, Fe_2O_3 0.24, CaO 3.25, SrO 12.87, P_2O_5 16.74, SO_3 17.38, H_2O 12.54			
d 3.22, ω 1.635, ϵ 1.649	1.640	1.654	+0.014

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
66. Arrojadite, $(Na, K)_5(Fe, Mn, Ca)_{16}(PO_4)_{12}(F, OH)\cdot H_2O$			
P_2O_5 40.10, Al_2O_3 2.67, FeO 28.29, MnO 15.82, MgO 1.04, CaO 2.47, Li_2O 0.09, Na_2O 6.41, K_2O 1.74, H_2O 0.91, F 0.80			
d 3.553, α 1.664, β 1.670, γ 1.675	1.670	1.675	+0.005
67. Graftonite, $(Fe, Mn, Ca)_3(PO_4)_2$			
P_2O_5 39.85, Al_2O_3 0.20, FeO 30.85, MnO 21.92, MgO 0.10, Li_2O 0.05, Na_2O 0.28, CaO 6.03, H_2O 0.60, F 0.20			
d 3.775, α 1.709, β 1.714, γ 1.736	1.720	1.725	+0.005
68. Lazulite, $MgAl_2(PO_4)_2(OH)_2$			
MgO 11.96, FeO 2.80, CaO 0.08, Al_2O_3 32.53, Fe_2O_3 0.49, TiO_2 0.16, P_2O_5 46.08, H_2O 5.90			
d 3.118, α 1.610, β 1.634, γ 1.644	1.629	1.629	0.000
69. Scorzalite, $FeAl_2(PO_4)(OH)_2$			
MgO 2.93, FeO 17.07, MnO 0.10, CaO 0.03, Al_2O_3 30.83, Fe_2O_3 0.13, TiO_2 0.10, P_2O_5 42.71 H_2O 6.10			
d 3.327, α 1.636, β 1.666, γ 1.676	1.659	1.666	+0.007
70. Zincian rockbridgeite, $(Fe'', Mn)(Fe'''_{4-y}, Zn_y)(PO_4)_3(OH)_{5-y}\cdot {}_yH_2O$			
P_2O_5 33.74, FeO 10.86, MnO 2.11, Li_2O 0.01, Na_2O 0.13, ZnO 5.20, Fe_2O_3 41.20, H_2O 6.75			
d 3.51, α 1.82, β 1.83, γ 1.88	1.843	1.839	−0.004
71. Montebrasite, (Li, Na)Al(PO$_4$)(OH, F)			
Li_2O 9.68, Na_2O 0.43, Al_2O_3 35.31, P_2O_5 47.70, F 5.42, K_2O 0.10, MgO 0.33, H_2O 3.31			
d 3.085, α 1.594, β 1.608, γ 1.616	1.606	1.615	+0.009
72. Adamite, $Zn_2(AsO_4)(OH)$			
ZnO 57.05, As_2O_5 39.15, H_2O 3.54, SiO_2 0.26			
d 4.435, α 1.722, β 1.742, γ 1.763	1.742	1.736	−0.006
73. Brazilianite, $NaAl_3(PO_4)_2(OH)_4$			
Na_2O 8.29, K_2O 0.20, Al_2O_3 42.77, Fe_2O_3 0.03, TiO_2 0.05, P_2O_5 38.71, H_2O 9.95			
d 2.985, α 1.602, β 1.609, γ 1.623	1.611	1.614	+0.003
74. Ludlamite, $Fe_3(PO_4)_2\cdot 4H_2O$			
FeO 49.59, MgO 0.97, P_2O_5 33.20, H_2O 16.24			
d 3.14, β 1.650, β 1.667, γ 1.688	1.668	1.669	+0.001
75. Monazite, (Ce, La)PO$_4$			
$(Ce, La)_2O_3$ 67.23, ThO_2 2.98, SiO_2 0.70, P_2O_5 28.97, PbO 0.12			
d 4.98, α 1.779, β 1.782, γ 1.833	1.798	1.781	−0.017

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
76. Monazite			
$(Ca, La)_2O_3$ 65.28, Y_2O_3 5.13, P_2O_5 29.59			
d 5.173, α 1.785, β 1.787, γ 1.840	1.804	1.822	+0.018
77. Variscite, $AlPO_4 \cdot 2H_2O$			
Al_2O_3 32.38, Fe_2O_3 0.06, Cr_2O_3 0.18, P_2O_5 44.71, H_2O 22.67			
d 2.53, α 1.563, β 1.588, γ 1.594	1.582	1.570	−0.012
78. Scorodite, $Fe'''(AsO_4) \cdot 2H_2O$			
Fe_2O_3 34.75, As_2O_5 49.45, TiO_2 0.02, Sb_2O_5 0.06, SiO_2 0.30, H_2O 15.42			
d 3.278, α 1.784, β 1.795, γ 1.814	1.798	1.779	−0.019
79. Mansfieldite, $Al(AsO_4) \cdot 2H_2O$			
Fe_2O_3 0.88, Al_2O_3 23.30, As_2O_5 56.43, P_2O_5 0.59, Sb_2O_5 0.12, TiO_2 0.91, H_2O 17.77			
d 3.031, α 1.622, β 1.624, γ 1.642	1.629	1.628	−0.001
80. Hurlbutite, $CaBe_2(PO_4)_2$			
CaO 21.99, BeO 21.44, P_2O_5 56.57			
d 2.877, α 1.595, β 1.601, γ 1.604	1.600	1.598	−0.002
81. Fluorapatite, $Ca_5(PO_4)_3F$			
CaO 55.59, P_2O_5 42.22, F 3.78			
d 3.18, ω 1.6325, ϵ 1.630	1.632	1.648	+0.016
82. Chlorapatite, $Ca_5(PO_4)_3Cl$			
CaO 53.85, P_2O_5 40.88, Cl 6.81			
d 3.17, ω 1.6684, ϵ 1.6675	1.669	1.686	+0.017
83. Rossite, $CaV_2O_6 \cdot 4H_2O$			
CaO 18.09, V_2O_5 58.67, H_2O 23.24			
d 2.45, α 1.710, β 1.770, γ 1.840	1.773	1.782	+0.009
84. Scheelite, $CaWO_4$			
CaO 19.47, WO_3 80.53			
d 6.10, ω 1.920, ϵ 1.936	1.925	1.921	−0.004
Silicates			
85. Anthophyllite, $(Mg, Fe, Al)_7(Si, Al)_8O_{22}(OH, F)_2$			
SiO_2 42.90, TiO_2 0.49, Al_2O_3 17.82, Fe_2O_3 1.03, FeO 18.36, MnO 0.14, MgO 15.58, Na_2O 1.52, K_2O 0.03, F 0.31, H_2O 1.95			
d 3.277, α 1.656, β 1.667, γ 1.678	1.667	1.666	−0.001
86. Anthophyllite			
SiO_2 45.47, TiO_2 0.44, Al_2O_3 15.86, Fe_2O_3 2.94, FeO 15.34, MnO 0.07, MgO 17.62, CaO 0.14, Na_2O 0.28, H_2O 1.84			
d 3.216, α 1.655, β 1.663, γ 1.672	1.663	1.672	+0.009

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
87. Anthophyllite			
SiO_2 46.06, TiO_2 0.53, Al_2O_3 14.95, Fe_2O_3 0.62, FeO 17.45, MnO 0.04, MgO 18.30, CaO 0.07, Na_2O 0.47, H_2O 1.51			
d 3.245, α 1.652, β 1.660, γ 1.669	1.661	1.660	−0.001
88. Anthophyllite			
SiO_2 48.38, TiO_2 0.41, Al_2O_3 13.23, Fe_2O_3 1.28, FeO 14.56, MgO 20.51, CaO 0.04, Na_2O 0.11, H_2O 1.48			
d 3.259, α 1.648, β 1.654, γ 1.662	1.655	1.667	+0.012
89. Anthophyllite			
SiO_2 50.32, TiO_2 0.43, Al_2O_3 8.05, Fe_2O_3 2.18, FeO 18.34, MgO 17.55, CaO 0.74, Na_2O 0.70, H_2O 1.69			
d 3.279, α 1.654, β 1.659, γ 1.667	1.660	1.670	+0.010
90. Anthophyllite			
SiO_2 57.16, Al_2O_3 1.40, FeO 8.73, MnO 0.09, MgO 28.88, CaO 1.48, Na_2O 0.66, H_2O 1.60			
d 3.106, α 1.618, β 1.637, γ 1.635	1.630	1.637	+0.007
91. Anthophyllite			
SiO_2 57.05, Al_2O_3 1.94, FeO 11.10, MnO 0.11 MgO 26.78, CaO 0.64, Na_2 0.27, K_2O 0.06, H_2O 2.05			
d 3.102, α 1.616, β 1.628, γ 1.641	1.628	1.633	+0.005
92. Amphibole, $NaCa_2(Mg, Fe)_5(Al, Si)_7O_{22}(OH, F)_2$			
SiO_2 44.61, TiO_2 0.63, Al_2O_3 16.72, Fe_2O_3 0.72, FeO 8.20, MnO 0.11, CaO 10.36, MgO 15.26, Na_2O 1.72, K_2O 0.11, H_2O 1.51, F 0.08			
d 3.17, α 1.646, β 1.653, γ 1.668	1.656	1.656	0.000
93. Amphibole			
SiO_2 45.10, TiO_2 2.05, Al_2O_3 13.39, Fe_2O_3 1.58, FeO 9.60, MnO 0.09, MgO 13.69, CaO 11.15, Na_2O 1.55, K_2O 0.21, H_2O 1.49, F 0.18			
d 3.17, α 1.661, β 1.669, γ 1.678	1.669	1.668	−0.001
94. Amphibole			
SiO_2 48.07, TiO_2 0.57, Al_2O_3 11.16, Fe_2O_3 0.61, FeO 11.31, MnO 0.08, MgO 13.34, CaO 11.67, Na_2O 0.91, K_2O 0.14, H_2O 2.05, F 0.16			
d 3.15, α 1.648, β 1.664, γ 1.678	1.663	1.676	+0.013
95. Amphibole			
SiO_2 45.12, TiO_2 1.70, Al_2O_3 9.47, Fe_2O_3 3.45, FeO 14.48, MnO 0.07, MgO 10.46, CaO 11.50, Na_2O 1.45, K_2O 0.79, H_2O 1.37, F 0.23			
d 3.51, α 1.667, β 1.678, γ 1.685	1.677	1.741	+0.064

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
96. Amphibole			
SiO_2 39.51, TiO_2 1.46, Al_2O_3 12.16, Fe_2O_3 4.10, FeO 23.15, MnO 0.09, MgO 4.42, CaO 9.97, Na_2O 1.81, K_2O 1.38, H_2O 1.26, F 1.20			
d 3.42, α 1.690, β 1.702, γ 1.711	1.701	1.709	+0.008
97. Muscovite, $KAl_2Si_4O_{10}(OH, F)_2$			
SiO_2 46.75, Al_2O_3 34.73, Fe_2O_3 0.71, FeO 0.77, MgO 0.92, TiO_2 0.21, CaO 0.13, Na_2O 0.47, K_2O 10.61, BaO 0.13, F 0.16, H_2O 4.48			
d 2.82, α 1.555, β 1.589, γ 1.590	1.578	1.582	+0.004
98. Dalyite, $K_2ZrSi_6O_{15}$			
SiO_2 61.91, ZrO_2 21.72, K_2O 14.62, Na_2O 1.75			
d 2.84, α 1.575, β 1.590, γ 1.601	1.589	1.575	−0.014
99. Biotite, $K(Mg, Fe)_3(Si, Al)_4O_{10}(OH)_2$			
SiO_2 34.95, Al_2O_3 19.15, TiO_2 2.86, Fe_2O_3 0.88, FeO 20.81, MnO 0.10, MgO 8.42, CaO 0.26, Na_2O 0.24, K_2O 8.54, H_2O 3.79			
d 3.06, α 1.595, β 1.649, γ 1.649	1.631	1.640	+0.009
100. Kaersutite, $Ca_4(Na, K)_2(Mg, Fe)_7Al_6TiSi_{12}O_{46}(OH, F)_2$			
SiO_2 41.39, TiO_2 5.69, Al_2O_3 14.21, Fe_2O_3 3.32, FeO 5.69, MnO 0.08, MgO 13.64, CaO 11.60, Na_2O 2.29, K_2O 1.72, H_2O 0.12, F 0.42			
d 3.215, α 1.670, β 1.692, γ 1.701	1.688	1.698	+0.010
101. Searlesite, $Na_2B_2Si_4O_{12}\cdot 2H_2O$			
SiO_2 58.79, B_2O_3 16.92, Na_2O 15.29, H_2O 8.89, Al_2O_3 0.04, Fe_2O_3 0.04, MgO 0.03			
d 2.46, α 1.516, β 1.531, γ 1.535	1.527	1.534	+0.007
102. Hornblende, $W_{2-3}(XY)_5(Z_4O_{11})_2(OH, F, Cl)_2$			
SiO_2 39.80, Al_2O_3 11.39, Fe_2O_3 5.93, FeO 14.22, MgO 9.62, CaO 9.68, Na_2O 1.57, K_2O 1.60, H_2O+ 2.59, H_2O- 0.25, TiO_2 1.47, F 1.29, Cl 0.58, MnO 0.68			
d 3.211, α 1.666, β 1.689, γ 1.693	1.683	1.683	0.000
103. Hornblende			
SiO_2 38.38, Al_2O_3 11.01, Fe_2O_3 5.79, FeO 24.63, MgO 1.97, CaO 9.54, Na_2O 1.81, K_2O 1.72, H_2O 1.28, TiO_2 2.37, F 0.96, Cl 0.60, MnO 0.48			
d 3.445, α 1.694, β 1.717, γ 1.723	1.711	1.728	+0.017
104. Hornblende			
SiO_2 41.37, Al_2O_3 10.43, Fe_2O_3 3.86, FeO 16.33, MgO 8.05, CaO 10.29, Na_2O 1.59, K_2O 1.46, H_2O 1.80, TiO_2 2.91, F 1.17, Cl 0.60, MnO 0.76			
d 3.258, α 1.680, β 1.692, γ 1.696	1.689	1.693	+0.004

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
105. Pyroxene, $Ca(Mg, Fe)Si_2O_6$ SiO_2 52.49, Al_2O_3 2.72, Fe_2O_3 2.59, FeO 12.66, MgO 8.73, CaO 16.76, Na_2O 0.78, K_2O 0.28, H_2O 0.94, TiO_2 1.33, MnO 0.72 *d* 3.315, α 1.689, β 1.695, γ 1.716	1.700	1.700	0.000
106. Augite, $Ca(Mg, Fe)Si_2O_6$ SiO_2 50.13, Al_2O_3 4.31, Fe_2O_3 1.97, FeO 8.90, MgO 12.04, CaO 20.50, Na_2O 0.52, H_2O 0.74 *d* 3.315, α 1.689, β 1.695, γ 1.716	1.700	1.700	0.000
107. Ferrosalite, $CaFeSi_2O_6$ SiO_2 50.73, Al_2O_3 1.06, Fe_2O_3 0.53, FeO 18.57, MgO 5.70, CaO 22.86, Na_2O 0.16, K_2O 0.02, H_2O 0.12, TiO_2 0.07, MnO 0.18 *d* 3.413, α 1.708, β 1.714, γ 1.736	1.719	1.708	−0.011
108. Hedenbergite, $CaFeSi_2O_6$ SiO_2 48.41, Al_2O_3 0.30, Fe_2O_3 1.50, FeO 22.97, MgO 1.06, CaO 21.33, Na_2O 0.14, K_2O 0.03, H_2O 0.46, TiO_2 0.08, MnO 3.72 *d* 3.535, α 1.722, β 1.730, γ 1.750	1.734	1.734	0.000
109. Antigorite, $Mg_6Si_4O_{10}(OH)_8$ SiO_2 43.64, Al_2O_3 1.03, Cr_2O_3 0.02, Fe_2O_3 0.90, FeO 0.81, MnO 0.04, MgO 41.03, NiO 0.16, CaO 0.05, Na_2O 0.01, K_2O 0.03, H_2O 12.27 *d* 2.603, α 1.561, β 1.566, γ 1.567	1.565	1.575	+0.010
110. Kornerupine, $R_{40}(Si, B)_{18}O_{86}$ SiO_2, 30.28, B_2O_3 3.51, Al_2O_3 40.92, Fe_2O_3 0.42, FeO 8.53, MgO 14.89, CaO 0.06, Na_2O 0.08, H_2O 0.97, TiO_2 0.19, P_2O_5 0.09, Cr_2O_3 0.06 *d* 3.37, α 1.681, β 1.694, γ 1.695	1.690	1.677	−0.013
111. Kornerupine SiO_2 30.6, B_2O_3 2.8, Al_2O_3 37.7, Fe_2O_3 3.3, FeO 4.8, MgO 20.8 *d* 3.335, α 1.669, β 1.681, γ 1.682	1.677	1.675	−0.002
112. Chloritoid, $(Fe, Mg)_2(Al, Fe)_2(Al, Si)_4O_{10}(OH)_4$ SiO_2 25.12, Al_2O_3 40.25, Fe_2O_3 3.22, FeO 19.62, MnO 1.05, MgO 3.89, K_2O 0.09, H_2O 6.76 *d* 3.52, α 1.719, β 1.721, γ 1.725	1.722	1.734	+0.012
113. Ferrocarpholite, $H_4(Fe, Mg)Al_2Si_2O_{10}$ SiO_2 37.59, Al_2O_3 29.39, Fe_2O_3 2.07, TiO_2 0.22, MgO 2.52, MnO 0.14, FeO 17.98, H_2O 10.08 *d* 3.04, α 1.628, β 1.644, γ 1.647	1.640	1.652	+0.012

TABLE 3—(*continued*)

Mineral data	$(\alpha+\beta+\gamma)/3$ or $(2\omega+\epsilon)/3$	$dK+1$	Deviation
114. Allanite, $X_2Y_3Z_3O_{12}OH$			
SiO_2 28.66, Al_2O_3 9.95, Fe_2O_3 10.20, FeO 7.29, TiO_2 2.00, MgO 0.58, MnO 6.71, CaO 10.03, ThO_2 0.95, UO_2 0.01, $(Ce, La)_2O_3$ 22.30, H_2O 1.32			
d 3.95, α 1.791, β 1.815, γ 1.822	1.809	1.806	−0.003
115. Manganpyrosmalite, $(Mn, Fe)_8(Si_6O_{15})(OH, Cl)_{10}$			
MnO 39.25, FeO 12.48, MgO 0.74, ZnO 1.95, SiO_2 34.27, As_2O_5 0.13, Cl 3.82, H_2O 8.22			
d 3.13, ω 1.669, ϵ 1.631	1.656	1.663	+0.007
116. Wollastonite, $CaSiO_3$			
CaO 48.29, SiO_2 51.71			
d 2.915, α 1.616, β 1.629, γ 1.631	1.625	1.629	+0.004
117. Pseudowollastonite, $CaSiO_3$			
CaO 48.29, SiO_2 51.71			
d 2.905, α 1.610, β 1.610, γ 1.654	1.625	1.627	+0.002
118. Phenacite, Be_2SiO_4			
SiO_2 54.45, BeO 45.55			
d 3.00, ω 1.654, ϵ 1.668	1.659	1.663	+0.004
119. Fayalite, Fe_2SiO_4			
SiO_2 29.48, FeO 70.52			
d 4.34, α 1.835, β 1.877, γ 1.886	1.866	1.837	−0.029
120. Tremolite, $Ca_2Mg_5Si_8O_{22}(OH)_2$			
SiO_2 59.15, CaO 13.81, MgO 24.82, H_2O 2.22			
d 2.9, α 1.599, β 1.613, γ 1.625	1.612	1.611	−0.001
121. Acmite, $NaFeSi_2O_6$			
SiO_2 52.01, Fe_2O_3 34.57, Na_2O 13.42			
d 3.55, α 1.776, β 1.819, γ 1.836	1.810	1.824	+0.014

INDEX TO DATA OF TABLE 3

1. PALACHE, C., BERMAN, H., AND FRONDEL, C., (1944), Dana's System of Mineralogy: 7th Ed., Vol. I, John Wiley and Sons: N. Y., p. 499, anal. 1.
2. *Ibid.*, p. 507, anal. 1.
3. *Ibid.*, p. 691, anal. 11.
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(*Continued from page 761*)

calculated and experimental n values for all but one entry, #95, where $dK+1$ deviates from $(\alpha+\beta+\gamma)/3$ by $+0.064$. The chemical composition of this amphibole, #95, suggests that the density of 3.51 is in error on the high side. Amphibole #96, for example, contains much more combined iron and titanium than amphibole #95 and has a lower density of 3.42.

The density of amphibole #95 calculated from $(n-1)/d=K$ is 3.21 and suggests that the measured value 3.51 is either a typographical or experimental error. On the other hand, the deviations of the calculated from the measured mean n values for rutile, anatase, and brookite (nos. 11, 12, 13, Table 3) are +0.029, +0.011, and +0.021 and must result from differences in bonding in the 3 different structures of the poly-

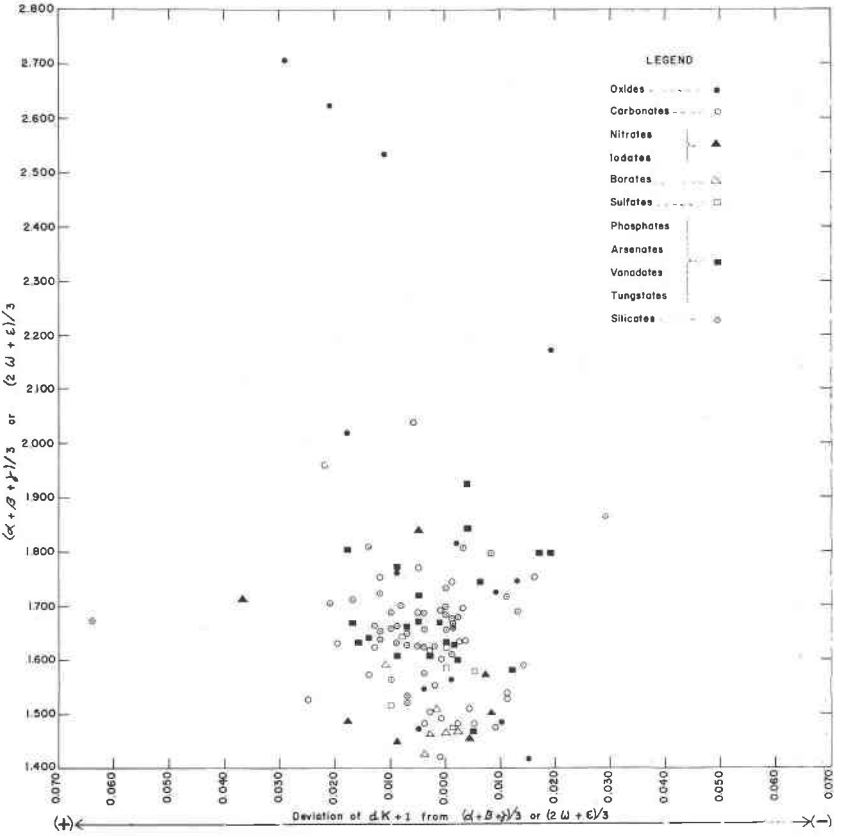

FIG. 1. ±Deviation of $dK+1$ from $(\alpha+\beta+\gamma/3)$ or $(2\omega+\epsilon)$ for 121 minerals.

morphs. Inasmuch as all 3 polymorphs show a positive deviation of $dk+1$, there is good possibility that Larsen's value, $k_{TiO_2}=.397$, is slightly on the high side.

The writer concludes that, on the basis of data for 121 minerals from the modern literature, the rule of Gladstone and Dale, $[(n-1)/d=K]$, holds surprisingly well for most minerals. Here is a rapid, workable method for (1) evaluating much of the old mineralogical data in the

literature, (2) checking new data before publication, (3) calculating a reasonably reliable approximation of the mean index of refraction or density where one or the other cannot be measured, and (4) locating an inaccurate chemical determination where the density, indices of refraction, and analytical determinations are otherwise accurate, e.g. use of the rule, $[(n-1)/d=K]$, revealed that an 88 per cent summation for a preliminary analysis of sahamalite, (Mg, Fe)(Ce, La)$_2$(CO$_3$)$_4$, resulted from an error in the CO_2 determination. This was accomplished by determining that the specific refractive energy (k) of the missing 12 per cent was approximately that of CO_2 (Jaffe, Meyrowitz, and Evans, 1953).

If this paper encourages more widespread use of the neglected rule of Gladstone and Dale, it will have served its purpose.

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