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DISCREPANCIES BETWEEN OPTIC AXIAL ANGLES OF OLIVINES MEASURED OVER DIFFERENT BISECTRICES*

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ABSTRACT

More than 100 measurements of 2V were made on forsteritic olivines from two picrite sills in Soay, Scotland, using both double and single axis conoscopic methods. The mean of double axis measurements agrees closely with the mean of single axis measurements, but a significant difference exists between the means of double axis measurements made over different bisectrices. Measurements of $(-)2V_\alpha$ are smaller than $(-)2V_\gamma$ by about 2.5°. The difference can be explained if the recorded angles are smaller than the true angles when high radial angles are measured on the universal stage, and calculations show that refraction and displacement of light within the central layers of the sphere could produce this effect. Optical measurements of minerals with 2V near 90° may be unreliable unless corrections are made for the errors introduced by the high angles of tilt which are involved in the measurements.

INTRODUCTION

The chemical composition of olivines occurring in igneous rocks is commonly estimated by the measurement of refractive indices or 2V. The measurements of 2V which are recorded in this paper were made in order to compare the results obtained by single axis measurements with the results obtained by double axis measurements made over different bisectrices. The means of double axis measurements are in close agreement with the means of single axis measurements, but a significant difference exists between double axis measurements made over different bisectrices. It is concluded that a single measurement of 2V may be in error by as much as 3°, corresponding to about 6 molecular per cent of fayalite. The errors result from the high angles of tilt of the universal stage which are involved in the measurement of 2V's near 90°.

The olivines measured occur in two picrite sills which outcrop on the Island of Soay in western Scotland. Slight zoning was detected in some crystals but it is restricted to the narrow, outer margins of the olivine.

PREVIOUS OBSERVATIONS

Game (1941) measured refractive indices and optic axial angles of olivines from Ubekendt Ejland, West Greenland, and reported that:

> "a set of values obtained about one bisectrix usually show fair mutual agreement but differ by as much as 7° or 8° from another set obtained about the other bisectrix (single axis measurements), whereas the mean of the two sets shows good agreement with the value inferred from the measurement of β. . . . no explanation can be offered."

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Frankel (1942), in studies of olivines of Karroo dolerites, found that the sets of single axis 2V measurements made about α and γ varied as much as 7° from each other and that the mean value of each set did not show good agreement with the value suggested from measurement of β. The mean of approximately the same number of single axis measurements taken about both α and γ was in good agreement with double axis measurements and the value obtained from β. He referred to the results of Game and concluded that the two studies demonstrated that single axis measurements of 2V may be quite unreliable.

Johnston (1953), in a study of the olivines of the Garbh Eilean Sill, used the more accurate conoscopic method of Hallimond (1950) and, whenever possible, confined his observations to double axis measurements. This ensured a higher degree of accuracy than is possible with the orthoscopic method. He found that the measurement of $2V_\alpha$ was consistently smaller than $2V_\gamma$, the average difference being 2.4°. The average of an approximately equal number of determinations about each bisectrix was taken as the true 2V of the olivines. The persistence of a difference between sets of double axis measurements using the more precise conoscopic method suggests that the differences noted by Game and Frankel cannot be attributed solely to the inaccuracy of single axis measurements.

Wilkinson (1956) made optical determinations of the olivines from a teschenite sill. Double axis measurements were made always across the acute bisectrix, but the presence of magnesian olivine with large optic axial angles involving high angles of rotation around A_4 discouraged such determinations and in the majority of cases 2V was determined by single axis methods. Agreement was "moderately good" between the mean values of double axis and single axis measurements, and "good correspondence" was found in compositions inferred from refractive indices and 2V. No comparison was possible between double axis measurements made over α and γ, and no discrepancy was found between single axis measurements made over α and γ.

Methods and Measurements

There appears to be no general agreement concerning the use of the terms "direct" and "indirect" as applied to the measurement of 2V. Measurements made about one optic axis are frequently referred to as "indirect" but the writer follows Fairbairn and Podolsky (1951) in describing both single and double axis measurements as direct methods. Indirect methods then include extinction angle procedures and relative retardation methods. Techniques used were as follows:

(A) On suitably orientated grains direct double axis determinations were made using the conoscopic method. In addition, stereographic projections of optical data were completed for each crystal, the orthoscopic method being used to locate the bisectrix (Fig. 1*a*).

(B) Using the stereograms constructed in (A), direct single axis determinations were made on these crystals. The angle between the bisectrix and the optic axis making the smaller radial angle with the center of the projection was doubled to give 2H (Fig. 1*a*).

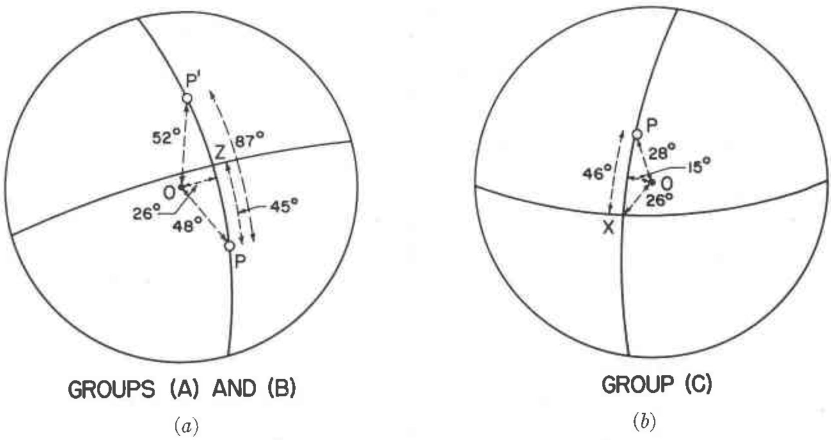


FIG. 1. Stereographic projections of olivine crystals illustrating the measurement of 2H by the three methods (A), (B) and (C).

(*a*) Crystal 50*a* in Table 1. For method (A), $2H_\gamma = PP' = 87°$. For method (B), $2H_\gamma = 2PZ = 90°$.

(*b*) Crystal 50i in Table 1. Method (C), $2H_\alpha = 2PX = 92°$.

(C) On grains not suitably orientated for double axis measurements direct single axis determinations were made using stereographic projections (Fig. 1*b*). Optic axes were located by the conoscopic method.

The measurements, made on 64 crystals in eight specimens from Sills 1 and 1A, are listed in Tables 1 and 2. Two or three thin sections were used from each specimen. Two examples will illustrate the form of tabulation. Figure 1*a* is the stereogram for crystal 50a in Table 1. The tilt of the optic axial plane (OAP) is 26°. In the double axis measurement (group A), $OP' = 52°$ is the greatest radial angle measured and $PP' = 87°$ is the value of $2H_\gamma$. In the single axis measurement (group B) OP is the greatest radial angle measured and $PZ = 45°$ is the value of H_γ, giving $2H_\gamma = 90°$. OP is always less than OP′ so that the single axis measurements involve smaller angles of tilt of the universal stage than do the

TABLE 1. MEASUREMENT OF OLIVINES IN SILL 1

Spec.	Tilt of OAP.	2H						(−)2V			
		Double Axis			Single Axis			2-Axis		1-Axis	
		OP′	α	γ	OP	α	γ	α	γ	α	γ
50 a	26	52		87	48		90		94		91
b	24	52		86	44		86		95		95
c	10	48		87.5	40		88		93.5		93
d	21				38		88				93
e	19				34		90				91
f	23				33	92				91	
g	16				30	93				92	
h	10				30	91				90	
i	15				28	92				91	
48 a	34	54		85	50		88		96		93
b	30	54		86	47		86		95		95
c	20	52	93		47	95		92		94	
d	4	51.5		85	34		88		96		93
e	2	45		87	42		87		94		94
f	4	44.5		87.5	43		88		93.5		93
47 a	5.5	53	93		40	93		92		92	
b	7	49	93		44	91		92		90	
c	22.5				32		90				91
d	5				24		90				91
e	2				24	90				89	
46 a	7.5	56	92.5		37	92		91.5		91	
b	10	54	92		38	95		91		94	
c	11	53		88.5	38		89		92.5		92
d	4	48	93.5		45.5	95		92.5		94	
e	13				35		89				92
f	5				26	93				92	
45 a	22.5	54	92		39	92		91		91	
b	12	54	92.5		40	93		91.5		92	
c	27	53	92		49	91		91		90	
d	15.5	52		86	39		87		95		94
e	25				38	92				91	
f	20				32		86				95
g	19				32		90				91
h	14				32		88				93
i	0.5				31.5	94				93	
j	7				28		88				93
k	4.5				24		86				95

OAP: optic axial plane. OP′ and OP: the radial angles occurring in each measurement.

TABLE 2. MEASUREMENT OF OLIVINES IN SILL 1A

Spec.	Tilt of OAP.	2H						(−)2V			
		Double Axis			Single Axis			2-Axis		1-Axis	
		OP′	α	γ	OP	α	γ	α	γ	α	γ
73 a	4	55	90		35	92		89		91	
b	13.5	53	93		43	94		92		93	
c	5	53	92		43	92		91		91	
d	5	52	91.5		40	92		90.5		91	
e	2	52		90	39		92		91		89
f	10	51	92		43	94		91		93	
g	28.5				37	90				89	
h	2.5				28.5		87				94
i	10				28	93				92	
j	4				23		90				91
74 a	11	54	91.5		40	92		90.5		91	
b	13	53	92		39	92		91		91	
c	19	53	91.5		44	90		90.5		89	
d	26	50		87.5	48		89		93.5		92
e	0.5	48.5		88.5	40		88		92.5		93
f	12	48		88	43		88		93		93
g	8				26	91				90	
h	1				27		88				93
75 a	2	53.5	91.5		38	94		90.5		93	
b	22	52.5		88	44		89		93		92
c	15	52	91		42.5	92		90		91	
d	4.5	51	91		40	92		90		91	
e	12	51		88	39		90		93		91
f	17.5	49		89.5	45		90.5		91.5		90.5
g	14.5				36	95				94	
h	20.5				35	93				92	
i	5				33		92				89

OAP: the optic axial plane. OP and OP′: the radial angles occurring in a given measurement.

double axis measurements. Figure 1*b* is the stereogram for crystal 50i in Table 1 (group C). The tilt of the optic axial plane (OAP) is 15°. OP is the greatest radial angle measured and PX = 46° is the value of H_α, giving $2H_\alpha = 92°$.

Values of 2H were converted to 2V using the refractive index of the hemispheres (n) and the refractive index β of the olivines. In group (A), the radial angles measured from the centers of the projections to the optic axes were corrected, and 2V was then measured on the great circle passing through the two corrected points (Emmons 1943). By this method

2V was obtained from 2H with an accuracy of $\pm 0.5°$. Conversion of H to V directly using the correction $\sin V = n \cdot \sin H/\beta$ was found to give the same result for 2V within $\pm 0.5°$. Therefore, the simple conversion of H to V was used in groups (B) and (C). The converted value of $2H_\alpha$ gives $(-)2V_\alpha$ and the converted value of $2H_\gamma$ gives $(+)2V_\gamma$. For comparative purposes the supplement of $(+)2V_\gamma$ is listed as $(-)2V_\gamma$. Thus, in Tables 1 and 2, values of $(-)2V$ are listed in four columns for determinations over α and γ by both double and single axis methods.

Angles of tilt of the universal stage were kept as small as possible. In the double axis measurements the tilt of the plane was generally less than 25° and the greatest angle of tilt was 34°. The greatest radial angle involved in a measurement, however, usually exceeded 50°, whereas in the single axis measurements for the same grains the greatest radial angle, with one exception, was always less than 50°. The significance of this fact will be discussed below. In the single axis measurements of group (C) 38° was the greatest tilt of the stage, but most angles were less than 25°.

The accuracy of the measurements varies with the method employed. In group (A) the optic axes may be located with an accuracy of $\pm 0.25°$ by the concoscopic method (Hallimond 1950) giving a possible error of $\pm 0.5°$ for 2H in a given crystal. In groups (B) and (C) the optic axes may be located with an accuracy of $\pm 0.25°$ and the optic symmetry plane containing the optic normal can be found to within $\pm 1°$, using the orthoscopic method. Hallimond (1950) and Turner (1942) state that the accuracy of measurements is variable for different directions within the indicatrix, and some planes were found which appeared to give complete extinction through a range of 4° of tilt, i.e. $\pm 2°$ accuracy. For such planes, the mean of several readings was taken, and the maximum error was probably reduced to $\pm 1°$. The estimated possible error for 2H of a given crystal by this method is therefore $\pm 2.5°$. These experimental errors will be increased slightly by the index correction.

Table 3 compares the values of $(-)2V$ obtained over α and γ for the three groups of measurements. The arithmetic means of $(-)2V$ and the standard deviation of the mean for each column have been calculated. With 99 per cent certainty, the true value of $(-)2V$ lies within the range Arithmetic Mean $\pm 2.6 \times$ Standard Deviation of the Mean, and this may be accepted with confidence as the maximum error of each mean. The values of $2.6 \times$ standard deviation, listed in Table 3, are very close in magnitude to the estimated experimental errors for single measurements.

Discussion of Results

The compositions of the olivines were estimated from the optical data collated by Poldervaart (1950). The molecular percentages of fayalite

Table 3. Comparison of (−)2V Obtained by Different Methods

(A) Double axis measurements. (B) Single axis measurements for same grains as in (A). (C) Single axis measurements with no radial angles greater than 40°

	Sill 1						Sill 1A					
	(A)		(B)		(C)		(A)		(B)		(C)	
	α	γ	α	γ	α	γ	α	γ	α	γ	α	γ
		94		91	91	93	89		91		91	89
		95		95	92	93	92		93		91	94
		93.5		93	90	91	91		91		89	91
					91		90.5		91		92	
		96		93		93	91		93			
		95		95				91		89		
		96		93			90.5		91		91	93
		94		94			91		91		91	93
		93.5		93			90.5		89		90	
	92		94					93.5		92		
	92		92		92	91		92.5		93		
	92		90		89	91		93		93		
	91.5		91		91	92	90.5		93		93	91
	91		94		94	92	90		91		91	89
	92.5		94		92		90		91		94	
		92.5		92				93		92	92	
	91		91		91	94		93		91		
	91.5		92		92	95		91.5		90.5		
	91		90		91	91						
		95		94	93	93						
						93						
						95						
$\bar{x}$	91.6	94.5	92	93.3	91.5	92.6	90.5	92.5	91.4	91.5	91.4	91.4
2.6×s	0.5	0.9	1.4	0.9	0.9	1.0	0.6	0.9	0.9	1.4	1.0	2.0
Av.	93.0		92.7		92.1		91.5		91.5		91.4	

$\bar{x}$ Arithmetic mean; s, Standard deviation of the mean; Av., Average of the means of (−)2V obtained by measurements over α and γ.

only are given below. In Sill 1, the average values of (−)2V for groups (A), (B), and (C) respectively correspond to: Fa_7, $Fa_{7.5}$, and Fa_8. Agreement is excellent. In group (A) the mean values for $(-)2V_\alpha$ and $(-)2V_\gamma$ give the compositions $Fa_{9.5}$ and $Fa_{3.5}$, so the discrepancy between the measurements over different bisectrices, 2.9°, corresponds to about 6 molecular per cent of fayalite.

In Sill 1A, the average values of (−)2V for the three groups of measurements give the same composition, namely Fa_{10}. In group (A) the mean values for $(-)2V_\alpha$ and $(-)2V_\gamma$ give the compositions Fa_{12} and Fa_8. The discrepancy between the measurements about different bisectrices, 2.0°, corresponds to about 4 molecular per cent of fayalite, which is smaller than in Sill 1.

A few measurements of β were made on grains pried from thin sections (± 0.003). Two measurements from Sill 1 give compositions $Fa_{6.5}$ and Fa_7, agreeing closely with the mean 2V determinations. Three measurements from Sill 1A give a mean composition of Fa_7 (range $Fa_{5.5}$ to Fa_8), which differs from the mean 2V determination by 3 per cent fayalite. More reliance must be placed upon the 2V determinations because the results of many 2V measurements are consistent, and only five measurements of β were made.

There may be compositional variations among the olivines within each sill but the low values of the errors for the means of 2V measurements indicates that such differences, if present, are too small to be distinguished.

In both sills, the averages of (−)2V obtained by each of the three methods agree within 1°, and the greatest differences between the means of measurements over α and γ are found in columns (A) of Table 3. Yet the latter results were obtained by a more accurate method than those in columns (B) and (C). The significance of the differences may be tested statistically. The variance of the difference between two independent variables is equal to the sum of their variances (Moroney 1951), and the standard deviation of the variables is the square root of the variance. Considering the arithmetic means obtained by different methods as independent variables, the standard errors of the differences of means obtained by measurements over α and γ have been calculated and they are compared with the observed differences in Table 4. A difference of more than two standard errors between means is probably significant and a difference of more than three standard errors between the means is highly significant; the probability that such a difference should arise by chance is less than one half of one per cent. The observed differences between the means obtained by method (A), in both sills, are more than four times the standard errors of the differences and it is very improbable that they are due to chance. In Sill 1A, it is obvious that the differences obtained by methods (B) and (C) are not significant. In Sill 1, the differ-

TABLE 4. DIFFERENCES BETWEEN THE MEANS OF (−)2V MEASURED OVER α AND γ

	Sill 1			Sill 1A		
Observed Difference Between Means	2.9	1.3	1.1	2.0	0.1	0.0
Standard Error of the Difference	0.4047	0.6799	0.5120	0.4139	0.9536	1.031

ences obtained in methods (B) and (C) are about two standard errors and they may be significant.

The values of (−)2V obtained by method (A) are plotted in Fig. 2 against the greatest radial angle (OP′) involved in the measurement. A statistical test is hardly necessary to prove that a significant difference exists between measurements made over α and γ.

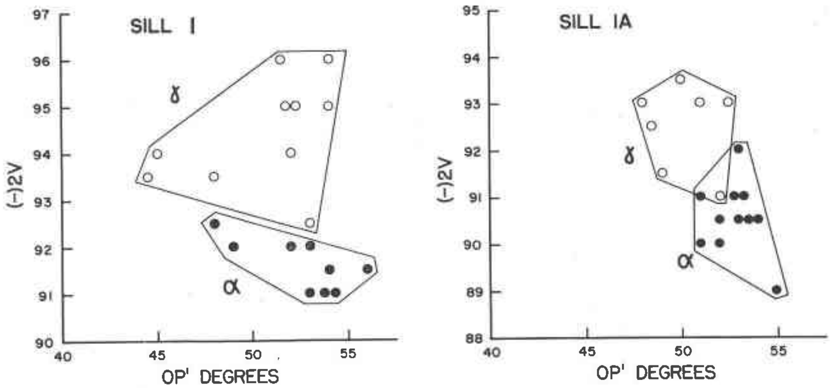


FIG. 2. Measurements of (−)2V obtained by method (A) plotted against the greatest radial angle OP′ involved in the measurement (see Fig. 1*a*). The results are taken from Tables 1 and 2. In both sills, the measurements over α are separated from those over γ.

This confirms that the means of supposedly less accurate single axis measurements (B) give more consistent results *for the same grains* than do the means of the conoscopic double axis measurements (A). The results in columns (B) were obtained from measurements involving smaller angles of tilt of the universal stage than those in columns (A), as were those in columns (C) obtained for different grains. The average values must be regarded as the best values and thus, for higher angles of tilt of the stage $(-)2V_\alpha$ is smaller, and $(-)2V_\gamma$ is larger than the expected value. Since $(-)2V_\gamma$ is the supplement of $(+)2V_\gamma$ it is clear that for higher angles of tilt of the stage (method A) both the measured angles $2H_\alpha$ and $2H_\gamma$ are smaller than the mean value of 2H obtained when lower angles of tilt are involved (methods B and C). The value of 2H within the upper hemisphere depends upon the refractive index of the hemispheres, n, 2V and β, all of which are fixed quantities in a given measurement. Therefore, for large tilts of the stage, the observed angle between the optic axes is smaller than the true 2H, i.e. the angle between the rays in the upper hemisphere. This means that there must be refraction at the surface of the upper hemisphere and the emergent ray cannot be normal to the surface. Since the emergent angle H′ is smaller than the true angle

H, the light rays must be displaced from the center of the sphere as illustrated diagramatically in Fig 3*a*, and it is in the center of the sphere that the cause of the observed discrepancy must have its origin.

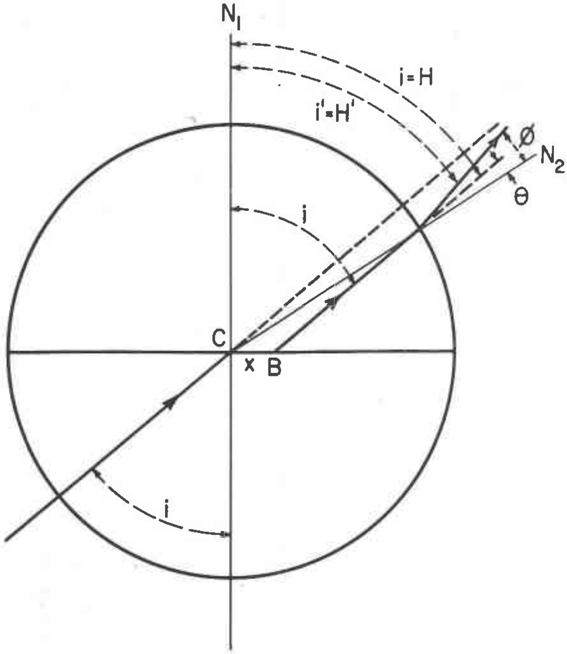
The theoretical treatment of the universal stage as a uniform sphere with a thin mineral plate at its center is, of course, simplified and with increasing angles of tilt the treatment becomes less rigorous. When a thin section is mounted on the stage there are seven refracting layers between the stage glass and the upper hemisphere and their effect upon the light rays cannot be neglected for high angles of tilt. The path of a single light ray has been traced through the layers for different angles of tilt of the stage in an attempt to find the cause of the displacement CB (Fig. 3) which could account for the observed discrepancy.

Consider a light ray with angle of incidence i passing through the mount in such a way that without refraction it would pass through the center C of the sphere, i.e. the center of the mineral plate if the stage is correctly adjusted (Fig. 3*b*). In each layer, the light ray will have an angle of refraction r, and $r>i$ for all layers except the mineral plate which, in this example, has a refractive index greater than that of the sphere. In a given layer of thickness t, the ray will be displaced laterally by a distance $\Delta x = t(\tan r - \tan i)$. The total displacement of the ray in passing through the seven layers will be $x = \Sigma t(\tan r - \tan i)$, and this corresponds to CB in Fig. 3. The thickness of the microscope slide is large compared to the other layers, and $x = t_s(\tan r_s - \tan i)$ approximately, where t_s and r_s are the thickness of and the angle of refraction in the glass slide. Knowing that the refractive index of the slide is 1.516 and $t_s = 1$ mm., values of x have been calculated for different values of i, which is the angle of tilt of the stage. The calculations are plotted in Fig. 4*a*. From the angles of incidence and the calculated values of x approximate values of θ, the angle of incidence of the emergent ray at the glass-air interface of the upper hemisphere (of radius 12 mm.), are given by the expression:

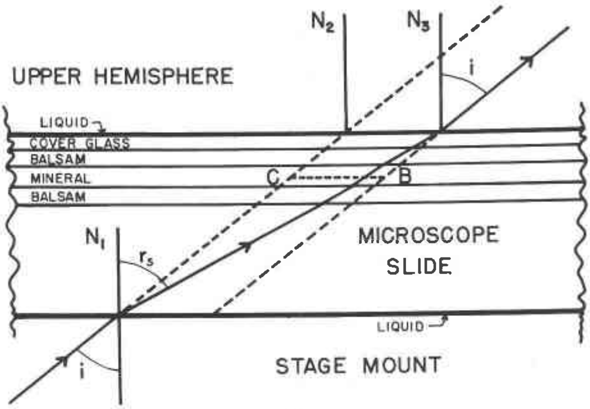
$$\theta = \frac{180 \cdot x \cdot \cos i}{(12 - x \sin i)} \text{ degrees}$$

(see Fig. 3*a*). The angle of emergence is $\phi = \sin^{-1}(1.649 \sin \theta)$ and from Fig. 3*a* it can be seen that the error of an angular measurement, for an angle of incidence (angle of tilt) i, is $(i - i') = (\phi - \theta)$. Calculated values of $(\phi - \theta)$ have been plotted against i in Fig. 4*b*, and for a given angle of tilt the observed angle i' will be smaller than the true angle i by an amount which can be read from the graph.

When the radial angle from the azimuth of the sphere to the point of measurement is 35°, the calculated error of measurement, $(i - i')$, is 0.25°.



(*a*)



(*b*)

FIG. 3. (*a*) Diagrammatic representation of the path of a light ray through the universal stage. The angle of incidence is $i = \mathrm{H}$, and the emergent angle, $i' = \mathrm{H}'$ is smaller than i (see text). The emergent ray, therefore, is not normal to the glass-air interface of the upper hemisphere, and there must be a displacement $\mathrm{CB} = x$ in the central layers of the sphere.

(*b*) Diagrammatic representation of the seven layers at the center of the universal stage, between the stage mount and the upper hemisphere. The center of the sphere is at the center of the mineral plate, C. r_s is the angle of refraction within the microscope slide, and the distance CB is the displacement of the light ray indicated in Fig. 3*a*. N_1, N_2 and N_3 are normals.

For angles less than 35° the error is negligible but for angles greater than 35° it increases markedly, passing through 0.5° for an angle of tilt of 46° and reaching 1° for an angle of tilt of 56°. These errors are much increased when allowance is made also for the displacive effect of the stage mount. It is tacitly assumed in text books that the stage glass has the same refractive index as the hemispheres, but this is not so. The stage glass used in the present measurements has a refractive index of 1.560 and thickness 2.46 mm. (measured by R. Johnston). If the displacive effect of this

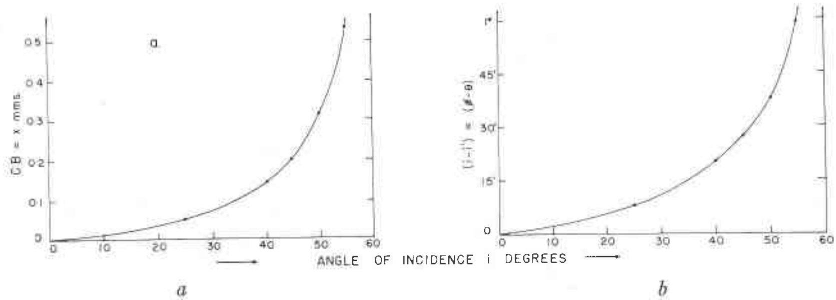


FIG. 4. (*a*) The calculated displacement CB (Fig. 3) plotted against the angle of incidence (angle of tilt) *i*.

(*b*) The calculated angular error $(i-i')$ plotted against the angle of incidence *i*. There is a rapid rate of increase of both values when *i* exceeds 40°.

plate is added to that of the microscope slide, a ray with angle of incidence 45° would have an angular error of 2° when it emerged from the sphere instead of the 27′ which would be produced by the microscope slide alone (Fig. 4*b*).

Examination of the path of a single light ray demonstrates an increasing error with increasing tilt of the stage, but it does not give a realistic picture of what actually happens when a measurement is made on the universal stage. Figure 5 shows a beam of parallel light rays, *abc*, entering the sphere with the central ray, *b*, directed towards the center of the sphere. Only those rays which emerge almost parallel to the incident beam will reach the eye of the observer. The ray *b* is incident normally but the other rays will be refracted slightly on entering the sphere, the amount of refraction increasing towards the edges of the beam, *a* and *c*. The beam as a whole is displaced by refraction within the central layers. Each ray within the beam has a different effective displacement from the center of the sphere and each ray therefore has a different error $(\phi-\theta)$. The rays in the side *c* of the beam will have a value of $(\phi-\theta)$ greater than, and the rays in the side *a* of the beam less than that for the center of the beam, *b*. Figure 5 shows that, except for low angles of tilt of the stage, the rays emerging parallel to the microscope axis are not those which were directed towards the center of the sphere, i.e. the center of the beam, *b*. The rays emerging parallel to the microscope axis are those from the side *a* of the incident beam. These rays are refracted when they enter the sphere and their angles within the hemispheres do not equal the angle of incidence, which is the measured angle. With increasing angle of tilt, rays further from the center of the incident beam will be received by the microscope

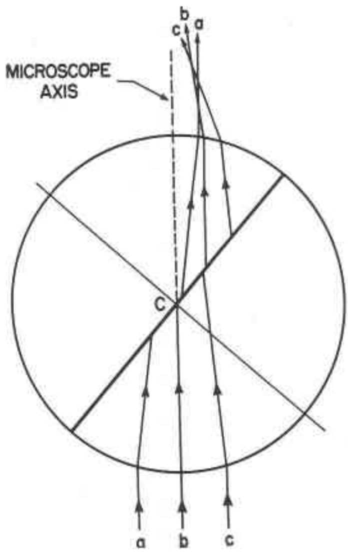


FIG. 5. Diagrammatic representation of the refraction and displacement of a beam of parallel rays entering the universal stage with a high angle of tilt. The center ray *b* is directed towards the center of the sphere, C. The outer rays of the beam are refracted slightly on entering the sphere. The beam as a whole is displaced within the central layers of the sphere (Fig. 3) and the effective displacement of rays in different parts of the beam varies. In the example shown, the ray *a* is displaced the least, and this ray emerges nearly parallel to the microscope axis. The rays *b* and *c* are refracted through successively greater angles on emergence. The angle measured is the angle of tilt, i.e. the angle of incidence of the central ray, *b*. The rays observed will be those most nearly parallel to the microscope axis. i.e. those near *a*. The ray *a* has a different angle within the hemispheres compared to that of *b*.

and the difference between the recorded angle (angle of incidence) and the true angle 2H (the angle of the observed rays within the hemispheres) also increases. The recorded angle is smaller than the true angle (Fig. 5). This is a direct result of the increasing displacement within the central layers of the sphere, and it is clear that accurate universal stage measurements cannot be expected when high angles of tilt are involved unless a correction is made for the displacement within the central layers as well as for refraction within the mineral plate.

Conclusions

(1) No significant difference was found between the means of single axis measurements made over different bisectrices in Sill 1A. In Sill 1, the differences are of the order of the estimated experimental errors and they may be statistically significant. The means of single axis measurements agree closely with the means of double axis measurements.

(2) In double axis measurements a significant difference exists between measurements made over different bisectrices. The average of the means of measurements over α and γ are in close agreement with the single axis measurements.

(3) The discrepancy in the double axis measurements may be explained if the observed angle is smaller than the true angle and it has been shown that refraction within the microscope slide and stage mount could produce this effect for high angles of tilt.

These conclusions are not in accord with those of Game (1941) and Frankel (1942). The difference between the means of $2V_\alpha$ and $2V_\gamma$ is of the same order as that noted by Johnston (1953). Wilkinson (1956) found good agreement between double axis and single axis measurements, and this is probably due to the fact that he avoided double axis measurements involving high angles of rotation.

The results presented emphasize the importance of using low angles of tilt of the universal stage whenever possible, and they demonstrate the nature and magnitude of the error introduced when high angles cannot be avoided. The introduction of errors with high angles of tilt has frequently been noted in the literature. Turner (1942) discussed refractive index corrections and stated that the errors cannot wholly be eliminated if high angles are involved, but according to Hallimond and Taylor (1950) little or nothing has been said in most text books about the refractive effect of the glass-air interfaces of the hemispheres. They discussed the errors arising if the upper hemisphere is not correctly centered. Piller (1957) gives a quantitative treatment of the error which occurs when the specimen or cover-glass is of the wrong thickness, causing a vertical displacement of the upper hemisphere with reference to the horizontal axis.

High angles of tilt of the universal stage, although avoided whenever possible, must frequently be used in optical studies. This is particularly true for the double axis determination of 2V's near 90°. By using the double axis conoscopic method in order to obtain results of higher accuracy, a further error is introduced and this can be detected only because the method is so accurate for a given measurement; the difference between $(-)2V_\alpha$ and $(-)2V_\gamma$ is the only indication that such an error exists. If one measurement made over α or γ were assumed to be correct the result could be wrong by as much as 3°, corresponding to an error of about 6 molecular per cent of fayalite in the estimated composition of an olivine (see Fig. 2). The accuracy of such measurements is therefore not as high as claimed by Hallimond (1950) unless corrections are made for the high angles of tilt.

Fairbairn and Podolsky (1951) stressed the importance of precision and accuracy in stage measurements and regretted the dearth of published data on the subject. They recorded the compositional variation occurring among plagioclase feldspars within the same rock and concluded that unless accurate determinations are made such variations may not be distinguished. Yet the supposedly accurate measurements plotted in Fig. 2, with estimated experimental errors of $\pm 0.5°$, occupy ranges of 5°. This could be interpreted as due to compositional variations among the olivines, amounting to 10 molecular per cent of fayalite, but the systematization of results indicates that it is due to additional errors produced by the high angles of tilt which had to be used. To the request of Fairbairn and Podolsky for greater precision of measurement may be added a plea for further investigation of the errors involved in measurements.

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