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EXTINCTION MEASUREMENTS FOR THE DETERMINATION OF 2V WITH THE UNIVERSAL STAGE

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ABSTRACT

Several alternative procedures are given for calculating the optic axial angle $2V$ from extinction measurements made with the universal stage. The formulae used are those derived by Garaycochea and Wittke (1963) for the calculation of $2V$ from the extinction curve of a crystal mounted on a spindle stage.

INTRODUCTION

It is well known that it is impossible to carry out a direct determination of $2V$ with the universal stage when neither of the optic axes can be brought into coincidence with the axis of the microscope. This condition arises when the optic axial plane is nearly parallel to the plane of the thin section, but it may be encountered also when the obtuse bisectrix is nearly normal to it, and the mineral has a low or moderate 2V. Difficulties in making accurate measurements may also arise whenever it is necessary to incline the normal to the section at angles greater than 40° to the microscope axis in order to locate an optic axis; this is particularly so when it is necessary to make a large correction for the refractive index difference between the crystal and the hemispheres.

In coarse-grained rocks the problem can usually be solved by selecting a more suitably oriented crystal; but occasions arise when it is desired to determine the optic axial angle on a particular crystal, either because it is to be used subsequently for refractive index determination, or because it is regarded as having special importance. A graphical solution to this problem was developed by Berek (1923) and improved greatly by Dodge (1934), but the accuracy of these procedures declines considerably when the size of the optic axial angle falls much below 50°. In many cases results differing by as much as $\pm 4°$ from the true value may be obtained, and with lower values of 2V even larger errors may be produced.

We have found that the formulae derived by Garaycochea and Wittke (1963) for calculating the optic axial angle of a crystal mounted on a spindle stage can also be applied very conveniently to the universal stage, either instead of, or as a complement to, Dodge's procedures. The data required for applying Garaycochea and Wittke's formulae may be obtained very simply by making a few extinction measurements, plotting them on the stereographic projection of the optical orientation and then measuring a few angles on the projection. There are a number of alternative ways in which this can be done; these are set out below.

PROCEDURE

First, the principal vibration directions X, Y and Z[1] are determined by any of the usual procedures, and their positions are plotted on a stereographic projection. Initially, it is convenient to take the plane of the projection parallel to the thin section and to mark on the primitive circle the zero azimuth of the A_1 axis. If possible, it is desirable to refine the positions of these principal vibration directions either by using conoscopic methods or by means of the extinction curve method (Joel and Muir, 1958a, p. 868–870), since they act as reference points for all the subsequent measurements and any considerable errors in their location will thus lead to further errors in the value of 2V.

In some cases it may also be advisable, in order to sharpen the extinctions, to use monochromatic light (or at least a color filter) and a Nakamura plate; this may improve the accuracy in the extinction readings.

In order to determine the optic axial angle, any of the following alternative procedures may then be followed.

(a) Set the universal stage into the Normal position (all axes set at their zero positions, with A_2 running N-S and A_4 parallel to the E-W crosswire). Rotate to extinction about A_5 and plot the two extinction (vibration) directions P and P' on the primitive circle of the projection. Now 2V may be calculated using formula (7) of Garaycochea and Wittke:

$$\tan^2 \mathrm{V_z} = \frac{\cos P\mathrm{Z}\,(\cos P_0\mathrm{Z} - \cos P\mathrm{Z}\cos PP_0)}{\cos P\mathrm{X}\,(\cos P_0\mathrm{X} - \cos P\mathrm{X}\cos PP_0)} \qquad (1)_7$$

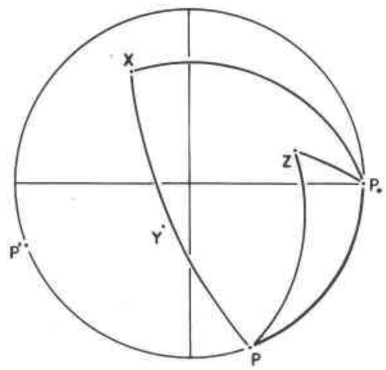
The five angles that have to be measured to use this formula (1) are shown in Fig. 1. No complication at all arises if one or more of these angles is obtuse: the corresponding negative cosine is inserted in the formula, leading always to a positive value for $\tan^2 \mathrm{V_z}$.

In Fig. 1, P_0 is plotted on the primitive circle at the right hand end of the horizontal diameter since it represents the direction of the A_4 axis which plays the part of the spindle-stage axis; and the same holds for all the projections shown in this paper except that of Fig. 4. The subscripts attached to the reference numbers of the formulae refer to the corresponding numbers in the paper by Garaycochea and Wittke. In all these formulae, either P or P', the two plotted vibration directions, may be taken as P.

[1] We have kept the notation of Garaycochea and Wittke (1963) who kept that of Joel and Garaycochea (1957) who in turn used the notation customary in analytic geometry. It would be most confusing to change it in the present paper as we constantly refer to the formulae of Garaycochea and Wittke. Hence the X, Y, Z and not α, β, γ.

(b) Set the universal stage into the Normal position, rotate about A_4 to any desired position and set to extinction by means of a rotation about A_5. Plot the vibration directions P and P' on the corresponding great circle (Fig. 2), measure the five angles shown, and use formula (1) as described in section (a) above.

The advantage of (b) over (a) occurs when extinction on A_5 is unsharp in (a) due to low partial birefringence, or when conditions are unfavorable for any other reason. A rotation about A_4 should then be tried to find a



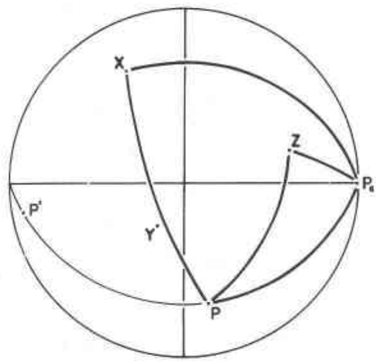


FIG. 1. Determination of 2V by means of formula 1 with $\alpha_4=0$; the five angles to be measured are PP_0, PX, PZ, P_0X and P_0Z.
FIG. 2. Determination of 2V by means of formula 1 with $\alpha_4 \neq 0$; the five angles to be measured are PP_0, PX, PZ, P_0X and P_0Z.

wave normal where extinction is more precise. By using different settings of A_4 several solutions for the value of 2V may be obtained, and these can be averaged (see also, Discussion).

(c) Set the universal stage into the Normal position and rotate about A_4 until a position of extinction is reached. Plot the two vibration directions on the corresponding great circle: in this case one of them will be the point P_0 itself, while the other one will be a point U at 90° from P_0 (Fig. 3).[1] Now use formula (10) of Garaycochea and Wittke, with the four angles indicated in Fig. 3:

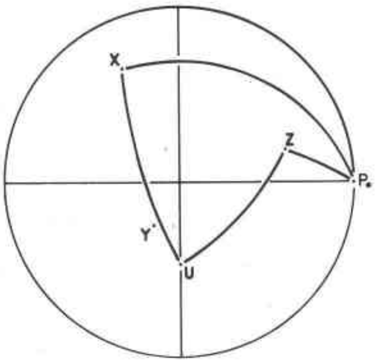
$$\tan^2 V_z = \frac{\cos UZ \cos P_0Z}{\cos UX \cos P_0X} \qquad (2)_{10}$$

If the vibration direction represented by the point U cannot be reached, due to the restriction of useful rotations about A_4 to not more than about 45°, then a different setting of A_1 (or A_2) may be chosen that

[1] This point U is a special point of the extinction curve and it has several interesting properties (Joel and Garaycochea, 1957; Garaycochea and Wittke, 1963).

enables the corresponding point U to be reached. In this way different directions of the rotation axis P_0 (A_4) relative to the indicatrix are selected; a new position of P_0 will of course give rise to a different point U. A convenient way of finding a new point U is to rotate about A_1 to extinction with the stage in the normal position; thus the point U will be found on the primitive. This amounts to setting P_0 into coincidence with one of the vibration directions of Fig. 1, the other one becoming coincident with the point U.

(d) Formula (2) can also be applied directly to vibration directions



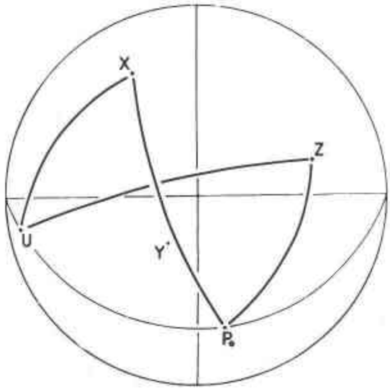


FIG. 3. Determination of 2V by means of formula 2; the four angles to be measured are P_0X, P_0Z, UX and UZ.

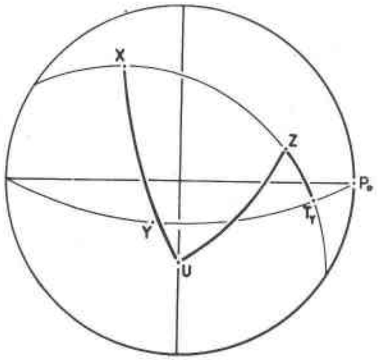
FIG. 4. Determination of 2V by means of formula 2. P_0 and U are in this case the two vibration directions of any given wave front; the four angles to be measured are P_0X, P_0Z, UX and UZ.

such as the ones plotted in Fig. 2: one can think of appropriate rotations around some of the axes of the universal stage (they need not be carried out) that will bring one of the two vibration directions of a wavefront, say P, into coincidence with P_0; the other one will then become the point U. Consequently, all that really need be done, after having plotted the two vibration directions, is to label them as P_0 and U (or vice-versa) and to measure the four angles required by formula (2) as shown in Fig. 4. This is obviously much more convenient than using formula (1).

(e) Let us call T_y the point of intersection of the great circles XZ and P_0Y, Fig. 5. (Y and T_y are the two vibration directions in the wavefront P_0Y, that is, the two vibration directions that would be obtained if the Y axis were brought into the plane of the microscope stage by a rotation around A_4.) This point T_y can also be used for obtaining 2V; the following formula can be applied with the three angles shown in Fig. 5:

$$\tan^2 V_z = \left| \frac{\cos UZ}{\cos UX} \cot ZT_y \right| \tag{3}_{12}$$

(f) Once the point U has been located for any given setting of the crystal as indicated in (c) above, a point G can be determined as the intersection of the great circle XZ with the great circle of which U is the pole.[1] The value of the optic axial angle may then be calculated by means of the



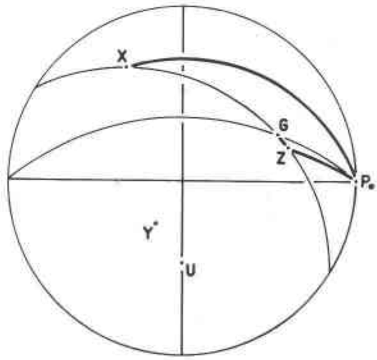


FIG. 5. Determination of 2V by means of formula 3; the three angles to be measured are UX, UZ and ZT_y.
FIG. 6. Determination of 2V by means of formula 4; the three angles to be measured are P_0X, P_0Z and ZG.

following formula, using the three angular measurements shown in Fig. 6:

$$\tan^2 V_z = \left| \frac{\cos P_0Z}{\cos P_0X} \tan ZG \right| \tag{4}_{17}$$

(g) If the points T_y and G have been located as explained in (e) and (f) above, the angle 2V may be obtained from the two measurements shown in Fig. 7, which further simplifies the calculation. Either of the following two formulae may be used:

$$\tan^2 V_z = |\tan ZG \cot ZT_y| \tag{5}_{18}$$

$$\cos 2V_z = \left(\frac{\sin (ZT_y - ZG)}{\sin (ZT_y + ZG)}\right)^{\pm 1} \tag{6}_{20}$$

The appropriate sign in the exponent of formula (6) must be selected in order to ensure a cosine whose absolute value is not greater than 1.

(h) It is also possible to use formulae (2) to (6) with no more measurements than those shown in Fig. 4; one of the two vibration directions in any given wave front can be taken to be the point P_0 (which is now no

[1] The theory of this point G has been given by Garaycochea and Wittke (1963).

longer on the primitive circle but in a general position), and the other one becomes the point U. The points T_y and G if required are then determined as explained in sections (e) and (f). It must be remembered that each new position of P_0 gives rise, in general, to new positions of U, G and T_y.

In this way, a very wide range of points can be selected as P_0 without the need of carrying out any further operations on the universal stage (except for A_4), that is, without changing the orientation of the ellipsoid relative to A_4. This range includes all the points of the extinction curve corresponding to that particular orientation of the ellipsoid relative to

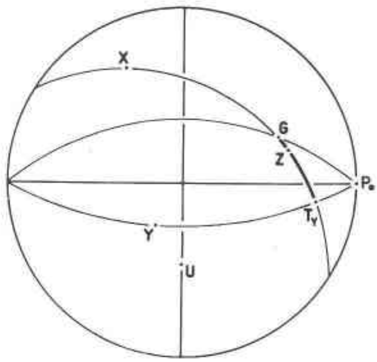


FIG. 7. Determination of 2V by means of formulae 5 or 6, the two angles to be measured are ZG and ZT_y.

A_4. (Of course, with the universal stage only part of the extinction curve is accessible.) If it is desired to select some other directions for P_0 it becomes necessary to use one or more of the other axes of the stage.

Finally, and this applies to all the procedures (a) to (h), it is useful to remember that once $\tan^2 V_z$ has been calculated, the value of $2V_z$ can be obtained from the following expression:

$$\cos 2V = \frac{1 - \tan^2 V}{1 + \tan^2 V}$$

In section (a) it was said that no complication at all arises if one or more of the angles measured on the projection is obtuse. This holds true for all the procedures (a) to (h). In the case of formula (1), the corresponding negative cosines have to be used. As to formulae (2), (3), (4) and (5), one may do the same, that is, use these negative cosines (and tangents and cotangents), the final result in any case being a positive value for $\tan^2 V_z$; alternatively, one may replace in them any obtuse angle by its supplement, thus avoiding negative signs altogether. No sign difficulty can arise out of formula (6).

Discussion

Once the principal vibration directions have been located, operations (a) and (b) require the very minimum of manipulation of the universal stage (hardly any), but the calculation makes use of five measurements. The disadvantages of these procedures lie not with the number of measurements to be taken, since these can be carried out very rapidly and easily, but in the possibility that errors in the location of X, Y, Z and P may affect considerably the final value of 2V through the various arithmetical operations of formula (1). Usually, these procedures (a) and (b) can lead to quite inaccurate results, and it is advisable to avoid them.

Procedures (c) and (e) require a little more manipulation with the universal stage; but the calculations using formulae (2) and (3) are more convenient, as the measurements (four and three of them respectively) are involved in only three and two arithmetical operations, with a consequent increase in accuracy.

Procedures (f) and (g) make use of the point G with formulae (4), (5) and (6). The operations are as simple as in (c) and (e) and are in general affected by the same type or errors since the determination of G follows that of U.

Procedure (d) seems to be a reasonable compromise as to simplicity in manipulation, plotting and calculation. It is also very versatile as it enables one to use several points P_0 selected among a wide range; and this without having to rotate any axis of the stage except A_4.[1]

Finally, the procedures outlined in (h) combine the simplicity of (d), as far as measurements go, with the further possibilities offered by the use of the points T_y and G.

There are thus several alternative procedures available, which afford many checks for the value of 2V, and it seems that a good start would be to begin with procedure (d) using formula (2) as shown in Fig. 4; several wavefronts can be observed using the A_4 axis, and their extinctions are measured by means of the rotatable microscope stage A_5.

As to the effect that the experimental errors and the graphical errors may have on the calculated value of 2V, this will depend in each formula on the magnitudes of the angles to be measured and the trigonometric function involved. For instance, an error of half a degree in an angle of 86° will affect its cosine by about 12%, while the same error in angles of 48° and 7° affects the cosine by only 1% and 0.1% respectively. The ex-

[1] Consequently, one can also use the procedures (d) and (h) with extinction measurements taken on a spindle stage (a one-axis instrument).

ample of Fig. 3 shows this clearly: in this case $P_0X=115°$, $P_0Z=30.5°$, $UX=116.5°$, $UZ=86.5°$; so that (formula 2) $\tan^2 V_z=0.279$, $\cos 2V_z=0.564$, $2V_z=55.7°$. A decrease of half a degree in UZ increases 2V by 3.2°, while the same decrease in UX decreases 2V by only 0.4°. It becomes obvious that inaccurate readings in the neighborhood of 90° should be avoided if formula (2) is used. Similar analyses can be made regarding the other formulae.

Some simple rules follow regarding the regions that should be avoided in order to achieve accurate results. For instance, one should not use a point U that is close to any of the three symmetry planes of the ellipsoid. Also, if U is distant from Y, the point G will be located more precisely since the intersection of the corresponding two great circles will be more sharply defined. If G and T_y are to be used, it should be noticed that when different positions of P_0 are chosen, G and T_y will both move towards Z or both move away from it. As it will be necessary to avoid the regions around 0° and 90° for the values of ZG and ZT_y, one may be able to select first a point T_y on the great circle XZ and then a suitable point P_0 on the circle YT_y.

The procedures for plotting the vibration directions in the various cases that may arise—according to which axes of the stage are used—have been explained by Joel and Muir (1958a, p. 871–874). It is useful to remember that in those cases where rotations around A_1 and A_2 are required in order to achieve a suitable orientation of the indicatrix, the subsequent plotting can be simplified by changing the plane of projection, from the one normal to the A_1 axis, to the one normal to A_3.

It was mentioned at the beginning of this paper that the directions of the axes X, Y and Z of the ellipsoid have to be located first, before determining the optic axial angle 2V. However, an interesting feature of the present method is that in order to determine the directions of the two optic axes it is not necessary to know which of the two bisectrices is X and which is Z (of course, Y must be identified). Indeed, if either of the two bisectrices is assumed to be Z, and V_z is calculated, and then two points are marked on the optic axial plane at distances V_z from Z, then these points are (except for the experimental errors) the poles of the two optic axes. This is so because an interchange between X and Z in the formulae used for calculating V_z produces a value that is the complement of the original one. The optic axes are thus determined without any ambiguity, even without knowing whether the crystal has a positive or a negative indicatrix.

It follows from the formulae used that if the bisectrices X and Z have been correctly identified, the value obtained for 2V will be the optic axial angle measured around Z. The angle will be acute or obtuse according to whether the crystal has a positive or negative indicatrix; (or vice-versa if the bisectrices have not been correctly identified).

Examples

1. Labradorite An_{62}; phenocrysts in porphyritic basalt, St. John's Point, County Down, Northern Ireland. $2V_z=81°$ on accurately determined readings (Muir, 1955; page 551); but individual crystals may show a complex form of slight zoning to more sodic compositions that may result in values of $2V_z$ as low as 77° being obtained in some parts. The crystal selected for study showed this zoning and had the Y axis of the indicatrix nearly normal to the plane of the thin section (at an angle of about 27° from the normal to the section). On this crystal extinction readings were taken which were accurate to ±half a degree or better. The hemispheres with $n_h=1.554$ were used and no refractive index corrections were required. The necessary angular distances were measured on the stereographic projection and they were used with formulae (1) to (6) as outlined in paragraphs (a), (c), (e), (f) and (g) of Procedure. The results obtained vary between 78.2° and 79.5°. The indirect determination of 2V by Dodge's modification of the Berek method gave $2V_z=78°$.

2. Andalusite, for which the reported value of $2V_z$ is 95°. A crystal was selected to which the direct methods for determining 2V could not be applied (it turned out to have one optic axis nearly parallel to the thin section and the other one inclined to the plane of the section at about 29°). Most extinction readings were accurate to ± half a degree; the 1.648 hemispheres were used and therefore no refractive index corrections were required. Extinction readings were taken on several wave fronts, and for each wave front the procedure outlined in paragraph (d) was followed: P and P' were taken to be P_0 and U, and formula (2) was used (Fig. 4). The individual values thus obtained for $2V_z$ were in the range from 93° to 98°, with an average between 94° and 96°. To one of these wave fronts—for which procedure (d) gave a value of $2V_z$ of 93°—the procedure outlined in paragraph (h) of Procedure was then applied: as one of the vibration directions is the point U and the other one the point P_0, the points T_y and G were determined. With formulae (3) to (6) the following results were obtained for $2V_z$: 93.4°, 93.8°, 94.5° and 94.5°. As a further check, part of a n_0 curve was then drawn using points of two extinction curves (Joel, 1963); its semi-diameters were 19° and 29° from which $2V_z=95.5°$.

Refractive Index Corrections

It is usually necessary, in order to obtain the maximum accuracy in universal stage work, to correct for refractive index differences between the crystal and the hemispheres. However, no correction is required if both the maximum birefringence $(\gamma-\alpha)$ and the difference $|n_c-n_h|$ between the average index n_c of the crystal and the index n_h of the hemispheres are less than about 0.03, provided that tilts of no more than 20° are used; under these conditions the deviation of a wavenormal on passing from the crystal to the upper hemisphere is never more than 0.6°, generally much less. Where the necessity for refractive index correction arises, a distinction should be be made between crystals of low and high birefringence. The application of this correction to the plotting of the vibration directions has been discussed by Joel and Muir (1958a, p. 874; 1958b, p. 881–882).

The application of the correction is quite a simple matter if the crystal has a sufficiently low birefringence to allow the estimated average index to be used, especially when the A_2 axis is not used, since inclinations around A_4 only need then be corrected; the plotting in this case is made easier if instead of applying the correction to the stereogram it is made to the A_4 setting of the stage, in the opposite sense, in order to make use of one of the great circles inscribed on the net.

With crystals of high birefringence significant errors may be incurred by using an assumed average index, and ideally the correction to the wavenormal should be applied separately for the two vibrations, each with its correct refractive index. If these indices cannot be estimated more accurately than ± 0.01 by means of an approximate knowledge of the principal refractive indices and the orientation of the indicatrix, or by other means, then the average index must be used; this will affect the accuracy of any individual measurement, but the final errors can be reduced by reading the extinction positions for several wavefronts and calculating 2V for each of them.

Conclusions

Several alternative procedures for calculating the angle 2V from extinction measurements made on the universal stage have been described and discussed. They work well even when the optic axial plane lies close to the plane of the thin section, as shown by the examples.

The great advantages of having a number of alternative procedures are that more than one of them may be applied to any problem for any given orientation of the ellipsoid relative to A_4; and that any single procedure may be applied with different orientations of the ellipsoid selected in turn by means of rotations about any of the axes of the stage other than A_4. (The plotting will be simpler if A_1 is used for this purpose.) In this way many checks are available for the final value of $2V$.

Acknowledgments

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