

approximately the melting point (1890° C.) indicates non-stability with a large pressure of gaseous magnesium and carbon monoxide resulting at equilibrium. One can postulate a "spittle-on-the-hot-iron effect" as a likely phenomenon which might explain the intact boules. Since the forsterite-graphite reaction would yield quantities of gas, it could be conceived that the liquid boule drop might be in contact with the graphite only intermittently, being in general cushioned on a gaseous layer of its own reaction products. This would assume no reaction with the gaseous products and a high surface tension. Indeed, calculation of the thermodynamic stability of forsterite in contact with carbon monoxide shows it to be stable at temperatures around the melting point, and the surface tension estimated from the additivity of that of the oxides (Kingery, 1959) is approximately 500 ergs/cm², a relatively high value.

Attempts to grow iron-containing orthosilicate (olivine) crystals by this technique have not been successful, with polycrystallinity and evidence of metallic iron present in the resulting masses. Although we have not as yet tried the technique extensively, we believe that other silicate systems with the right combination of high surface tension and thermodynamic stability may be amenable to sizeable crystal production by this method.

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ON MERTIE'S ANALYTIC CLASSIFICATION AND QUADRIPLANAR CHARTING OF ANALYSES WITH NINE OR MORE COMPONENTS

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INTRODUCTION

According to Mertie (1961) the nine components of a chemical analysis can be reduced to three *unique* numerical indices which are the eigenvalues of the matrix which has the chemical components as its elements (Mertie, p. 617).

The eigenvalues are the solution of the characteristic equation:

$$\begin{aligned}
 f(\lambda) = & +\lambda^3 - (a_{11} + a_{22} + a_{33})\lambda^2 + [(a_{11}a_{22} + a_{22}a_{33} + a_{11}a_{33}) - (a_{12}a_{21} + a_{23}a_{32} + a_{13}a_{31})]\lambda \\
 & - [(a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{13}a_{22}a_{32} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{21})] \\
 = & \lambda^3 + b\lambda^2 + c\lambda + d = 0.
 \end{aligned}
 \tag{1}$$

It was stated that "a critical examination of the make-up of coefficients b, c, and d will convince the most sceptical person that the cubic equation in lambda obtained from the percentages of one analysis cannot be duplicated by similar data from any other analysis." The following discussion easily proves that the three eigenvalues obtained from a given chemical analysis can be duplicated by similar data from other chemical analyses.

DISCUSSION

I. Let A be the matrix formulated from the components of a given chemical analysis according to Mertie's method (p. 616). If P is any non-singular matrix and

$$A' = PAP^{-1} \tag{a}$$

the eigenvalues of A' are identical to those of A. If the transformation matrix P is chosen such that the elements of A' are positive and their sum equal to 100, it implies that the same three eigenvalues of A' represent a completely different analysis. Theoretically an infinite number of such matrices P is possible. For example we consider the simple group of elementary permutation matrices, viz.

$$\begin{vmatrix}
 100 & & & & & & & & & \\
 & 100 & & & & & & & & \\
 & & 010 & & & & & & & \\
 & & & 010 & & & & & & \\
 & & & & 001 & & & & & \\
 & & & & & 100 & & & & \\
 & & & & & & 010 & & & \\
 & & & & & & & 001 & & \\
 & & & & & & & & 100 & \\
 & & & & & & & & & 100
 \end{vmatrix}$$

Transforming the matrix representing the analysis of a biotite granite, given by Mertie, we find the following analyses having exactly the same eigenvalues (the first one as given by Mertie):

TABLE I.

	% SiO ₂	% Al ₂ O ₃	% Fe ₂ O ₃	% FeO	% MgO	% CaO	% Na ₂ O	% K ₂ O	% R
1	71.66	14.49	1.46	1.10	0.87	1.97	3.06	4.13	1.26
2	71.66	1.46	14.49	3.06	1.26	4.13	1.10	1.97	0.87
3	0.87	1.10	1.97	14.49	71.66	1.46	4.13	3.06	1.26
4	0.87	1.97	1.10	4.13	1.26	3.06	14.49	1.46	71.66
5	1.26	3.06	4.13	1.46	71.66	14.49	1.97	1.10	0.87
6	1.26	4.13	3.06	1.97	0.87	1.10	1.46	14.49	71.66

It will be noted that the effect of these permutation transformations is an interchanging of rows and columns. They were chosen for this reason to constitute a practical contradiction to Mertie's statement.

II. Let

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{and} \quad B = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}$$

with characteristic equations:

$$f_A(\lambda) = \lambda^3 + b\lambda^2 + c\lambda + d = 0$$

and

$$f_B(\lambda) = \lambda^3 + b'\lambda^2 + c'\lambda + d' = 0$$

For identical eigenvalues the following equations must hold:

$$b = b', \quad c = c', \quad d = d' \quad \text{and} \quad \sum_{i=1}^3 \sum_{j=1}^3 a_{ij} = \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} = 100$$

As the coefficients of λ in the characteristic equations $f_A(\lambda)$ and $f_B(\lambda)$ are functions of the elements a_{ij} and b_{ij} respectively, we have only four equations from which the nine unknown variables b_{ij} (where $i, j = 1, 2, 3$) can be solved. Theoretically an infinite number of solutions (b_{ij}) is possible.

To obtain a practical solution which can represent a chemical analysis it will be necessary to round off the elements (b_{ij}) to 2 decimal places. Since any chemical analysis is subject to variations of at least 0.01% in each component, the above mentioned rounding off is feasible in spite of the fact that it changes the eigenvalues. Furthermore it should be borne in mind that a small variation of the elements of a non-symmetric matrix can induce a relatively large variation in its eigenvalues. Table II illustrates these arguments. In Table II there are the nine components and the corresponding three eigenvalues of four chemical analyses which could have been obtained by the same analyst on one sample, and that of a chemical analysis (No. 5) calculated by the above method to give the same eigenvalues as that of analysis No. 1. This calculated chemical analysis was rounded off to two decimal places. It is obvious that the eigenvalues of the first four samples which have for all practical purposes the same chemical composition, range between 42.21995–42.24355 for λ_1 , 14.73089–14.75235 for λ_2 , and 8.01961–8.03574 for λ_3 . Since the eigenvalues of Sample No. 5 lie within these intervals this sample should have practically the same chemical composition as samples 1, 2, 3 and 4 according to Mertie.

As Mertie's method may, therefore, lead to wrong conclusions we are of

TABLE II.

Sample No.	Analyses										Eigenvalues		
	% SiO ₂	% Al ₂ O ₃	% Fe ₂ O ₃	% FeO	% MgO	% CaO	% Na ₂ O	% K ₂ O	% R	λ_1	λ_2	λ_3	
1	40.00	2.00	5.00	15.00	15.00	5.00	5.00	3.00	10.00	42.22894	14.73637	8.03469	
2	39.98	2.00	5.01	15.01	15.00	5.00	5.00	3.00	10.00	42.21336	14.73089	8.03574	
3	40.00	1.98	5.00	15.00	15.00	5.00	5.00	3.02	10.00	42.21995	14.75235	8.02770	
4	40.02	1.99	5.01	14.99	15.01	4.99	5.00	3.01	9.98	42.24355	14.74684	8.01961	
5	8.40	4.00	0.50	5.00	40.47	5.00	16.50	4.00	16.13	42.22905	14.74011	8.03084	

the opinion that this method cannot be used even though the chance of an accidental duplication in the terms of an analysis may be small.

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COMMENT ON "ERRORS IN THE MEASUREMENT OF 2V WITH THE UNIVERSAL STAGE," BY M. MUNRO

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In Munro's interesting paper (*Am. Mineral.* **48**, 308-323, 1963) it is concluded that a difference in refractive index of the segments on the one hand and the central assembly (central plate and thin section) on the other, may cause an error of measurement. This error reaches a maximum when a closed substage diaphragm is used in the orthoscopic procedure (his Table 2). A few additional remarks may here be made.

The paper refers exclusively to the Leitz Universal stage. It is, however, of interest to note that the universal stage made by Zeiss (Oberkochen) is not provided with a central plate: the thin section is placed directly between the two segments. As the Leitz central plate (*ca.* 2.5 mm) is about twice as thick as the glass slide of the thin section, the error mentioned above should be considerably less when working with the Zeiss stage.

This advantage, however, is counteracted by the fact that Zeiss recently changed the shape of the lower segments with $n=1.555$ and $n=1.649$. Contrary to the description still provided with the instrument, these segments are now true hemispheres without any slice ground off to account for the thickness of the glass slide of the thin section. Apparently (pers. comm. of the manufacturer) the change was brought about to improve the illumination for conoscopic work. When working orthoscopically, a thin illuminating beam is now deflected on tilting the stage because the horizontal axes of the stage (1) do not pass through the center of curvature of the lower segment (2) (Fig. 1). At first sight, I was inclined to think that the Zeiss stage was better adapted for conoscopic than for orthoscopic work. Judging from Munro's Table 2, however, we