Extinction angles for monoclinic amphiboles or pyroxenes: a cautionary note

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Abstract

For monoclinic crystals with \( b = Y \), extinction angles measured relative to a \{110\} cleavage trace will equal the optic orientation angle \( Z \land c \) (or \( X \land c \)) only for \( (010) \) sections. Such sections are best recognized from display of a centered optic normal figure and, as long known, are not necessarily the section in the [001] zone that exhibits the maximum extinction angle relative to the \{110\} cleavage trace. Such maximum extinction angles are here proved to exceed \( Z \land c \) (or \( X \land c \)) for optic orientations where \( b = Y \), if the obtuse bisectrix is within 45° of \( c \). Indeed, if \( c \) also happens to lie in or near a circular section of the optical indicatrix, maximum extinction angles for sections in the [001] zone may approach 45° regardless of the value of \( Z \land c \) (or \( X \land c \)). A hornblende from Mt. Monadnock, New Hampshire—for which Winchell and Winchell (1951) cite \( 2V_\varepsilon = 137° \) and \( Z \land c = 21° \)—closely approaches this special case because its angles \( V_\varepsilon \) and \( Z \land c \) are almost complementary.

Introduction

For monoclinic amphiboles and pyroxenes in thin sections, the maximum extinction angle, if measured from grains oriented so that their \{110\} cleavage appears as a single set of mutually parallel traces, is usually assumed to have been measured from a \( (010) \) section and thus to represent the angle \( Z \land c \) (or \( X \land c \)). Frequently forgotten are the cautions of long ago (Daly, 1899; Duparc and Pearce, 1907; Rosenbusch and Wülfling, 1921-24) and of recent times (Hartshorne and Stuart, 1970). These note that, for crystal sections parallel to \( c \), the maximum extinction angle relative to \{110\} cleavage traces may occur for sections at a significant angle \( \phi \) to \( (010) \) and, in such case, may significantly exceed \( Z \land c \) (or \( X \land c \)). In the appendix, we prove that this will occur for monoclinic crystals with \{hk0\} cleavage and \( b = Y \) only if the obtuse bisectrix lies within 45° of \( c \). Such orientations occur for several common amphiboles and pyroxenes. Table 1, to be discussed next, permits petrographers to estimate the degree to which the maximum extinction angle will exceed the optic orientation angle \( Z \land c \) (or \( X \land c \)) if measured from sections parallel to \( c \) at varying angles \( \phi \) relative to \( (010) \).

Discussion

In Table 1 the column heads—namely, 5, 10 . . . to, at most, 40—represent the angle (in degrees) between the obtuse bisectrix \( (0B) \) and the \( c \) axis. The vertical column at left represents \( \phi \), the dihedral angle between \( (010) \) and the crystal section parallel to \( c \) from which the extinction angle \( E \) was measured relative to the \{hk0\} cleavage traces. Values in the body of the Table 1 represent \( E \) for different combinations of the obtuse optic angle \( 2V_\varepsilon \), \( \phi \) and \( 0B \land c \).

Vertical columns headed by those values of \( 0B \land c \) which are complements of \( V_\varepsilon \) represent special orientations where \( b = Y \) and where the \( c \) axis lies within a circular section of the optical indicatrix. For these columns wherein

\[
(0B \land c) + V_\varepsilon = 90°
\]

the angle \( E \), regardless of the value of \( 0B \land c \), approaches 45° as \( \phi \) approaches 90°. For \( \phi \) precisely equal to 90°, the section will lie parallel to \( (010) \). In such a case, to the extent that Equation (1) holds for the crystal, this section will be perpendicular to an optic axis and thus will have no well defined extinction position. However, a plane at a small angle \( \delta \) to \( (100) \) will have a defined extinction angle (Fig. 1). As shown, the two circular sections intersect this particular plane at the two black dots. Following the Biot-Fresnel rule, the hollow point \( v \), which bisects the angle between the two black dots, represents the privileged direction for the dashed plane. Note that this privileged direction is almost at 45° to the \( c \) axis, and thus to the trace of the \{110\} cleavage on this dashed plane.

For a crystal that exactly conforms to Equation (1), a plot of \( E \) versus \( \phi \) discloses (Fig. 2A) that the true angle \( 0B \land c \) corresponds to the minimum value for \( E \), not the
Table 1. Extinction angles relative to (hk0) cleavage for (hk0) planes at varying interfacial angles \( \phi \) to (010) and for different optic orientations c:0B and 2V angles.

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( 2V = 100^\circ )</th>
<th>( 2V = 170^\circ )</th>
<th>( 2V = 110^\circ )</th>
<th>( 2V = 160^\circ )</th>
<th>( 2V = 120^\circ )</th>
<th>( 2V = 150^\circ )</th>
<th>( 2V = 130^\circ )</th>
<th>( 2V = 140^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td>25.0</td>
<td>30.0</td>
<td>35.0</td>
<td>40.0</td>
<td>5.0</td>
</tr>
<tr>
<td>10</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td>25.1</td>
<td>30.1</td>
<td>35.1</td>
<td>40.1</td>
</tr>
<tr>
<td>20</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td>25.2</td>
<td>30.2</td>
<td>35.2</td>
<td>40.2</td>
</tr>
<tr>
<td>30</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td>25.3</td>
<td>30.3</td>
<td>35.3</td>
<td>40.3</td>
</tr>
<tr>
<td>40</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td>25.4</td>
<td>30.4</td>
<td>35.4</td>
<td>40.4</td>
</tr>
</tbody>
</table>

The extinction angles for phi equal 0°, here designated with slightly bolder type, precisely equal the c:0B angle which the petrographer seeks. The columns headed with a bold 5.0o thus show how extinctions observed for an (hk0) plane vary as their phi angle relative to (010) in turn varies from 0°, 10° . . . to 90°.

* The extinction angles closely approach 45° and are for phi angles slightly less than 90°. If phi exactly equals 90°, this extinction angle becomes indeterminate.

max. This holds true if we exclude from consideration sections for which \( \phi \) nearly or exactly equals 90°. Fortunately, such sections are readily recognized (low birefringence and near-centered optic axis figures). By contrast the desired sections, at or near \( \phi \) equal 0°, will display a centered optic normal figure (and thus maximum retardation). However, even if \( \phi \) equals as much as 20° (cf. Table 1), the measured E angles will exceed 0B \( \wedge \) c by only 6% at most.

The data of Winchell and Winchell (1951) for a hornblende from Mt. Monadnock, New Hampshire—\( 2V_z = 137° \) and \( Z \wedge c = 20° - 21° \)—provide a practical example.
SU AND BLOSS: EXTINCTION ANGLES

Fig. 1. Stereographic projection of a monoclinic hornblende crystal from Mt. Monadnock, New Hampshire, using the data—
$Z \wedge c = 21^\circ$; $2V_x = 43^\circ$—given by Winchell and Winchell (1951, p. 435). For this crystal, the $c$ axis lies practically within a circular section of the indicatrix because

$Z \wedge c + V_z \approx 89.5^\circ$

The dashed line represents a plane parallel to $c$ which lies at an angle $\phi$, namely $90^\circ - \delta$ where $\delta$ is extremely small, to the optic plane (010). This dashed line intersects one circular section (CS$_1$) at $c$ and the other (CS$_2$) at $i$. Consequently $v$, the vibration direction for light normally incident on the plane represented by this dashed line, is the bisector of the angle between $c$ and $i$. Point $c$ also represents the intersection between said plane and any {hk0} cleavage. Accordingly, $E$ equals the extinction angle for light normally incident on said plane if measured relative to an {hk0} cleavage trace. For this special case, note that extinction angle $E$ has approached $45^\circ$ even though $Z \wedge c$ only equals $21^\circ$.

Accepting $Z \wedge c$ as $21^\circ$, our calculations of $E$ versus $\phi$, if plotted (Fig. 2B), disclose that $E$ drops off precipitously to $0^\circ$, as $\phi$ exceeds (ca.) $85^\circ$.

Amphibole crystals in oil (or Canada balsam) mounts likely lie on a {110} cleavage. If so, this plane of rest will be at an angle $\phi$ of (ca.) $62^\circ$ to the (010) plane. For amphiboles with $b = Y$ and with $c$ within $45^\circ$ of the obtuse bisectrix, the extinction angles measured from such mounted grains will thus exceed $Z \wedge c$ (or $X \wedge c$). Comparing the $E$ angles for $\phi = 60^\circ$ to those for $\phi = 0^\circ$ in Table 1 indicates the extent of the expected discrepancy. Similarly, for grain mounts of pyroxenes, the values for $\phi = 45^\circ$ should be compared to the values obtained by interpolating halfway between those at $\phi = 40^\circ$ and $\phi = 50^\circ$ in Table 1.

Acknowledgments

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References


Daly, R. A. (1899) On the optical characters of the vertical zone of amphiboles and pyroxenes; and on a new method of determining the extinction angles of these minerals by means of cleavage pieces. Proceedings of the American Academy of Arts and Sciences. 34, 311–323.
Appendix

Previously we stated without proof that, for monoclinic crystals with \{hk0\} cleavage and $b = Y$, the extinction angle $E$ can exceed $Z \pm c$ (or $X \pm c$) only if the obtuse bisectrix is at less than 45° to the $c$ axis. To prove this, let $c$ and $Z$ (Fig. 3A) represent the $e$ axis and one bisectrix of the optical indicatrix for a monoclinic crystal with $b = Y$ and \{hk0\} cleavage. The bold ellipse represents an equivibration curve (Wright, 1923, p. 785; Bloss, 1981, p. 100-106) which by definition represents the locus of all points that represent radii of equal length within this crystal’s optical indicatrix. The dashed line represents the direct projection of a plane, in the [001] zone, at a random angle $\phi$ to the (010) plane. This dashed plane intersects the equivibration curve at $c$ and $P$. Because $c$ and $P$ are radii of equal length in the indicatrix, the hollow dot representing the line that bisects the angle between $c$ and $P$ is necessarily a privileged direction for light normally incident on this dashed plane. By varying $\phi$ between 0° and 90°, the dashed plane can represent the entire family of planes parallel to [001]. Note that, for any plane in this family, the angle $c \wedge P$ equals twice the extinction angle $E$ as measured relative to the \{hk0\} cleavage for light normally incident on the dashed plane.

In Figure 3A, $\theta_1$, $\theta_2$ and $\theta_3$ respectively represent angles $c \wedge Z$, $P \wedge Z$, and $P_0 \wedge X$—where $P_0$ represents the intersection between the indicatrix’s $XY$ plane and the plane containing $P$ and $Z$. Each pair of lines—$c$ and $Z$, $P$ and $Z$, $P_0$ and $X$—defines a plane which intersects the indicatrix in an ellipse (Figs 3B, 3C and 3D). Hence, from the equation for an ellipse, the lengths of radii of the indicatrix corresponding to $c$, $P_0$ and $P$ can be written

$$c^{-2} = \gamma^{-2} \cos^2 \theta_1 + \alpha^{-2} \sin^2 \theta_1 \quad (2)$$

$$P_0^{-2} = \alpha^{-2} \cos^2 \theta_3 + \beta^{-2} \sin^2 \theta_3 \quad (3)$$

$$P^{-2} = \gamma^{-2} \cos^2 \theta_2 + P_0^{-2} \sin^2 \theta_2 \quad (4)$$

The right hand side (r.h.s.) for Equation 3 can substitute for $P_0^{-2}$ in Equation 4 and, since radii $c$ and $P$ are equal (because both plot on the same equivibration curve), the r.h.s. of Equations 2 and 4 may be equated. Dividing the resultant equation by $(\alpha^{-2} - \gamma^{-2})$ and then substituting $\sin^2 V_Z$ for $(\alpha^{-2} - \beta^{-2})/(\alpha^{-2} - \gamma^{-2})$, we obtain

$$\sin^2 \theta_2 = \frac{\sin^2 \theta_1}{1 - \sin^2 \theta_1 \sin^2 V_Z} \quad (5)$$

or

$$\cos^2 \theta_2 = \frac{\cos^2 \theta_1 - \sin^2 \theta_3 \sin^2 V_Z}{1 - \sin^2 \theta_1 \sin^2 V_Z} \quad (6)$$

Our proof is simplified, but not negated, if we equate $\theta_1$ to 90° so that equation 6 becomes

$$\cos^2 \theta_2 = \frac{\cos^2 \theta_1 - \sin^2 V_Z}{\cos^2 V_Z} \quad (7)$$
For $\theta$, equal 90°, point $P$ moves to position $P_2$ (Fig. 3A) and from the spherical right triangle $cZP_2$,

$$\cos 2E = \cos \theta_1 \cos \theta_2$$

(8)

where $2E$ represents twice the extinction angle for the plane containing radii $c$ and $P_2$. If $E$ is to exceed $\theta_1$, then necessarily

$$\cos 2\theta_1 > \cos 2E$$

and, in consequence of this and equation 8,

$$\cos^2 2\theta_1 > \cos^2 \theta_1 \cos^2 \theta_2$$

(9)

Table 2 summarizes the algebraic manipulations whereby, as a consequence of inequality (9), we show that

$$\tan^2 \theta_1 + \tan^2 \theta_2 > 2$$

(10)

The angle $\theta_1$, equal to $Z \wedge c$ in our example, by convention is less than 45°. Accordingly, for inequality (10) to hold, $V_2$ must exceed 45°. In other words, extinction angle $E$ can exceed $\theta_1$, if $\theta_1 = 90^\circ$, only if the indicatrix bisectrix closest to $c$ is the obtuse bisectrix.

Table 2. Algebraic steps that lead from Equation 9 to Equation 10 in the text

<table>
<thead>
<tr>
<th>Step</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\cos^2 2\theta_1 &gt; \cos^2 \theta_1 \cos^2 \theta_2$</td>
</tr>
<tr>
<td>2.</td>
<td>$4\cos^2 \theta_1 - 4\cos^2 \theta_1 + 1 &gt; \cos^2 \theta_1 \cos^2 \theta_2 (\cos^2 \theta_1 - \sin^2 \theta_2)$</td>
</tr>
<tr>
<td>3.</td>
<td>$\cos^2 \theta_1 &gt; 4\cos^2 \theta_1 \sin^2 \theta_1 \cos^2 \theta_2 + \cos^2 \theta_1 \sin^2 \theta_2$</td>
</tr>
<tr>
<td>4.</td>
<td>$\cos^2 \theta_1 &gt; \cos^2 \theta_1 (1-\sin^2 \theta_1) + \cos^2 \theta_1 \sin^2 \theta_2 - 4\cos^2 \theta_1 \cos^2 \theta_2 \sin^2 \theta_2 &gt; 0$</td>
</tr>
<tr>
<td>5.</td>
<td>$\cos^2 \theta_1 &gt; \cos^2 \theta_1 \sin^2 \theta_2 - \cos^2 \theta_1 \cos^2 \theta_2 - 4\cos^2 \theta_1 \cos^2 \theta_2 \sin^2 \theta_2 &gt; 0$</td>
</tr>
<tr>
<td>6.</td>
<td>$\cos^2 \theta_1 &gt; \cos^2 \theta_1 \sin^2 \theta_2 - 4\cos^2 \theta_1 \cos^2 \theta_2 \sin^2 \theta_2 &gt; 0$</td>
</tr>
<tr>
<td>7.</td>
<td>$\sin^2 \theta_2 &gt; 0$, $\cos^2 \theta_1 &gt; 0$ and $\cos^2 \theta_2 &gt; 0$, we have $\cos^2 \theta_1 + \cos^2 \theta_2 &gt; 4$</td>
</tr>
<tr>
<td>8.</td>
<td>$\tan^2 \theta_1 + \tan^2 \theta_2 &gt; 2$</td>
</tr>
</tbody>
</table>

Eq 9  
Eq 10