# Computer simulation of feldspar structures 

J. Purton<br>Department of Chemistry, University of Keele, Keele, Staffordshire ST5 5BG, United Kingdom<br>C.R.A. Catlow<br>Davy Faraday Research Laboratory, The Royal Institution, 21 Albemarle Street, London W1X 4BS, United Kingdom


#### Abstract

Lattice-energy minimization calculations have been performed for the feldspars low albite, maximum microcline, and anorthite using an ionic Born-model potential. Analysis of earlier work shows that, although the bond lengths are simulated with reasonable accuracy using a two-body rigid-ion model, the bond angles display significant differences from the experimental results. The inclusion of three-body terms and polarization effects overcomes these problems. The T-O-T bond angles are particularly sensitive to the nature of the short-range cation-O potential.


## Introduction

Feldspars are important rock-forming minerals present in a wide variety of igneous, metamorphic, and sedimentary rocks. Their ubiquitous nature has led to considerable research on their structural, physical, and chemical properties (see, for example Smith, 1974a, 1974b; Ribbe, 1983). Despite the recent interest in computer simulation of minerals and other inorganic compounds, there has only been a limited amount of work on feldspars. The most basic requirement of any simulation is the ability to reproduce the crystal structure. Thus the present paper reports a simulation study of feldspar structures as a precursor to investigating other physical and chemical properties (e.g., defects, hydrolysis, and $\mathrm{Si}-\mathrm{Al}$ ordering). Our methodology is based on the use of three-body shell model potentials, which are employed in energy minimization calculations. We pay special attention to the contrast between our results and those of earlier studies (Post and Burnham, 1987) in which only two-body models were employed.

## Method

Our techniques are based on lattice-energy minimization. We employ the THBREL code, which calculates lattice energies by a direct summation of two- and threebody short-range terms, and an Ewald summation of the Coulombic interactions. The lattice-energy calculations are coupled with a Newton-Raphson minimization procedure. The observed crystal structures used as the starting point for the calculations are those of Harlow and Brown (1980), Brown and Bailey (1964), and Wainwright and Starkey (1971) for low albite, maximum microcline, and anorthite, respectively. The Al atoms were assumed to be fully ordered into the Tlo site of albite and microcline and the $\mathrm{T} 1(0 \mathrm{z} 00), \mathrm{T} 1(0 \mathrm{zi} 0), \mathrm{T} 1(\mathrm{~m} 000), \mathrm{T} 1(\mathrm{~m} 0 \mathrm{i} 0)$, $\mathrm{T} 2(0000), \mathrm{T} 2(\mathrm{mz} 00)$, and $\mathrm{T} 2(\mathrm{mzi})$ sites of anorthite. The relaxation was carried out to zero pressure, allowing both
atom coordinates and cell dimensions to vary simultaneously. Calculations were performed with two types of potential for all of the minerals. The first was a rigid-ion two-body model (2BRI) in which the electrostatic forces are counterbalanced by short-range repulsive forces. The electrostatic or Coulomb ( $U_{\mathrm{c}}$ ) energy is given by

$$
\begin{equation*}
U_{\mathrm{c}}=\frac{1}{2} \sum_{i}^{\text {one cell all cells }} \sum_{j \neq i} q_{i} q_{j} r_{i j}^{-1} \tag{1}
\end{equation*}
$$

where $q$ is the formal charge and $r_{i j}$ the interionic dis-

Table 1. Short-range potentials

| Two-body rigid-ion model |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Interaction | $\mathrm{A}(\mathrm{eV})$ | $\rho(\AA)$ | $\mathrm{C}\left(\mathrm{eV} \AA^{6}\right)$ | Method | Refer- |
| ence |  |  |  |  |  |

Note: The methods of derivation are empirical (EMP), modified electron gas (MEG), and Hartree-Fock (HF). References: $\mathrm{a}=$ Post and Burnham (1986); $b=$ Sanders et al. (1984); c = James (1979); $d=$ Catlow (1977); $\mathrm{e}=$ Walker (personal communication); $\mathrm{f}=$ Parker (1983); and $\mathrm{g}=$ Lewis and Catlow (1985).

Table 2. Calculated bond lengths and bond angles of low albite compared with the observed results of Harlow and Brown (1980)

|  | Obs |  |  | 2BRI |  | 3BSM1 |  | 3BSM2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a |  | 8.142 |  | 8.375 |  | 8.461 |  | 8.146 |
| b |  | 12.785 |  | 12.978 |  | 12.889 |  | 12.748 |
| c |  | 7.159 |  | 7.220 |  | 7.142 |  | 7.070 |
| $\alpha$ |  | 94.19 |  | 95.38 |  | 91.10 |  | 93.68 |
| $\beta$ |  | 116.61 |  | 116.57 |  | 116.12 |  | 116.66 |
| $\gamma$ |  | 87.68 |  | 87.34 |  | 87.86 |  | 88.02 |
| Lattice energy/eV |  |  |  | 44.81 |  | -1926.86 |  | -1929.87 |
| T10-0a1 |  | 1.747 |  | 1.721 |  | 1.739 |  | 1.742 |
| T10-Obo |  | 1.745 |  | 1.719 |  | 1.723 |  | 1.731 |
| T10-Oco |  | 1.735 |  | 1.721 |  | 1.740 |  | 1.735 |
| T10-Odo |  | 1.745 |  | 1.740 |  | 1.731 |  | 1.739 |
| $\mathrm{Oa}-\mathrm{Ob}$ | 2.713 | 100.75 | 2.811 | 109.56 | 2.714 | 103.26 | 2.684 | 101.21 |
| $\mathrm{Oa}-\mathrm{Oc}$ | 3.042 | 120.03 | 2.802 | 108.97 | 2.957 | 116.43 | 2.994 | 118.90 |
| $\mathrm{Oa}-\mathrm{Od}$ | 2.735 | 101.88 | 2.877 | 112.44 | 2.721 | 103.25 | 2.698 | 101.63 |
| $\mathrm{Ob}-\mathrm{Oc}$ | 2.888 | 112.21 | 2.930 | 116.79 | 2.881 | 112.58 | 2.867 | 111.61 |
| $\mathrm{Ob}-\mathrm{Od}$ | 2.878 | 111.16 | 2.809 | 108.57 | 2.864 | 112.03 | 2.883 | 112.41 |
| Oc -Od | 2.852 | 110.08 | 2.657 | 100.31 | 2.824 | 108.91 | 2.859 | 110.50 |
| T1m-Oa1 |  | 1.596 |  | 1.558 |  | 1.582 |  | 1.590 |
| T1m-Obm |  | 1.600 |  | 1.577 |  | 1.600 |  | 1.591 |
| T1m-Ocm |  | 1.621 |  | 1.600 |  | 1.629 |  | 1.630 |
| T1m-Odm |  | 1.618 |  | 1.597 |  | 1.626 |  | 1.614 |
| $\mathrm{Oa}-\mathrm{Ob}$ | 2.608 | 107.82 | 2.632 | 112.62 | 2.561 | 107.21 | 2.584 | 108.66 |
| Oa Oc | 2.587 | 116.43 | 2.787 | 110.01 | 2.706 | 114.85 | 2.682 | 112.77 |
| $\mathrm{Oa}-\mathrm{Od}$ | 2.624 | 105.04 | 2.599 | 112.53 | 2.557 | 105.68 | 2.555 | 105.79 |
| $\mathrm{Ob}-\mathrm{Oc}$ | 2.664 | 108.23 | 2.610 | 113.96 | 2.629 | 108.99 | 2.604 | 107.92 |
| Ob-Od | 2.525 | 111.31 | 2.657 | 105.43 | 2.683 | 112.56 | 2.683 | 113.70 |
| Oc-Od | 2.480 | 108.03 | 2.621 | 101.75 | 2.628 | 107.64 | 2.626 | 108.11 |
| T20-Oa2 |  | 1.632 |  | 1.591 |  | 1.629 |  | 1.641 |
| T2o-Obo |  | 1.592 |  | 1.565 |  | 1.572 |  | 1.580 |
| T20-Ocm |  | 1.617 |  | 1.580 |  | 1.629 |  | 1.620 |
| T2o-Odm |  | 1.619 |  | 1.597 |  | 1.626 |  | 1.617 |
| $\mathrm{Oa}-\mathrm{Ob}$ | 2.660 | 111.15 | 2.617 | 112.07 | 2.659 | 112.34 | 2.663 | 111.54 |
| $\mathrm{Oa}-\mathrm{Oc}$ | 2.565 | 104.28 | 2.701 | 116.85 | 2.540 | 102.41 | 2.553 | 103.08 |
| $\mathrm{Oa}-\mathrm{Od}$ | 2.611 | 107.06 | 2.542 | 105.76 | 2.604 | 106.31 | 2.593 | 105.52 |
| $\mathrm{Ob}-\mathrm{Oc}$ | 2.661 | 112.06 | 2.558 | 108.88 | 2.650 | 111.73 | 2.657 | 112.24 |
| $\mathrm{Ob}-\mathrm{Od}$ | 2.652 | 111.60 | 2.508 | 104.96 | 2.674 | 113.37 | 2.655 | 112.31 |
| Oc-Od | 2.653 | 110.34 | 2.562 | 107.49 | 2.666 | 110.01 | 2.677 | 111.58 |
| T2m-Oa2 |  | 1.645 |  | 1.608 |  | 1.655 |  | 1.660 |
| T2m-Obm |  | 1.621 |  | 1.607 |  | 1.632 |  | 1.625 |
| T2m-Oco |  | 1.596 |  | 1.544 |  | 1.587 |  | 1.583 |
| T2m-Odo |  | 1.602 |  | 1.572 |  | 1.583 |  | 1.592 |
| $\mathrm{Oa}-\mathrm{Ob}$ | 2.628 | 107.21 | 2.563 | 105.72 | 2.630 | 106.30 | 2.644 | 107.24 |
| $\mathrm{Oa}-\mathrm{Oc}$ | 2.588 | 105.97 | 2.677 | 116.24 | 2.547 | 103.52 | 2.576 | 105.17 |
| $\mathrm{Oa}-\mathrm{Od}$ | 2.635 | 108.51 | 2.567 | 107.67 | 2.646 | 109.58 | 2.654 | 109.37 |
| $\mathrm{Ob}-\mathrm{Oc}$ | 2.637 | 110.14 | 2.548 | 107.93 | 2.622 | 109.07 | 2.612 | 109.04 |
| Ob-Od | 2.643 | 110.22 | 2.527 | 105.30 | 2.668 | 112.11 | 2.642 | 110.44 |
| $\mathrm{Oc}-\mathrm{Od}$ | 2.688 | 114.45 | 2.601 | 113.18 | 2.682 | 115.53 | 2.681 | 115.19 |
| T10-Oa1-T1m |  | 141.45 |  | 148.69 |  | 140.45 |  | 137.15 |
| T2m-Oa2-T2o |  | 130.08 |  | 143.49 |  | 134.49 |  | 129.10 |
| T10-Obo-T20 |  | 139.66 |  | 147.57 |  | 146.87 |  | 139.58 |
| T1m-Obm-T2m |  | 161.20 |  | 65.79 |  | 153.41 |  | 158.77 |
| T10-Oco-T2m |  | 129.88 |  | 142.12 |  | 130.52 |  | 130.46 |
| T1m-Ocm-T2o |  | 135.85 |  | 149.97 |  | 143.70 |  | 134.69 |
| T10-Odo-T2m |  | 133.95 |  | 141.90 |  | 139.39 |  | 134.36 |
| T1m-Odm-T2o |  | 151.84 |  | 60.44 |  | 153.41 |  | 150.01 |

Note: For each tetrahedron the O-O bond lengths are followed by the O-T-O bond angles (lengths in $\AA$; angles in degrees). Lattice energy is for the unit cell.
tance. The method for calculating the Coulombic interaction of the ions uses the Ewald transformation to allow rapid convergence. Details of the implementation of the method in our lattice energy codes are given by Catlow and Norgett (1976).

It is, of course, necessary to include short-range interactions that are described by effective potentials. Buckingham potentials have been used exclusively within this study to model the central force component of these interactions. The form of this potential is given by

$$
\begin{equation*}
U_{\mathrm{R}}=\frac{1}{2} \sum_{i}^{\text {one cell all cells }} \sum_{j \neq i} \mathrm{~A}_{i j} \exp \left(-r_{i j} / \rho_{i j}\right)-\mathrm{C}_{i j} r^{-6} . \tag{2}
\end{equation*}
$$

The exponential term arises from repulsive interaction of the electron clouds as the atoms approach. The $r^{-6}$ term accounts for short-range dispersive attractions. It is common practice to use a cutoff value beyond which the shortrange forces are assumed to be negligible. The value used throughout this paper is $10.0 \AA$.

Interatomic potentials can be obtained either by fitting

Table 3. Calculated bond lengths and bond angles of microcline compared with the observed results of Brown and Bailey (1964)

|  | Obs | 2BRI | 3BSM1 | 3BSM2 |
| :---: | :---: | :---: | :---: | :---: |
| $a$ | 8.560 | 8.657 | 9.162 | 8.580 |
| b | 12.964 | 13.062 | 12.829 | 12.897 |
| $c$ | 7.215 | 7.305 | 7.263 | 7.139 |
| $\alpha$ | 90.39 | 92.83 | 90.40 | 90.36 |
| $\beta$ | 115.50 | 115.59 | 115.37 | 116.04 |
| $\gamma$ | 87.42 | 87.24 | 88.27 | 87.85 |
| lattice energy/eV |  | -2042.95 | -1921.61 | -1927.52 |
| T10-Oa1 | 1.738 | 1.722 | 1.741 | 1.736 |
| T10-Obo | 1.739 | 1.718 | 1.727 | 1.721 |
| T10-Oco | 1.745 | 1.738 | 1.748 | 1.742 |
| T10-Odo | 1.741 | 1.736 | 1.737 | 1.730 |
| O-T1-O | 109.43 | 109.42(4.4-9.2) | 109.44(0.1-1.3) | 109.42(0.1-2.3) |
| O-0 | 2.841 | 2.817(2.0-6.9) | 2.837(0.1-1.3) | 2.825(0.0-1.5) |
| T1m-Oa1 | 1.592 | 1.551 | 1.578 | 1.579 |
| T1m-Obm | 1.608 | 1.578 | 1.610 | 1.603 |
| T1m-Ocm | 1.629 | 1.599 | 1.644 | 1.631 |
| T1m-Odm | 1.627 | 1.606 | 1.636 | 1.629 |
| O-Tim-O | 109.48 | 109.38(3.9-6.9) | 109.46(0.0-1.5) | 109.49(0.5-1.7) |
| O-0 | 2.636 | 2.582(0.3-6.5) | 2.640(0.1-0.5) | 2.629(0.2-1.5) |
| T20-Oa2 | 1.614 | 1.586 | 1.622 | 1.623 |
| T2o-Obo | 1.574 | 1.559 | 1.571 | 1.571 |
| T20-Ocm | 1.633 | 1.583 | 1.646 | 1.632 |
| T20-Odm | 1.621 | 1.601 | 1.628 | 1.628 |
| O-T20-0 | 109.42 | 109.39(0.4-10.7) | 109.36(0.0-2.7) | 109.37(0.2-1.5) |
| O-O | 2.636 | 2.582(1.0-4.9) | 2.640(0.2-1.9) | 2.629(0.0-1.1) |
| T2m-Oa2 | 1.644 | 1.609 | 1.650 | 1.651 |
| T2m-Obm | 1.617 | 1.607 | 1.639 | 1.635 |
| T2m-Oco | 1.593 | 1.547 | 1.594 | 1.588 |
| T2m-Odo | 1.592 | 1.563 | 1.583 | 1.581 |
| O-T2m-0 | 109.38 | 109.38(0.2-9.1) | 109.39(0.2-2.8) | 109.34(0.4-1.3) |
| O-O | 2.628 | 2.580(1.2-4.4) | 2.637(0.2-1.8) | $2.631(0.1-1.0)$ |
| T10-Oa1-T1m | 144.69 | 151.31 | 143.20 | 141.00 |
| T2o-0a2-T2m | 138.47 | 149.62 | 145.30 | 136.58 |
| T10-Obo-T2o | 151.16 | 155.28 | 147.33 | 147.78 |
| T1m-Obm-T2m | 155.95 | 164.37 | 149.89 | 151.64 |
| T10-Oco-T2m | 130.71 | 138.32 | 129.97 | 130.20 |
| T20-Ocm-T1m | 130.68 | 148.77 | 130.43 | 131.78 |
| T10-Odo-T2m | 139.99 | 146.48 | 138.23 | 140.59 |
| T20-Odm-T1m | 143.19 | 154.79 | 139.73 | 141.97 |

Note: The $\mathrm{O}-\mathrm{O}$ distances and $\mathrm{O}-\mathrm{T}-\mathrm{O}$ bond angles are given as an average for each $\mathrm{TO}_{4}$ tetrahedron with the minimum and maximum percentage differences in brackets (lengths in $\AA$; angles in degrees). Lattice energy is for the unit cell.
the observed crystal properties or from theoretical methods. Among the latter, electron gas (EG) and modified electron gas (MEG) methods have enjoyed considerable success. The MEG potentials of Post and Burnham (1986) are employed in the present study. Indeed in the twobody rigid-ion calculations (2BRI), all parameters were based on Post and Burnham's study, whereas in the threebody shell model calculations, discussed below, both empirical and MEG potentials were used. All potential parameters employed in this study are reported in Table 1. Note that in modeling interactions between the framework species and the extra-framework cations, short-range terms are included only for cation-O interactions; interactions between tetrahedral atoms and extra-framework species are assumed to be purely Coulombic.
The second model (3BSM) is similar to the first but includes three-body and polarization terms. The latter are described using the shell model of Dick and Overhauser (1958). Sanders et al. (1984) demonstrated the inadequacy of two-body models when applied to $\alpha-\mathrm{SiO}_{2}$ and showed that it is necessary to include a bond-bending term to account for covalent forces acting within the crystal. The
bond-bending term (which is coupled to the shell) has the form

$$
\begin{equation*}
U_{\mathrm{B}}=\frac{1}{2} \sum_{i, j, k} \mathrm{k}_{\mathrm{b}, i j k}\left(\theta_{i j k}-\theta_{\mathrm{o}}\right)^{2} \tag{3}
\end{equation*}
$$

where $\left(\theta_{i j k}-\theta_{0}\right)$ is the deviation of the bond angle from an ideal bond angle and $\mathrm{k}_{\mathrm{b}}$ is a harmonic force constant. In the work of Sanders et al. (1984) the three-body terms were applied around the $\mathrm{O}-\mathrm{Si}-\mathrm{O}$ bond angles in $\alpha$-quartz. These authors suggested that this term describes "the rigidity that is conferred upon the $\mathrm{SiO}_{4}$ tetrahedra by the component of $\mathrm{sp}^{3}$ hybridized, covalent bonding around the Si atom." It is assumed that the electronic environment of the Si and O atoms in feldspars is not significantly different from that in quartz and that the potentials derived by Sanders et al. (1984) can be used without modification in this study. Jackson and Catlow in their study of zeolites (1988) demonstrated that this three-body force constant can also be used for O-Al-O bond angles in aluminosilicates.

The potentials between framework O atoms and extra-

Table 4. Observed and calculated bond lengths for anorthite

|  | Obs | Calc |  | Obs | Calc |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T1(0000)-Oa(1000) | 1.645 | 1.650 | T1(0z00)-Oa(1z00) | 1.760 | 1.753 |
| -Ob(0000) | 1.619 | 1.632 | - $\mathrm{Ob}(0 \mathrm{zOO})$ | 1.743 | 1.734 |
| -Oc(0000) | 1.582 | 1.579 | -Oc(0z00) | 1.709 | 1.710 |
| -Od(0000) | 1.616 | 1.627 | -Od(0z00) | 1.776 | 1.773 |
| T1(00i0)-Oa(10i0) | 1.632 | 1.630 | T1(0zio)-Oa(1zio) | 1.772 | 1.765 |
| -Ob(00i0) | 1.606 | 1.617 | -ob(0zio) | 1.755 | 1.772 |
| -Oc(00i0) | 1.588 | 1.587 | -oc(0ziO) | 1.727 | 1.725 |
| -Od(00i0) | 1.626 | 1.640 | -Od(Ozi0) | 1.767 | 1.759 |
| T1(mz00)-Oa(1z00) | 1.647 | 1.657 | T1(m000)-Oa(1000) | 1.777 | 1.761 |
| -Ob(mz00) | 1.617 | 1.620 | - Ob(m000) | 1.705 | 1.677 |
| -Oc(mz00) | 1.617 | 1.633 | -Oc(m000) | 1.738 | 1.732 |
| -Od(mz00) | 1.571 | 1.567 | -Od(m000) | 1.779 | 1.764 |
| T1(mzio)-Oa(1xi0) | 1.644 | 1.657 | T1(m0io)-Oa(10i0) | 1.777 | 1.761 |
| -Ob(mzio) | 1.583 | 1.567 | -Ob(m0io) | 1.747 | 1.747 |
| -Oc(mzio) | 1.599 | 1.605 | -Oc(m0io) | 1.752 | 1.749 |
| -Od(mzio) | 1.626 | 1.630 | -Od(moio) | 1.702 | 1.676 |
| T2(0z00)-Oa(2z00) | 1.635 | 1.647 | T2(0000)-Oa(2000) | 1.760 | 1.780 |
| -Ob(0z00) | 1.620 | 1.624 | -Ob(0000) | 1.769 | 1.762 |
| -Oc(m0io) | 1.606 | 1.607 | -Oc(mzio) | 1.740 | 1.733 |
| -Od(m000) | 1.605 | 1.610 | -Od(mz00) | 1.698 | 1.685 |
| T2(0zio)-Oa(2zio) | 1.617 | 1.647 | T2(00i0)-Oa(20i0) | 1.769 | 1.777 |
| -Ob(0zio) | 1.628 | 1.640 | -Ob(00io) | 1.751 | 1.745 |
| - $\mathrm{Oc}(\mathrm{m000})$ | 1.614 | 1.603 | - Oc(mz00) | 1.754 | 1.754 |
| -Od(m0io) | 1.574 | 1.577 | -Od(mzio) | 1.727 | 1.723 |
| T2(m000)-Oa(2000) | 1.644 | 1.649 | T2(mz00)-Oa(2000) | 1.755 | 1.758 |
| -Ob(m000) | 1.581 | 1.577 | -Ob(mz00) | 1.748 | 1.736 |
| -Oc(0zio) | 1.607 | 1.602 | -Oc(00i0) | 1.716 | 1.719 |
| -Od(0z00) | 1.629 | 1.645 | -Od(0000) | 1.757 | 1.751 |
| T2(m0io)-Oa(20i0) | 1.641 | 1.648 | T2(mzio)-Oa(2zi0) | 1.759 | 1.759 |
| -Ob(m0io) | 1.618 | 1.622 | - Ob(mzio) | 1.711 | 1.682 |
| -Oc(0z00) | 1.586 | 1.589 | -Oc(0000) | 1.733 | 1.724 |
| -Od(0zi0) | 1.616 | 1.629 | -Od(00i0) | 1.775 | 1.771 |
| $\mathrm{Ca}(000)-\mathrm{Oa}(1000)$ | 2.608 | 2.479 | $\mathrm{Ca}(\mathrm{z} 00)-\mathrm{Oa}(1 \mathrm{zOO})$ | 2.496 | 2.442 |
| -Oa(1000) | 2.515 | 2.436 | -Oa(1200) | 2.733 | 2.578 |
| -Oa(2000) | 2.292 | 2.209 | -Oa(2z00) | 2.333 | 2.231 |
| -Ob(0000) | 2.378 | 2.285 | -Ob(0z00) | 2.443 | 2.324 |
| -Od(mz00) | 2.390 | 2.290 | -Ob(mz00) | 2.494 | 2.423 |
| -Od(m000) | 2.538 | 2.409 | -Oc(m0io) | 2.559 | 2.532 |
|  |  |  | -Od(0z00) | 2.372 | 2.363 |
| $\mathrm{Ca}(0 \mathrm{O})$-Oa(10i0) | 2.448 | 2.392 | $\mathrm{Ca}(\mathrm{zi} 0)-\mathrm{Oa}\left(1 \mathrm{zi}^{\text {io }}\right.$ ) | 2.454 | 2.481 |
| --aa(10i0) | 2.817 | 2.823 | -Oa(1zi0) | 2.616 | 2.353 |
| - $\mathrm{Oa}(2010)$ | 2.336 | 2.213 | -Oa(2zi0) | 2.300 | 2.213 |
| -Ob(00io) | 2.426 | 2.304 | -Ob(0zi0) | 2.405 | 2.253 |
| - Ob(m0io) | 2.494 | 2.368 | -Oc(m000) | 2.833 | 2.309 |
| -Oc(mz00) | 2.563 | 2.445 | -Od(Ozi0) | 2.440 | 2.501 |
| -Od(00io) | 2.434 | 2.305 | -Od(mzio) | 2.717 | 2.928 |

Note: The experimental data are from Wainwright and Starkey (1971); calculations employed the three-body shell model and the Ca-O potential of Post and Burnham (1986); lengths in $\AA$.
framework cations were of two types: first, empirical potentials were used from a variety of sources (Model 3BSM1); second, the Post and Burnham cation-O parameters were used (Model 3BSM2).
As noted, ionic polarization of the O atom can be simulated by the use of a shell model, in which a massless shell is connected by a harmonic spring to the core, in which the atomic mass is concentrated, thus:

$$
\begin{equation*}
U_{\mathrm{s}}=\mathrm{k}_{\mathrm{s}} r^{2} \tag{4}
\end{equation*}
$$

where $U_{\mathrm{s}}$ is the core-shell interaction energy, $\mathrm{k}_{\mathrm{s}}$ is the spring constant, and $r$ is the core-shell separation. The free-ion polarizability ( $\alpha$ ) is given by

$$
\begin{equation*}
\alpha=Y^{2} / k_{s} \tag{5}
\end{equation*}
$$

where $Y$ is the shell charge.

## Results

## Two-body models

Lattice-energy minimization calculations using the 2BRI model employed the potentials of Post and Burnham (1986, 1987). Detailed results for albite and microcline are presented in Tables 2 and 3, which give calculated and experimental bond lengths and bond angles for framework species. Detailed results are not reported for anorthite using the 2BRI model, but the level of agreement is very similar to that obtained with the other two structures.
For low albite the energy minimization produced bond lengths very similar to those given by Post and Burnham (1987). As those authors pointed out, the Al-O, $\mathrm{Si}-\mathrm{O}$, and $\mathrm{O}-\mathrm{O}$ bond lengths are a little too short, as is clear from the results presented in Tables 2 and 3. The unit-cell

Table 5. Cation-oxygen bond lengths for albite and microcline for the two three-body shell model calculations (3BSM1 and 3BSM2)

|  |  |  |  |
| :--- | :---: | :---: | :---: |
| Obs | 3BSM1 | 3BSM2 |  |
| Na-Oa1(0000) | 2.671 | 2.708 | 2.565 |
| Na-Oa1(000c) | 2.537 | 2.773 | 2.521 |
| Na-Oa2(0000) | 2.372 | 2.626 | 2.363 |
| Na-Oa2(000c) | 3.724 | 3.503 | 3.649 |
| Na-Oa2(000c) | 3.725 | 3.792 | 3.726 |
| Na-Ob(000c) | 2.461 | 2.482 | 2.456 |
| Na-Ob(m00c) | 3.465 | 3.158 | 3.317 |
| Na-Oc(0zi0) | 2.996 | 2.904 | 3.004 |
| Na-Oc(mzi0) | 3.266 | 3.315 | 3.215 |
| Na-Od(0000) | 2.437 | 2.755 | 2.415 |
| Na-Od(m000) | 2.996 | 2.919 | 2.993 |
| K-Oa1 | 2.877 | 3.192 | 2.833 |
| K-Oa1 | 2.881 | 3.295 | 2.850 |
| K-Oa2 | 2.750 | 3.334 | 2.744 |
| K-Oa2 | 3.402 | 3.187 | 3.386 |
| K-Obo | 2.961 | 3.378 | 2.930 |
| K-Obm | 3.136 | 3.511 | 3.083 |
| K-Oco | 2.907 | 3.007 | 2.892 |
| K-Ocm | 3.335 | 3.253 | 3.340 |
| K-Odo | 2.892 | 2.994 | 2.865 |
| K-Odm | 2.993 | 3.167 | 2.941 |

Note: Bond lengths in $\AA$.
parameters are, however, in reasonably good agreement with the observed values. However, the 2BRI model is not able to reproduce accurately the O-T-O and T-O-T bond angles. The distortions within the tetrahedra are not correctly reproduced, whereas the T-O-T angles are consistently too large. The calculations for microcline (Table 3 ) and anorthite exhibit similar characteristics. The maximum percentage differences between calculated and observed values for the O-T-O and T-O-T bond angles are $10.7 \%$ and $13.8 \%$, respectively, for microcline. The poor reproduction of T-O-T angles in $\alpha$-quartz (which are also too large, giving a structure similar to that of $\beta$-quartz) has been noted by numerous authors and has been attributed to the omission of angle-dependent forces within the simulation, as discussed by Sanders et al. (1984).

## Three-body models

Lattice-energy minimization of the albite, microcline, and anorthite structures using the 3BSM model gives bond lengths similar to the observed results (Tables 2, 3, and 4). Improved $\mathrm{Si}-\mathrm{O}$ and $\mathrm{Al}-\mathrm{O}$ bond lengths are obtained, as the empirical potentials give a better description of the short-range interactions. However, the $\mathrm{Si}-\mathrm{O}$ bond length within a Si-O-Al unit is slightly but consistently short, indicating that the $\mathrm{Si}-\mathrm{O}-\mathrm{Al}$ interaction is not perfectly modeled. More important, however, is the finding that by including the three-body potential, the $\mathrm{O}-\mathrm{O}$ bond lengths and the $\mathrm{O}-\mathrm{T}-\mathrm{O}$ and $\mathrm{T}-\mathrm{O}-\mathrm{T}$ bond angles are greatly improved. Calculations for all three minerals were carried out using the purely empirical potentials (3BSM1) and the potentials in which MEG parameters were used for the extra-framework cation-O interaction (3BSM2). $\mathrm{Si}-\mathrm{O}$ and $\mathrm{Al}-\mathrm{O}$ bond lengths within the 3BSM2 model tend to be slightly more accurate, especially for albite. For albite and microcline, the error for O-T-O bond an-

Table 6. Calculated and observed O-T-O and T-O-T bond angles (within fourfold rings) in anorthite

|  |  | $\mathrm{Oa}-\mathrm{Ob}$ | $\mathrm{Oa}-\mathrm{Oc}$ | Oa-Od | Ob-Oc | Ob-Od | Oc-Od |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T1(0000) | obs | 100.90 | 118.07 | 101.40 | 111.73 | 113.80 | 110.44 |
| T1(0000) | calc | 97.80 | 122.50 | 98.42 | 111.41 | 115.24 | 110.79 |
| T1(00i0) | obs | 103.09 | 116.38 | 102.05 | 110.92 | 113.56 | 110.51 |
| T1(00i0) | calc | 102.52 | 119.53 | 99.63 | 110.18 | 115.33 | 109.50 |
| T1 (mz00) | obs | 100.78 | 113.70 | 108.85 | 111.38 | 113.45 | 108.61 |
| T1(mz00) | calc | 98.42 | 115.19 | 107.58 | 111.67 | 116.90 | 107.15 |
| T1(mzi0) | obs | 106.15 | 112.27 | 102.08 | 113.14 | 112.06 | 110.54 |
| T1(mzi0) | calc | 106.74 | 112.67 | 99.49 | 114.00 | 112.47 | 110.53 |
| T2(0z00) | obs | 105.34 | 101.24 | 110.17 | 112.51 | 111.79 | 114.83 |
| T2(0z00) | calc | 103.90 | 98.89 | 109.73 | 112.89 | 112.22 | 117.41 |
| T2(0zi0) | obs | 109.19 | 102.50 | 110.99 | 112.86 | 107.14 | 114.10 |
| T2(0zio) | calc | 110.75 | 101.35 | 111.53 | 114.52 | 102.77 | 116.21 |
| T2(m000) | obs | 111.89 | 104.43 | 108.95 | 112.98 | 107.26 | 111.33 |
| T2(m000) | calc | 113.73 | 104.60 | 109.32 | 113.46 | 104.40 | 111.44 |
| T2(m0i0) | obs | 108.79 | 106.37 | 107.89 | 112.60 | 108.75 | 112.27 |
| T2(m0io) | calc | 107.75 | 107.97 | 106.75 | 111.80 | 109.73 | 112.59 |
| T1(0z00) | obs | 99.75 | 117.51 | 98.98 | 112.77 | 115.60 | 111.25 |
| T1(0z00) | calc | 97.34 | 120.44 | 95.71 | 111.81 | 122.73 | 112.52 |
| T1(0zi0) | obs | 97.25 | 121.03 | 97.16 | 113.20 | 116.44 | 110.66 |
| T1(0zio) | calc | 94.06 | 122.85 | 93.23 | 112.68 | 119.44 | 112.70 |
| T1(m000) | obs | 107.57 | 112.19 | 99.12 | 114.34 | 111.15 | 111.40 |
| T1(m000) | calc | 109.57 | 112.03 | 96.32 | 114.15 | 111.84 | 111.58 |
| T1(m0io) | obs | 99.28 | 113.08 | 108.52 | 113.59 | 113.26 | 108.83 |
| T1(m0i0) | calc | 96.62 | 113.29 | 109.83 | 112.82 | 113.93 | 109.82 |
| T2(0000) | obs | 108.87 | 104.12 | 107.84 | 112.63 | 108.08 | 115.02 |
| T2(0000) | calc | 109.17 | 102.61 | 107.97 | 113.33 | 108.55 | 114.85 |
| T2(00i0) | obs | 100.96 | - 99.94 | 107.64 | 112.74 | 115.95 | 116.74 |
| T2(00i0) | calc | 99.11 | 95.71 | 107.17 | 112.31 | 118.81 | 119.14 |
| T2(mz00) | obs | 108.59 | 105.69 | 103.15 | 110.45 | 111.95 | 116.30 |
| T2(mz00) | calc | 108.56 | 105.56 | 102.11 | 112.51 | 112.50 | 115.01 |
| T2(mzio) | obs | 110.32 | 105.24 | 107.35 | 111.99 | 108.71 | 113.09 |
| T2(mzi0) | calc | 111.68 | 105.27 | 107.69 | 111.69 | 109.31 | 111.10 |
| Atom |  |  | Calc angle |  |  | Obs angle | Calc angle |
| Ob(0000) |  |  | 126.37 | Ob( |  | 138.79 | 136.07 |
| Od(0000) |  |  | 133.33 | Od(0 |  | 123.67 | 120.39 |
| $\mathrm{Ob}(\mathrm{mzOc})$ |  |  | 141.45 | Ob (m | 0c) 1 | 169.93 | 174.66 |
| Od(mzoc) |  |  | 165.57 | Od(m | 0c) 1 | 139.70 | 136.16 |
| $\mathrm{Ob}(\mathrm{mOHO})$ |  |  | 140.67 | Ob (m |  | 165.67 | 168.18 |
| Od(m0io) |  |  | 174.02 | Od(m |  | 136.14 | 132.87 |
| Ob(0zic) |  |  | 120.40 | Ob(00ic) |  | 137.61 | 133.46 |
| Od(0zic) |  |  | 133.24 | Od(00ic) |  | 127.15 | 124.60 |

Note: Potential model as in Table 5; angles in degrees.
gles and $\mathrm{O}-\mathrm{O}$ bond lengths using 3BSM2 increases for the T10 tetrahedron and decreases for the T20 and T2m tetrahedra in both minerals.

The cation- $O$ bond lengths for albite and microcline are given in Table 5. The potentials from the MEG calculations are less repulsive, giving rise to shorter cation-O bond lengths. In general, they give more satisfactory results than the empirical models. Thus in albite the less repulsive cation-O potential causes a marked improvement in the T-O-T bond angles, although in microcline only a slight improvement is observed, possibly because the smaller $\mathrm{Na}^{+}$ion requires the framework to relax inward around it. Similarly, in anorthite the empirical models raise the symmetry to monoclinic. In contrast, the less repulsive MEG cation potential is able to distort the framework to its correct lower symmetry structure, as shown by the calculated bond angles for anorthite given in Table 6. Indeed the model can reproduce the strong distortions within the tetrahedra of anorthite, as shown by the calculated bond angles given in Table 6.

## DISCUSSION AND CONCLUSIONS

The ability to reproduce the $\mathrm{Al}-\mathrm{O}$ and $\mathrm{Si}-\mathrm{O}$ bond lengths by lattice-energy minimization is largely controlled by the accuracy of the Buckingham potential describing the twobody component of the short-range repulsive forces. The accurate modeling of bond angles and of the O-O bond length requires that angle-dependent terms and polarization be included in the potential. Thus, the 2BRI model is unsuccessful in producing accurate results. The inclusion of polarization effects and three-body terms for the O-T-O angle effects a considerable improvement. The 3BSM model is capable of simulating the large range of $\mathrm{T}-\mathrm{O}-\mathrm{T}$ and $\mathrm{O}-\mathrm{T}-\mathrm{O}$ bond angles observed in the feldspar minerals. We found that the T-O-T bond angle is very sensitive to the influence of the cation-O short-range potential, whereas the O-T-O bond angle is fairly insensitive.

The MEG potentials for cation-O interactions developed by Post and Burnham perform significantly better in this respect than do the various empirical cation-O potentials, and their use should be encouraged in future studies of feldspars.

Overall, the simulations reported in this paper show that feldspar structures can be accurately modeled by available three-body shell model potentials for framework interactions and MEG potentials for interactions between extra-framework cations and O atoms. More subtle problems relating to defects and to the energetics of various $\mathrm{Si}-\mathrm{Al}$ distributions are now amenable to study.

## References cited

Brown, B.E., and Bailey, S.W. (1964) Structure of maximum microcline. Acta Crystallographica, 17, 1391-1400.

Catlow, C.R.A. (1977) Point defect and electronic properties of uranium dioxide. Royal Society of London Proceedings, A353, 53, 3-561.
Catlow, C.R.A., and Norgett, M.J. (1976) Lattice structure and stability of ionic materials. United Kingdom Atomic Energy Authority Report, AERE-M2963.
Dick, 'B.G., and Overhauser, A.W. (1958) Theory of the dielectric constants of alkali halide crystals. Physical Review, 112, 90-113.
Harlow, G.E., and Brown, G.E. (1980) Low albite: An X-ray and neutron diffraction study. American Mineralogist, 65, 986-995.
Jackson, R.A., and Catlow, C.R.A. (1988) Computer simulation studies of zeolite structure. Molecular Simulation, 1, 207-224.
James, R. (1979) Disorder and non-stoichiometry in rutile and corundum structured metal oxides. United Kingdom Atomic Energy Authority Report, AERE-TP814.
Lewis, G.V., and Catlow, C.R.A. (1985) Potential models for ionic oxides. Journal of Physics C: Solid State Physics, 18, 1149-1161.
Parker, S.C. (1983) Computer modelling of minerals. Ph.D. thesis, University of London.
Post, J.E., and Burnham, C.W. (1986) Ionic modeling of mineral structures and energies in the electron gas approximation: $\mathrm{TiO}_{2}$ polymorphs, quartz, forsterite, diopside. American Mineralogist, 71, 142-150.

- (1987) Structure-energy calculations on low and high albite. American Mineralogist, 72, 507-514.
Ribbe, P.H. (Ed.) (1983) Mineralogical Society of America Reviews in Mineralogy, Vol. 2 (2nd edition). 362 p.
Sanders, M.J., Leslie, M., and Catlow, C.R.A. (1984) Interatomic potentials for $\mathrm{SiO}_{2}$. Journal of the Chemical Society, Chemical Communications, 1271-1273.
Smith, J.V. (1974a) Feldspar minerals I. Crystal structure and physical properties. Springer-Verlag, Heidelberg.
_- (1974b) Feldspar minerals II. Chemical and textural properties. Springer-Verlag, Heidelberg.
Wainwright, J.E., and Starkey, J. (1971) A refinement of the structure of anorthite. Zeitschrift für Kristallographie, 133, 75-84.

Manuscript received September 8, 1989
Manuscript accepted September 28, 1990

