The crystal structure of gearksutite, CaAlF₄(OH)·H₂O

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ABSTRACT

The discovery of unusually well-crystallized samples of gearksutite allowed determination of its crystal structure. The crystals belong to space group $P\overline{1}$, a=4.940(1), b=6.810(1), c=6.978(1) Å; $\alpha=101.12(1)$, $\beta=94.86(1)$, $\gamma=110.07(1)^\circ$; V=213.43(6) Å³; Z=2. The structure consists of layers of eightfold-coordinated calcium polyhedra connected through pairs of $[Al_2F_8(OH)_2]$ octahedra which are oriented identically, similar to those observed in other hydroxyl aluminum fluorides. A comparison with other minerals of similar composition is made. The structure data allowed us to reliably index the powder pattern and to show that the reflection intensities are strongly affected by preferred orientation.

INTRODUCTION

Gearksutite, $CaAlF_4(OH) \cdot H_2O$, was discovered by Hagemann (in Dana 1868) in the cryolite deposit of Ivigtut, Greenland. It has subsequently been found, as a late stage mineral in a wide variety of environments all over the world, commonly in white earthy masses. At the La Veneziana mine, Veneto, Italy (Boscardin et al. 1995), it is found as a secondary mineral, associated with Cu-alteration minerals such as ramsbeckite and posnjakite. At Vulcano island, Sicily, Italy, it is found as a fumarolic phase (Bernauer 1941). Raade and Haug (1980) have reported its presence as a late-stage, cavity filling phase in a soda-granite near Oslo, Norway.

Gearksutite crystals generally are small 5-10 μm. The simple composition of gearksutite recalls that of the monoclinic strontium hydroxyfluorides tikhonenkovite, SrAlF₄(OH)·2H₂O (Pudovkina and Pyatenko 1967) and acuminite, SrAlF₄(OH)· 2H₂O (Krogh Andersen and Ploug-Sørensen 1991). However, the generally small size of the crystals, 5-10 µm limit crystallographic study to powder diffraction patterns. Notwithstanding, two different indexing methods suggested the same monoclinic cell with the appropriate volume (427 Å³), we did not succeed in solving the structure by direct methods. Recently, Birch and Pring (1990) reported the presence of an interesting assemblage of fluorine minerals, such as calcium ralstonite, morinite, and gearksutite at the Cleveland mine, Tasmania, Australia. These authors described this mineral assemblage as formed during a hydrothermal low temperature stage of crystallization. They documented the fluorine minerals and especially gearksutite, showed an exceptionally well-developed crystal habit, compated with the normal habit of these minerals. The favorable opportunity stimulated us to undertake a single crystal study of gearksutite.

This paper also elucidates the relationships of gearksutite with other fluorides and correctly indexes its powder pattern.

STRUCTURE DETERMINATION

A colorless platy crystal fragment of gearksutite from the Cleveland Mine (Tasmania) was used for intensity data collection, performed on a Siemens P4 diffractometer. Table 1 summarizes the intensity data collection and of the crystal structure refinement.

The cell parameters were obtained by least squares fitting of 29 reflections with 2θ in the range $21-28^\circ$. The collected intensities were corrected for Lorentz and polarization effects and for absorption by means of a semiempirical method based on ψ scans. The structure was solved by direct methods (SHELXTL: Sheldrick 1992) and completed by standard Fourier procedure. The positions of the hydrogen atoms were located on the difference Fourier map and were not refined. In the last refinement cycles anisotropic displacements parameters were refined for all the non hydrogen atoms and the reliability factors listed in Table 1 were obtained. Final positional coordinates and equivalent isotropic thermal parameters B_{eq} are reported in Table 2.

Further details of the crystal structure determination in the form of CIF files have been deposited with FACHIN-FORMATIONSZENTRUM KARLSRUHE, D-76344 Eggenstein-Leopoldshafen (Germany), under the depository number CSD-410611. The relevant bond lengths and angles are collected in Table 3.

DESCRIPTION OF THE STRUCTURE

The crystal structure of gearksutite consists of a network of interconnected [CaF₆(OH₂)₂] polyhedra, hereafter CaX₈, and pairs of [Al₂F₈(OH)₂] octahedra, henceforth denoted as Al₂X₁₀. Following the classification of Hawthorne (1984) for aluminofluoride minerals, gearksutite can be classified in the group containing finite clusters, namely the Al₂X₁₀ pairs. The coordination geometry of Ca²⁺, as frequently happens for eightfold coordination,

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TABLE 1. Cyrstal data and refinement details

TABLE 1. Oyistai data and ici	moment details			
Empirical formula	CaAlF₄O₂H₃			
Formula weight	178.08			
Temperature	293(2) K			
Wavelength	0.71073 Å_			
Crystal system, space group	Triclinic, P1			
Unit-cell dimensions	$a = 4.940(1) \text{ Å} \alpha = 101.12(1)^{\circ}$			
	$b = 6.810(1) \text{ Å}$ $\beta = 94.86(1)^{\circ}$			
	$c = 6.978(1) \text{ Å}$ $\gamma = 110.07(1)^{\circ}$			
Volume Z	213.43(6) Å ³			
Calculated density	2, 2.771 Mg/m ³			
Absorption coefficient	1.679 mm ^{−1}			
F(000)	176			
Crystal size	$0.085 \times 0.14 \times 0.03 \text{ mm}$			
θ range for data collection	3.02 to 24.99 deg.			
Index ranges	$-5 \le h \le 1, -7 \le k \le 7, -8 \le l \le 8$			
Reflections collected / unique	$1028 / 748 [R_{int} = 0.0558]$			
Completeness to $2\theta = 24.99$	99.7%			
Refinement method	Full-matrix least-squares on F ²			
Data / restraints / parameters	748 / 0 / 73			
Goodness-of-fit* on F ²	1.117			
Final R indices* [I >2σ(I)]	$R_1 = 0.0362$, $wR_2 = 0.0731$			
R indices* (all data)	$R_1 = 0.0545$, $wR_2 = 0.0866$			
Largest diff, peak and hole	0.539 and -0.657 e. Å-3			

^{*} Goodness-of-fit = [Σ [w(F $_{c}^{2}$ -F $_{c}^{2}$)2]/(N-P)]^{1/2}, where N, P are the numbers of observations and parameters, respectively, R₁ = Σ | |F $_{o}$ | - |F $_{c}$ | | / Σ | F $_{o}$ |; w R $_{2}$ = [Σ [w(F $_{o}^{2}$ -F $_{c}^{2}$)2] | / Σ [w(F $_{o}^{2}$)2]]^{1/2}; w = 1 / [σ ² (F $_{o}^{2}$) + (0.0243Q)² + 0.57Q] where Q = [MAX(F $_{o}^{2}$,0) + 2F $_{o}^{2}$] / 3.

cannot easily be described in terms of an idealized polyhedron. A rational approach to this problem has been suggested by Muetterties and Guggenberger (1973), who proposed comparing a real coordination polyhedron with an idealized one based on dihedral angles between planes defined by the coordinating atoms. Table 4 lists the four δ and the two ϕ values calculated for the coordination polyhedron of Ca^{2+} in gearksutite together with those of the idealized polyhedral forms with eightfold coordination, namely the dodecahedron (Dod), the bicapped trigonal prism (BTP) and the square antiprism (SAp). Our calcium polyhedron may be described as a bicapped trigonal prism slightly distorted toward the square antiprism.

The calcium polyhedra are interconnected by sharing the two almost opposite edges F3-F3* and F4-F4 † , making infinite chains in the c direction (see bottom of Table 3 for the meaning of the

Table 2. Final atomic coordinates ($\mathring{A} \times 10^4$) and displacement parameters ($\times 10^3$)

			(- /	paramotoro	
*	U _{eq} *	Z	У	Х	
1)	10(1)	2225(1)	3876(1)	3562(2)	Ca
1)	8(1)	3384(2)	8281(2)	-55(3)	Al
1)	14(1)	2942(4)	6851(4)	2315(5)	F1
1)	13(1)	2080(4)	205(4)	1998(5)	F2
1)	13(1)	1146(4)	6746(4)	7342(5)	F3
1)	12(1)	5394(4)	3701(4)	2343(5)	F4
1)	15(1)	1499(5)	2524(5)	7835(7)	OW
1)	9(1)	5884(4)	9948(5)	2202(6)	01
1111	8(1 14(1 13(1 13(1 12(1 15(1	3384(2) 2942(4) 2080(4) 1146(4) 5394(4) 1499(5)	8281(2) 6851(4) 205(4) 6746(4) 3701(4) 2524(5)	-55(3) 2315(5) 1998(5) 7342(5) 2343(5) 7835(7)	AI F1 F2 F3 F4 OW

 $\underline{*} U_{eq} = (1/3)\Sigma_i\Sigma_iU_{ii}a_ia_ia_ia_i$

TABLE 3. Main bond distances (Å) in the gearksutite structure

Ca-F3*	2.285(3)	Al-F1	1.773(3)
Ca-F1	2.292(3)	Al-F2§	1.811(3)
Ca-F2	2.328(3)	Al-F3‡	1.815(3)
Ca-F3	2.485(3)	Al-F4	1.840(3)
Ca-F4	2.354(3)	Al-O1	1.900(3)
Ca-F4†	2.379(s3)	Al-O1#	1.890(3)
Ca-OW	2.626(3)		
Ca-OW‡	2.626(3)		

Note: Symmetry transformations used to generate equivalent atoms:

* = -x+1, -y+1, -z; || = -x, -y+1, -z+1; || = -x, -y+1, -z+1; || = -x, -y+2, -z+1; || = -x, -y+2, -z+1; || = -x, -y, -z; || ** = 1-x, -y, -z; || *= 1-x, 2-y, 1-z.

superscripts). Each polyhedron is related to its nearest neighbors by two inversion centers placed in the middle of the shared edges. Similar chains are present in the crystal structure of prosopite, Ca[Al₂F₄(OH)₄] (Giacovazzo and Menchetti 1969).

The two water molecules are placed on opposite apices of the polyhedra (the OW-Ca-OW[‡] angle being 140.3°) and the chains are joined by sharing the apices OW. Bidimensionally infinite layers of calcium polyhedra, as those shown in Figure 1, are so built. The layers are puckered and parallel to (010) plane.

Connected chains of nine-coordinated strontium polyhedra are also present in tikhonenkovite, but the sharing is not so extended as in gearksutite, being limited to chain pairs that form "ribbons," indefinitely extending along c.

The aluminum cation is octahedrally coordinated by four fluorines and two hydroxyl groups in the *cis* position. The octahedra

TABLE 4. Dihedral angles δ and ϕ suggested by Muetterties and Guggenberger for geometrically defining an eight-coordination polyhedron

	D ₂ -Dod	C _{2v} -BTP	gearksutite	D _{4d} -SAp
Torsion angles	b c p g	d a e	d h g a	h g e b d c a
δ ₁ a[b, c]d	29.5	21.8	21.7	0
δ_2 e[c, f]d	29.5	48.2	52.0	52.4
δ_3 e[g, f]h	29.5	0	10.4	0
δ_4 a[g, b]h	29.5	48.2	44.3	52.4
φ _A b[a, e]f	0	14.1	19.3	24.5
φ _B c[d, h]g	0	14.1	13.7	24.5

Notes: In gearksutite the labels a, b, c, d, e, f, g, and h correspond to OW[‡], F1, F3*, F3, F2, OW, F4, and F4[†], respectively. The edges defining φ angles have been reinforced. The f-g and b-c edges in the sketches of Bicapped Trigonal Prism (BTP) and Square Antrprism (SAp), respectively, are not real edges in those polyhedra. (See notes of Table 3.)

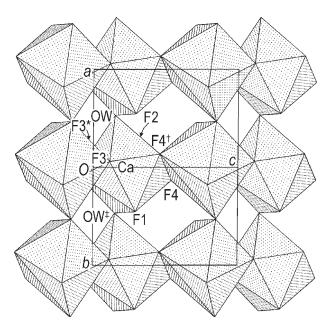


FIGURE 1. Wall of polyhedra $\{CaF_6(OH_2)_2\}$ projected in the b^* direction.

of each couple are paired by sharing the OH-OH# edge, related by the inversion center placed in the middle of the shared edge. Similar dimers with two bridging OH groups are also found in creedite, Ca₃Al₂F₈(OH)₂(SO₄)·2H₂O (Giuseppetti and Tadini 1983), acuminite and tikhonenkovite. When both fluorine and OH groups are present around the aluminum, the edge sharing of the two octahedra occurs through the hydroxyl groups, the Al₂X₁₀ groups join calcium hydrofluoride layers, by sharing all their eight fluorines with CaX₈ (Fig. 2).

The positions of hydrogen atoms were reliably suggested by the fact that the O1 atom shows only one, whereas OW atom only two, O-O or O-F distances less than 2.9 Å. Moreover, the residual electron density due to the hydrogen atoms was observed in the difference Fourier map between the most likely pairs of anions. The reported hydrogen positions may be taken with reasonable confidence, notwithstanding the low precision of the hydrogen location in X-ray structural analysis. On this basis, we describe a net of hydrogen bonds that contributes to stabilization of the gearksutite structure. The water molecule coordinating the Ca²+ ion is engaged as a donor in two hydrogen bonds with O1† [OW···O1† 2.709 Å] and F2** [OW···F2†† 2.837 Å], respectively, while the hydroxyl group bonds to the F(1††) atom [O(1)···F(1††) 2.765 Å].

Bond valence balance (Table 5) was computed according to Brese and O'Keefe (1991), apart from contribution of the hydrogen, calculated according to Brown and Altermatt (1985). Only small deviations from the exact charge balance were found, for the slightly underbonded F2 and F3, and for the overbonded O1.

Scrutiny of the intensity data shows that the $(1\overline{10})$ reflection is by far the strongest of all. This is because calcium cations are packed within a structural slab 0.28 Å thick around the $(1\overline{10})$ plane and F3 anions are located in a 0.54 Å thick slab. The layer of polyhedra parallel to $(1\overline{10})$ and 4.56 Å thick (Fig. 3), represents

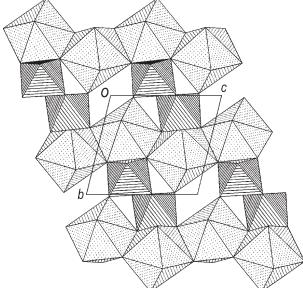


FIGURE 2. The structure of gearksutite projected in the *a* direction. The aluminum octahedra pairs bridging between the calcium walls can be seen.

TABLE 5. Bond valence (v.u.) in gearksutite

	F1	F2	F3	F4	01	OW	Σεν
Ca	0.30	0.27	0.18	0.25	_	0.17	1.80
			0.23	0.23		0.17	
ΑI	0.54	0.49	0.48	0.45	0.51	_	2.99
					0.52		
H1	0.12	_	_	_	0.88	_	1.00
HWA	_	_	_	_	0.20	0.80	1.00
HWB	_	0.10	_	_	_	0.90	1.00
$\sum_{\mathbf{a}^{v}}$	0.96	0.86	0.89	0.93	2.11	2.04	

a further way of describing the structure. Within this slightly puckered layer the connections between the cations are assured by F2, shared by one Al^{3+} and one Ca^{2+} , F3 and F4, shared by one Al^{3+} and two Ca^{2+} , and OH shared by two Al^{3+} . Each layer is connected to its nearest neighbors only by F1, bridging between one Ca^{2+} and one Al^{3+} , and by one H_2O shared between two Ca^{2+} . Hydrogen bonds too are unevenly distributed among intra- and interlayer connections with respect to these planes. In fact, while one half of water molecules give their hydrogen atoms for interlayer connection with the nearest $(1\overline{10})$ layers, the other half of water molecules and all the hydroxyl groups are engaged with bond acceptors belonging to the their own layer.

From the above considerations, one could expect that gearksutite crystals would display a platy habit and a good $(\bar{110})$ cleavage. The paucity of the material and the small size of the crystals allowed us to perform only a very limited number of tests, in which gearksutite crystals showed an imperfect $(\bar{110})$ cleavage, namely parallel to the calcium layers.

The crystal structures of creedite, acuminite and tikhonenkovite also can be conveniently described by means of layers, which have some common features with the $(\overline{110})$ layers of gearksutite.

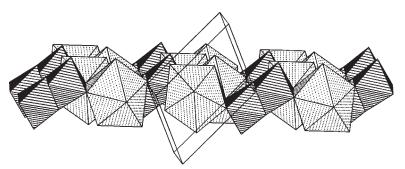


FIGURE 3. Perspective view of the plane 110. The orientation of the unit cell is also shown.

TABLE 6. Calculated and experimental powder patterns of gearksutite from Ivigtut, Greenland

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Note: Where overlapping occurs, the indices of the strongest reflection only are given for the sake of brevity.

In all these phases, the same octahedra pairs $[Al_2F_8(OH)_2]$ are linked to eightfold-coordinated Ca^{2+} in creedite and to nine-coordinated Sr^{2+} in the other two phases. Notwithstanding the similarity in chemical composition, the different means of connection between the large-radius cations and the aluminum polyhedra give rise to differently packed layers, markedly puckered in acuminite and tikhonenkovite. However, pairs of adjacent layers are linked in the same way in gearksutite, acuminite and tikhonenkovite, namely by one water molecule and by one fluorine. Structural similarities can also be noticed between gearksutite and the mineral artroeite, $PbAlF_3(OH)_2$, (Kampf and Foord 1995) in which the crystal structure consists of structural layers, parallel to $(10\overline{1})$, built up by $[Al_2F_6(OH)_4]$ linked to nine-coordinated lead cations. As in gearksutite, these layers are only slightly puckered and all the Al_2X_{10} pairs are is oriented.

Both in gearksutite and artroeite, the presence of slightly puckered structural layers correlates with the presence, in the powder pattern of the two minerals, of a very strong reflection with the same indices as the structural layers. This is not true for acuminite and tikhonenkovite, where markedly puckered layers are present.

X-RAY POWDER PATTERN OF GEARKSUTITE

X-ray powder data (Table 6) were collected on a Philips PW1050 diffractometer, using 0.02° 20 steps from 4 to 90° and counting 15 s per step, using samples coming from three different localities: Ivigtut (type locality), Chancellor mine, Colorado, and Vulcano island, Italy. The three patterns were not significantly different.

Indexing of the pattern was performed taking into account the single crystal intensity data set through the XPOW program, a part of the SHELXTL suite (Sheldrick 1992).

The above structural considerations together with the intensities calculated from single crystal data, suggested that the experimental patterns may be affected by strong (110) preferred orientation. This was confirmed by comparing the powder patterns obtained with the normal and side loaded mount. The effectiveness of side loading in randomizing the powder orientation was tested by Rietveld refinement of both the collected patterns.

The refinement was performed with the GSAS program (Larson and Von Dreele 1988). Together with background, scale factor, lattice parameters, and three profile coefficients in the 18 term pseudo-Voigt function, the preferred orientation was modeled following the formulation of Dollase (1986). The preferred orientation refinement coefficient converged to 0.742 for the

normal data collection (final reliability factor R = 0.126) and to 0.963 for the side loading collection (final R = 0.098).

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