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CONTINUOUS (DYNAMIC) MELTING THEORY REVISITED[¶]

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ABSTRACT

The concept of trace element behavior during mantle melting, in which melt is partially retained in the residue, was introduced by Langmuir *et al.* (1977), but without an explicit treatment of the process. Many subsequent workers have explored possible mechanisms, using both incremental (stepped) and continuous melting; most have discussed the melting of mantle to form basalt. Because calculations with the available models do not always give identical results, and because the assumptions embodied in those models are not always clear, this paper presents a complete statement of the formalism needed for modeling the melting of simple closed and open mantle systems, and the consequences of choosing particular values for model parameters.

Keywords: melting, continuous melting, step melting, trace elements, incompatible elements, fractionation.

SOMMAIRE

Le concept du comportement d'un élément trace au cours d'une fusion du manteau dans laquelle une partie du liquide est retenue dans le résidu a été introduit par Langmuir *et al.* (1977), mais sans traitement explicite du processus. Plusieurs chercheurs ont par la suite étudié des mécanismes possibles, en appliquant une fusion par étapes ou bien continue; la plupart ont traité de la fusion du manteau pour donner un liquide basaltique. Les calculs avec les modèles disponibles ne donnent pas toujours les mêmes réponses, parce que les suppositions de départ implicites ne sont pas toujours clairement exprimées. Cet article contient une expression complète du formalisme requis pour construire un modèle de fusion simple du manteau, soit en système ouvert ou fermé, et des conséquences d'un choix de valeurs particulières des paramètres du modèle.

(Traduit par la Rédaction)

Mots-clés: fusion, fusion continue, fusion par étapes, éléments traces, éléments incompatibles, fractionnement.

INTRODUCTION

Continuous and dynamic melting

In a study of mid-ocean-ridge basalts from the FAMOUS area in the north Atlantic, Langmuir *et al.* (1977) encountered puzzling features of the trace element distributions. These basalts show constant ratios of incompatible elements (*e.g.*, La/Ce, Ba/Nb, Zr/Nb, K/Ba, Ba/Th) and constant Mg/(Mg + Fe^{2+}); constant K/Rb and $^{87}Sr/^{86}Sr$ ratios were found by others. In contrast, there was considerable variation in La/Yb and in the rare-earth-element (*REE*) patterns, including crossing patterns.

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To explain these features, they proposed (p. 150) a process of *dynamic melting*, based on the principle that "... melt is continuously removed from the mantle with some melt always remaining in the residue" (H.J.B. Dick, oral commun., 1976). The process envisaged was described and model calculations were presented as sequences of stepped (incremental) melting episodes. The results were shown in the form of *REE* patterns, which were compared with those obtained from the rocks.

Unfortunately, few details of the calculations were included. For instance, they did not give [a] the assumed *REE* abundances of the mantle, [b] the assumed mineral proportions in the mantle, and [c] the mineral proportions melting. It was consequently unclear whether the process could be treated as modal or non-modal melting. In addition, the text is misleading, for they also introduced the concept of "*continuous melting*", which appears to be a different process from dynamic melting (their Figs. 9b, c), without sufficient explanation. As a consequence, it has been difficult to repeat the modeling that they described.

Succeeding authors have used various formal statements of how these processes might be understood, including Wood (1979), who also adopted an incremental batch-melting procedure. In the following, the dynamic process described (above) by Henry Dick will be referred to as "*continuous melting*" or "*continuous melting with retained or trapped melt*".

Fluid dynamics and fractionation

Mantle melting produces uprising magma, whose velocity exceeds that of the residual matrix, or otherwise it would not escape. However, the matrix is also moving convectively and melts by adiabatic decompression. The geochemical processes are thus influenced by the fluid dynamics of the system.

The processes taking place have been explored by McKenzie (1984) in a major paper that establishes the basic theory, in which melt, fertile mantle and mantle residue move at different velocities during melting. Later papers were devoted to (a) the physics of partly molten mantle (1985a) and (b) geochemical results of melting below ridges (1985b), and contain the claim that when melt moves out of the matrix producing it, then simple partition theory may not apply.

The physics of partly molten mantle has also been explored by Maaløe (1982), who proposed a somewhat similar process with ingenious plumbing, and by Maaløe & Johnston (1986), who described the accumulation of magma from "migmatized" (layered) partly molten mantle rising as a plume. As pressure decreases, melting begins, and the melt escapes by percolation through a compacting residuum, rising faster than the residue. Equations were developed for trace element partition as related to D values and melt fraction, F, as a function of depth.

Ribe (1985) followed a similar path. Melt formed by batch melting rises faster than its matrix, because of buoyancy; the result is that the rock-forming minerals melt at temperature minima to give rather constant major element concentrations, whereas trace element concentrations may vary widely. Navon & Stolper (1987) showed how a melt rising through the mantle might behave as a chromatographic column, so that trace elements would rise at different velocities, resulting in an inhomogeneous melt.

McKenzie & O'Nions (1991) made an exhaustive series of applications of inverse theory that claim to show that previous workers "lacked ... an understanding of the physical processes involved in the separation of the melt from its residue" (p. 1085). Qin (1992) examined melting as a grain-surface reaction, where the rates of diffusion of the trace element within the solid and within the melt determine the nature of the melting process. The validity of assuming equilibrium between grain and melt depends entirely on these rates. This has also been addressed by Bédard (1989), Bea (1991) and Bea *et al.* (1994) (see below). Sobolev & Shimizu (1993) applied continuous melting theory to the origin of "super-depleted" (SL) melt (glass) trapped in mantle materials. Ozawa & Shimizu (1995) invoked an open-system continuous melting regime to explain features of the Hayachine and Miyamori ultramafic complexes of Japan. Albarède (1995) discussed the formalism of continuous melting, but without considering the non-modal theory. Zou & Zindler (1996) adapted the melting model of Langmuir *et al.* (1977), as modified by McKenzie (1984), for fluid movement effects, as being more realistic than batch melting. Their discussion has been amplified in a more recent paper (Zou 1998).

If the mantle undergoes isentropic (decompression) melting, rather than isothermal or isobaric melting, then the characteristics of melt production over time are very different. Such studies (Asimow *et al.* 1995, 1997, Stolper *et al.* 1996) suggest that the degree of melting, F, is not an appropriate variable for following element fractionation.

A fundamental choice when developing appropriate theory is whether the melting system is to be treated as open or closed. When magma is being expelled the system is open, but if material is not being added from elsewhere, then the system may conform approximately to a closed model; this is the case for most of the work already mentioned. However, explicitly open-system models have also been proposed (Iwamori 1993, O'Hara 1995a, b, Ozawa & Shimizu 1995, Navon & Stolper 1987, Spiegelman 1996, Vernières *et al.* 1997).

In the following discussion, I will establish how trace elements might behave in the simpler melting situations where a fraction of melt is retained at the melting site, and resolve some of the ambiguities in published models. The assumptions embodied in those models have not always been made clear, and calculations with them do not always yield similar results. First of all, step melting will be examined, then continuous melting.

DEFINITIONS

Let:

W_o	initial mass of source
L, V_L	mass and volume of melt
W_s, V_W	mass and volume of solid
W_r, V_r	mass and volume of residue, *i.e.*, solid plus melt
w_o	mass of trace element in source
w^l	mass of trace element in melt
w^s	mass of trace element in solid
F	mass fraction of melt
f_p or ϕ	critical mass fraction of melt where permeability is established
α	mass fraction melt/solid where permeability is established
X^i_o	initial mass proportion of mineral *i*
X^i	later mass proportion of mineral *i*
ρ^s	density of rock matrix
ρ^l	density of liquid
Ψ	volume porosity
Q_j	melt mass proportion retained on step *j*, during incremental melting
c_o	initial concentration of trace element
c^l	concentration of fractionated element in melt
c^L	equilibrium concentration of element in melt
c^s	fractionated concentration of element in solid
c^S	equilibrium concentration of element in solid
c^r	concentration of element in residue
K^{i-l}	partition coefficient for an element between phases *i* and *l*
p^i	mass proportion of mineral *i* in melt
D	bulk partition coefficient of a trace element
β	mass proportion of material added in an open system, per unit of melting, F
γ	mass proportion of added material, β, contributed to the melt in an open system

STEP MELTING OF NON-POROUS ROCK WITH SOME MELT RETAINED

In addition to the continuous-melting models described below, incremental or step-melting processes may operate in rock fusion, as explored by Langmuir *et al.* (1977) and later by Wood (1979). Although less useful for modeling long-continued melting, such processes may apply to those natural systems which undergo intermittent melting with different parameters for each step.

Batch, or equilibrium, melting is assumed, Rayleigh-type melting (see later) being inappropriate. It is necessary to stipulate the melting proportion, F_j, as well as the degree of melt retention, Q_j, at each step *j* (Fig. 1). The proportions of minerals in the source may differ from the proportions that melt; the theory applying to this “non-modal melting” is readily adapted to “modal melting”, where the minerals melt in the proportions in which they are present, but the reverse is not the case.

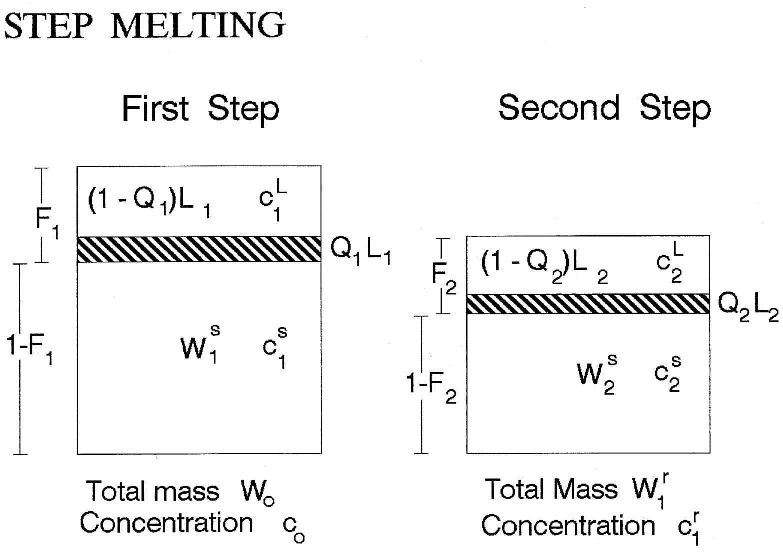

FIG. 1. Incremental batch melting, where a proportion of melt is retained in the source at each step. The source is here assumed to be solid, without porosity, and initially of mass W_o. A fraction F_1 melts, the proportion of retained melt at step 1 is Q_1, so the expelled melt has mass $L_1(1 - Q_1)$. The residue of rock and melt has mass $W_1{}^r$, which can be written R_1W_o, where $R_1 = (1 - F_1 + Q_1F_1)$; this residue undergoes batch melting for step 2. Trace element concentrations are explained in the text.

Mass fractions

Before considering trace element behavior, it is necessary to examine the evolution of the whole system in terms of the mass fractions generated on successive steps (see Fig. 1), because these quantities are needed for ensuing calculations.

Starting with a mass, W_o, of non-porous rock, melt accumulates interstitially without expulsion. When the permeability threshold is reached, at a melting fraction of F_1, the mass of melt is L_1, and it begins to escape. A proportion Q_1 is retained in the source as residual melt $L_1{}^r$, interstitial to the residual solid $W_1{}^s$, making a combined residue of mass $W_1{}^r$. A melt proportion $(1 - Q_1)$, or mass $L_1{}^{exp}$, is expelled. So, for step 1,

$$L_1 = F_1W_o\,; \qquad W_1^s = (1-F_1)W_o\,;$$
$$L_1^r = Q_1F_1W_o\,; \quad L_1^{exp} = (1-Q_1)F_1W_o$$
$$W_1^r = W_1^s + L_1^r = R_1W_o \quad \text{where} \quad R_1 = 1 - F_1 + Q_1F_1$$

On step 2, a fraction F_2 of the residual solid $W_1{}^s$ melts to form liquid L2. This is assumed to mix and equilibrate with the residual melt from step 1 to give L_2, some of which is expelled and some retained, where

$$L2 = F_2W_1^s\,; \quad W_2^s = (1-F_2)W_1^s = (1-F_2)(1-F_1)W_o$$
$$L_2 = L2 + L_1^r = \left[F_2(1-F_1) + Q_1F_1\right]W_o$$
$$L_2^{exp} = (1-Q_2)L_2\,; \quad L_2^r = Q_2L_2$$
$$W_2^r = W_2^s + L_2^r = \begin{bmatrix}(1-F_2)(1-F_1) + \\ Q_2F_2(1-F_1) + Q_2Q_1F_1\end{bmatrix}W_o$$

that is

$$W_2^r = \left[(1-F_1)R_2 + Q_2Q_1F_1\right]W_o$$

where

$$R_2 = 1 - F_2 + Q_2F_2$$

If the total degree of melting at the end of step 2 is f_2, then

$$(1-f_2)W_o = W_2^s = (1-F_2)(1-F_1)W_o$$

and so

$$f_2 = 1 - \frac{W_2^s}{W_o}$$

Similarly, the relevant quantities at the end of step 3 are:

$$L3 = F_3W_2^s\,; \quad W_3^s = (1-F_3)W_2^s$$
$$L_3 = L3 + L_2^r$$
$$L_3^{exp} = (1-Q_3)L_3$$
$$L_3^r = Q_3L_3$$
$$W_3^r = W_3^s + L_3^r$$
$$f_3 = 1 - \frac{W_3^s}{W_o}$$

The total degree of melting, f_n, at the end of step n may be found by generalizing the expression for f_3, that is

$$f_n = 1 - \frac{W_n^s}{W_o} = 1 - (1-F_n)(1-F_{n-1})\ldots(1-F_2)(1-F_1)$$

Most applications will assume that after melting begins, the melt accumulates without any expulsion taking place, until a permeability threshold is reached, at the end of step 1, *i.e.*, the retention is complete and Q_1 = 1.0. If the retention fraction after step 1 is taken as constant, and if the degree of melting on each step is taken as constant, and then these algebraic expressions become considerably simpler.

Partition coefficients

Next it is necessary to derive expressions for *bulk partition coefficients* in terms of the mineral/melt partition coefficients and the mineral abundances at sequential steps.

The mass of mineral *i* in the source is initially $W_o{}^i$, but at the end of step 1 it has decreased to $W_1{}^i$, by the amount going into the melt. So, using X for the mineral proportions in the solid, and p_i for the proportion in the melt,

$$W_1^i = W_o^i - p^iL_1$$
$$X_1^iW_1^s = X_o^iW_o - p^iL_1$$
$$X_1^i = \frac{X_o^iW_o - p^iL_1}{W_1^s} = \frac{X_o^i - p^i\,F_1}{1-F_1} \tag*{[1]}$$

Let the trace element concentrations in the source and in the solid, at the end of step 1, be c_o and $c_1{}^S$; similarly, for mineral *i*, the concentrations are $c_o{}^i$ and $c_1{}^i$, respectively. Then, summing for all the mineral phases,

$$c_o = \sum c_o^i\,X_o^i\,; \quad c_1^s = \sum c_1^i\,X_1^i$$

But, if K^{i-l} is the partition coefficient between mineral *i* and melt, and c_1^L, the melt concentration at the end of step 1, then assuming equilibrium

$$c_1^i = c_1^L \cdot K^{i-l}$$

and

$$c_1^S = \sum c_1^i \, X_1^i$$

or

$$c_1^S = c_1^L \cdot \sum X_1^i \, K^{i-l}$$

We then define a *bulk partition coefficient,* D_1, where

$$D_1 = \frac{c_1^S}{c_1^L} = \sum X_1^i \, K^{i-l}$$

and consequently, from eq. [1]

$$D_1 = \frac{D_o - PF_1}{1 - F_1} \qquad [2]$$

where

$$D_o = \sum X_o^i \, K^{i-l}, \quad P = \sum p^i \, K^{i-l}$$

Similarly, at the end of step 2 the mass balance for mineral *i* may be written

$$W_2^i = W_1^i - p^i L_2 \, ; \quad X_2^i W_2^s = X_1^i \, W_1^s - p^i L_2$$

$$X_2^i = \frac{X_1^i \, W_1^s - p^i L_2}{W_2^s} \qquad [3]$$

analogous to eq. [1], and this leads to a similar expression for the partition coefficient,

$$D_2 = \sum X_2^i \, K^{i-l} = \frac{D_1 W_1^s - PL_2}{W_2^s}$$

This equation applies to future steps, if the subscripts are changed appropriately, *e.g.*,

$$D_n = \frac{D_{n-1} W_{n-1}^s - PL_n}{W_n^s}$$

It should be noted that because X_n^i decreases at each step, the melting of a particular mineral assemblage will end when $X_n^i \leq 0$.

Trace element concentrations

When melting begins, the mass, w_o, of the trace element in the initial solid source material partitions into a solid residue portion, w^s, and a melt portion, w^l, so

$$w_o = w^S + w^L$$

or

$$c_o W_o = c_1^S \, W_1^s + c_1^L L_1$$

whence

$$c_1^L = \frac{c_o}{D_1 \left(1 - F_1\right) + F_1} \qquad [4]$$

and the mass balance for the residue, which has a concentration c_1^R, is similarly,

$$w_1^R = w_1^S + w_1^{RL}$$

$$c_1^R \, W_1^r = c_1^S \, W_1^s + c_1^L \, L_1^r$$

$$c_1^R = c_1^L \cdot \frac{D_1 W_1^s + L_1^r}{W_1^r} = c_o \cdot \frac{D_1 W_1^s + L_1^r}{W_1^r \left(D_1 W_1^s + L_1\right)}$$

It should be noted that the superscripts relating specifically to the trace element, rather than to the mass fractions, are capitalized to indicate equilibrium conditions.

On step 2, the trace element mass balance may be stated as follows:

$$w_2^S + w_2^L = w_1^R \, ; \quad c_2^S W_2^s + c_2^L L_2 = c_1^R W_1^r$$

$$c_2^L \left(D_2 W_2^s + L_2\right) = c_1^R W_1^r$$

$$c_2^L = c_1^R \cdot \frac{W_1^r}{D_2 W_2^s + L_2} \qquad [5]$$

For the residue, the mass balance is written as follows:

$$w_2^R = w_2^S + w_2^{RL} \, ; \quad c_2^R W_2^r = c_2^S W_2^s + c_2^L L_2^r$$

$$c_2^R = c_2^L \cdot \frac{D_2 W_2^s + L_2^r}{W_2^r}$$

$$c_2^R = c_1^R \cdot \frac{W_1^r}{W_2^r} \cdot \frac{D_2 W_2^s + L_2^r}{D_2 W_2^s + L_2} \qquad [6]$$

The approach used for Step 2 is readily adapted to subsequent steps, starting with the mass fractions

$$W_3^s + L = W_2^s;\quad W_3^s = (1 - F_3)W_2^s;\quad L = F_3W_2^s$$

$$L_3 = L + L_2^r;\quad L_3^{exp} = (1 - Q_3)L_3;\quad L_3^r = Q_3L_3$$

$$W_3^r = W_3^s + L_3^r$$

The partition coefficient has already been considered, so the trace element mass balance follows next:

$$w_3^S + w_3^L = w_2^R;\quad c_3^S W_3^s + c_3^L L_3 = c_2^R W_2^r$$

$$c_3^L = c_2^R \cdot \frac{W_2^r}{D_3W_3^s + L_3}$$

$$w_3^R = w_3^S + w_3^{RL};\quad c_3^R W_3^r = c_3^S W_3^s + c_3^L L_3^r$$

$$c_3^R = c_3^L \cdot \frac{D_3W_3^s + L_3}{W_3^r}$$

$$= c_2^R \cdot \frac{W_2^r}{W_3^r} \cdot \frac{D_3W_3^s + L_3^r}{D_3W_3^r + L_3}$$

Equations 3, 4, 5, and 6, and the expressions for c_3^L and c_3^R, with appropriate changes of subscript, may be applied to subsequent steps. They are more convenient to use than equivalent expressions using the variables F_n and Q_n, because they are constant in form from step to step.

It may be desirable to find the average concentration of a trace element in all the liquids expelled, assuming that they have accumulated and mixed. If the expelled mass after n steps is l_n, then

$$l_n = W_o - W_n^r$$

so, for example,

$$l_1 = W_o - W_1^r = W_o(1 - R_1),$$

TABLE 1. NORMATIVE MINERAL PROPORTIONS* IN PERIDOTITE AND DERIVED MELT

	Source	Melt
olivine	0.554	0.20
orthopyroxene	0.252	0.30
clinopyroxene	0.173	0.50
spinel	0.021	0.00
total	1.000	1.00

* weight fraction.

If the accumulated concentration of a trace element is $\overline{c_n}$ then

$$\overline{c_n}\, l_n = c_o W_o - c_n^R W_n^r$$

whence

$$\overline{c_n} = \frac{c_o W_o - c_n^R W_n^r}{W_o - W_n^r}$$

Most applications will assume that after melting begins, the melt accumulates without any expulsion taking place, until a permeability threshold is reached, at the end of step 1, *i.e.*, the retention is complete, and Q_1 is equal to 1.0. If the retention fraction after step 1 is taken as constant, and if the degree of melting on each step is taken as constant, and then these algebraic expressions become considerably simpler.

The melt fraction trapped in the source at the end of any step is obtained as the ratio $\frac{L_n^r}{W_n^r}$ and is not constant.

The relative masses of rock and trace element during successive melting steps of a peridotite, composed of olivine, orthopyroxene, clinopyroxene and spinel, with mass proportions given in Table 1, and partition coefficients for La in Table 2, are shown in Figure 2a. The total melt fraction on a step is L, made up of new melt (not shown) plus the residue from the previous step, before it splits up into residual (L[r}) and expelled (L[exp]) melt. W[s] is the solid remaining at the end of a step. It should be noted that, by taking $Q_1 = 1$, then *all* the melt produced in step 1 is retained, and it only begins to be expelled on step 2. Q_2 is chosen as 0.3, and

TABLE 2. SELECTED VALUES OF MINERAL/MELT PARTITION COEFFICIENTS FOR THE RARE-EARTH ELEMENTS

	Ol	Opx	Cpx	Spl
La	0.000053	0.000044	0.0536	0.00002
Ce	0.000105	0.00014	0.0858	0.00003
Pr	0.000251	0.00033	0.137	0.0001
Nd	0.000398	0.00052	0.1873	0.0002
Sm	0.00065	0.0016	0.291	0.0004
Eu	0.0008	0.0033	0.329	0.0006
Gd	0.0015	0.005	0.367	0.0009
Tb	0.0021	0.0067	0.405	0.0012
Dy	0.0027	0.0084	0.442	0.0015
Ho	0.005	0.0127	0.415	0.0023
Er	0.01	0.017	0.387	0.003
Tm	0.016	0.025	0.409	0.0038
Yb	0.027	0.033	0.43	0.0045
Lu	0.03	0.041	0.433	0.0053

The coefficients were selected from listings by Gurenko & Chaussidon (1995), Hart & Dunn (1993), Johnson *et al.* (1990), Jones (1995), Ozawa & Shimizu (1995), Yang *et al.* (1998), Zack *et al.* (1997).

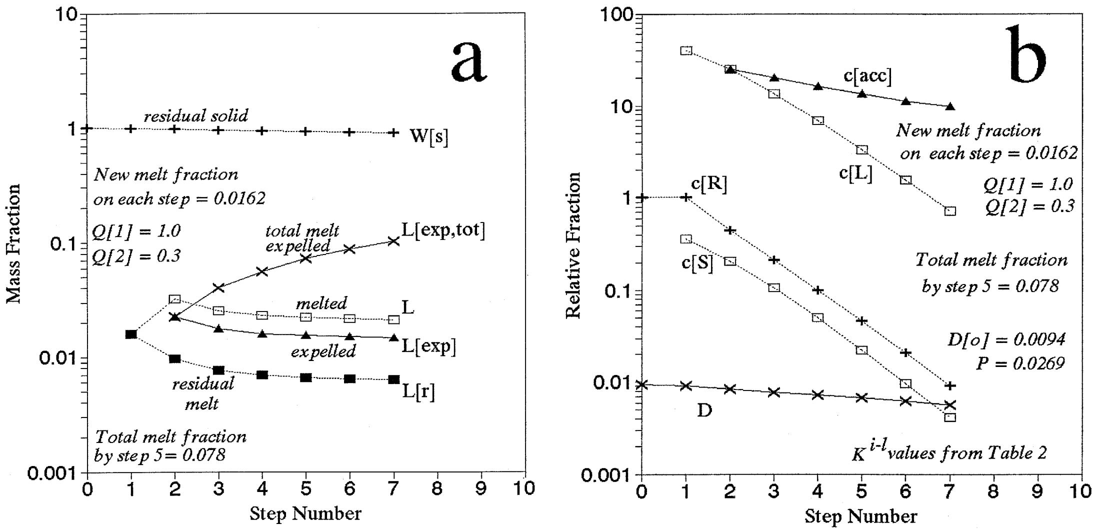

FIG. 2. a. Relative changes in mass from step to step during batch melting with trapped melt, setting Q_1 and Q_2 to 1.0 and 0.3, respectively; Q_2 remains constant in succeeding steps. Thus, all the first-step melt is retained, but on succeeding steps, 30% only is retained. The residue left at the end of a step melts and equilibrates on the next step, and part of the melt is expelled. L is the total melt on each step, made up of new melt (not shown) plus residual melt from the preceding step. L[r] and W[s] are the mass of liquid and solid remaining at the end of a step; L[exp] is the mass of melt expelled, and L[exp, tot] is the accumulated mass of expelled melt. The degree of melting, 1.62%, is the same on each step, and the ratio of residual melt to solid remains fairly constant. By step 7, the total degree of melting has reached 10.8%. b. Mineral proportions in Table 1 and partition coefficients for the incompatible element La in Table 2 are used to show relative concentrations for the trace element in the melt, solid and residual mass fractions shown in a), and also the average concentration c[acc] in the accumulated expelled melt fractions. Note that the concentration in the expelled melt only begins at step 2.

remains unchanged on succeeding steps. By making the degree of melting the same on each step, the ratio melt/solid remains relatively constant, as would be the case with constant permeability.

Taking this example as the melting of a mantle source, where melt accumulates until the degree of melting has reached a point where the source material becomes permeable and melt begins to be expelled, the changes in La concentration are shown in Figure 2b. The high initial concentration in the melt drops steeply in successive steps, the residual solid behaving similarly, one to two orders of magnitude below. The residue of melt plus solid has an intermediate La concentration, the solid being buffered by the trapped melt. Also shown are the steadily changing *D* values.

The permeability threshold has been arbitrarily taken to be 2% melt by volume, which corresponds roughly to a mass melting proportion of 0.0162; this is the value taken for f_p in Figure 2 and subsequently (see discussion in next section).

Figure 3 shows the behavior of all the *REE*, relative to unit initial concentrations, on three steps taken from Figure 2, using appropriate partition coefficients from Table 2. The light rare-earth elements (*LREE*) are enriched in the initial liquid, but by step 4 the effects of incompatibility in the *LREE* are being overcome, and the overall pattern is rather similar to that of the source, although about 10 times higher; by step 7, with over 10% total melting, the *LREE* are strongly depleted relative to the heavy rare-earth elements (*HREE*), in both melt and solid.

TRACE-ELEMENT-BASED CONTINUOUS MELTING MODELS AS "CLOSED" SYSTEMS

General considerations

The next topic is to consider how the progressive melting of a solid source in which some of the melt is retained determines the behavior of a trace element.

In models of trace element behavior, it is important to distinguish equilibrium behavior from mechanisms of fractionation. Within a mineral, the diffusion rate of an element toward the melting surface will determine whether equilibrium between crystal and melt is attainable (see, for example Qin 1992, Bea 1991, Bea *et al.* 1994). If the rate of diffusion is too slow, *"disequilibrium melting"* will occur (Prinzhofer & Allègre 1985, Bédard 1989), and the behavior of trace elements will not differ from that of major elements. The effects of diffusion will not be further considered in this section.

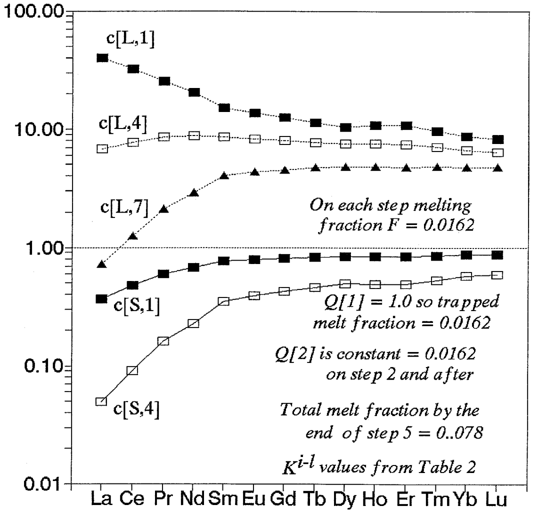

FIG. 3. Concentrations of *REE*, relative to $c_o = 1.0$ in expelled melt and residual solid at the end of steps 1, 4 and 7, in batch melting of a spinel peridotite (mineral proportions and partition coefficients as in Tables 1 and 2). The degree of melting is 1.62% on each step, the same as in Figure 2. On the first step, 100% of the melt is retained, *i.e.*, none is expelled, but in succeeding steps, 30% melt is retained. By the seventh step, the total melt fraction now amounts to 10.8%, as in Figure 2, of which most has been expelled (see text).

As the uprising mantle begins to melt adiabatically, drops of initial melt form at grain contacts and, as pointed out by Sobolev & Shimizu (1993, p. 183), the source "cannot lose melt until the degree of melting attains a certain critical level ..."

Maaløe (1982) had already described this critical level and referred to it as the "permeability threshold", which depends on several factors, including the stress field and the viscosities of melt and source. Estimates of this critical melt fraction, here denoted by f_p, at which permeability is established, range from one part in a thousand to several per cent.

The source is assumed to be non-porous, because of the high pressure at the seat of melting; this is undoubtedly true for the mantle, which is the main focus here, but may be less true for crustal melting. Nevertheless, discussions of mantle melting frequently invoke porosity, *e.g.*, Williams & Gill (1989, p.1609) defined "continuous melting" as "...continuous removal of magma from a progressively melting but dynamically static source having a finite porosity. That is, we assume that some fraction of liquid produced during melting remains with the matrix and is not extracted." This appears to be a different concept from ordinary porosity, which relates to pore space which is empty or filled only with fluid. Most writers about rock melting ignore this difference, and take the proportion of melt to define a volume porosity Ψ, so that with definitions as given above,

$$\psi = \frac{V_L}{V_L + V_W}$$

Where the mass fraction of liquid reaches a critical level, during melting, such that melt expulsion ensues, this level is sometimes called the mass porosity, ϕ or f_p. It is readily shown that

$$\phi = \frac{L}{L + W} = f_p = \frac{\psi\rho_l}{\psi\rho_l + \rho_s(1 - \psi)}$$

Some authors use ϕ for volume porosity, which can be confusing. Others prefer not to refer to porosity and, instead, use the ratio, α, of melt to solid, at the point where melt expulsion begins, where

$$\alpha = \frac{f_p}{1 - f_p}$$

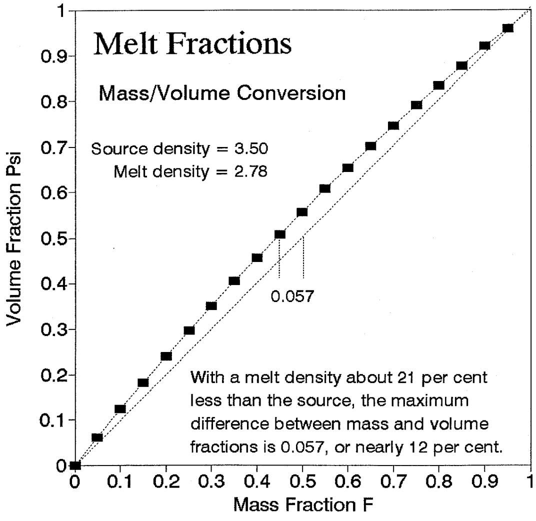

FIG. 4. Where the source and the liquid to which it is melting have different densities, their volume and mass fractions are not equal. If model parameters are expressed in volume units, as is commonly the case, then an adjustment will be needed for calculations using mass units. The effect is exaggerated here, because it is unlikely that a melt as low in density as 2.78 g cm^{-3} would come from a peridotite of density 3.5 g cm^{-3}.

The practical difference between volume and mass fractions is a small factor in most melting models, even assuming considerable difference in density between source and melt (Fig. 4).

Equilibrium relations are usually assumed to govern the concentration of a trace element before the melt begins to migrate; the subsequent *continuous (dynamic) melting*, with melt retained, may be a non-equilibrium reaction. The construction of a model therefore is made in terms of an equation for partial melting, starting from Rayleigh's formulation (1902), as adapted for a brief period of melting:

$$\frac{dw^s}{dW} = c^l \qquad [7]$$

where w^s is the mass of the trace element in the unmelted source material of mass W; c^l is the concentration in the liquid produced during this episode, regardless of any previous melt. Then the basic equation for a particular element will be obtained by finding an expression for

$$\frac{dc^s}{c^s} \text{ or } \frac{dc^l}{c^l}$$

which is amenable to integration in terms of some measure of evolution of the system; this may be *time* or *temperature* or, more usually, *F, the degree of melting*.

The degree of melting needs careful definition in circumstances where the source consists of an interstitial pre-existing melt in a solid matrix. The concept of "melting" may apply either to [a] the solid material only, or to [b] the whole source. In the first case, it is usual to assume that the melting solid and the new melt both equilibrate with the pre-existing melt, even if only momentarily. In the latter case it is necessary to modify the *D* value (see below), in order to treat the pre-existing melt as another source phase (with partition coefficient of unity); this "pre-existing melt" may in fact be solid material injected during some previous event *e.g.*, basalt veins in peridotite, which have subsequently melted.

One must also distinguish whether minerals melt in the proportions in which they occur in the source (*i.e.*, modal *versus* non-modal melting); this is of trivial importance with highly incompatible elements, but may not be ignored if the theory is to be used also for less strongly incompatible elements.

Discussion of four published formulations for "closed" system melting follows.

The McKenzie model

This model was put forward first in a fluid dynamics formulation (McKenzie1984), and subsequently (1985a, b) adapted specifically to trace element behavior during mantle melting. His treatment has subsequently been used by others (*e.g.*, Williams & Gill 1989) and by Eggins (1992), who adapted it to the melting of a rising mantle plume.

It may be noted that Williams & Gill (1989, p. 1609) define *continuous melting*, as quoted above. However, their *dynamic melting* denotes a model in which the rate of melting and the permeability remain the same, but "...the system of matrix and interstitial fluid is moving instead of static. Fertile material (undepleted by melt extraction) constantly moves into the source region..."

In attempting to compare the results from this theory with others, some adjustments have to be made. Although the solid was taken by McKenzie to be monomineralic with a single constant *D*-value, he noted (1984, eq. E30) that a polymineralic assemblage could be used, as is normally the case. In addition, no provision was made for variation of *D* with *F*, as occurs in non-modal melting. It is not easy to make such a provision (in the McKenzie treatment), but the following equations were developed for the non-modal case. McKenzie (1984) denoted volume porosity by ϕ but, to be consistent, Ψ is used here.

Then, assuming rapid intracrystalline diffusion of the trace element, constant porosity and constant melting rate, the application of eq. [7] to non-modal melting of a mineral assemblage gives

$$\frac{dc^l}{c^l} = -\frac{dF}{1-F} \cdot \frac{\left(1 - D_o - F + FP\right)\left(\psi\rho^l + \rho^s - \psi\rho^s\right)}{\rho^s D_o\left(1-\psi\right) + \psi\rho^l - F\left(\rho^s P\left[1-\psi\right] + \psi\rho^l\right)}$$

This expression may be integrated to give

$$\ln\frac{c^l}{c^l_o} = \frac{C}{2H}\ln\frac{HF^2 - F\left(E+H\right) + E}{E} + \left[\frac{C\left(E+H\right)}{2H\left(E-H\right)} - \frac{B}{E-H}\right]\ln\frac{E-HF}{E\left(1-F\right)} \qquad [8]$$

where

$$C = \left(1-P\right)\left(\rho^l\psi + \rho^s - \rho^s\psi\right)$$
$$B = \left(1-D_o\right)\left(\rho^l\psi + \rho^s - \rho^s\psi\right)$$
$$H = \rho^s P\left(1-\psi\right) + \psi\rho^l$$
$$E = \rho^s D_o\left(1-\psi\right) + \psi\rho^l$$

The term c_o^l represents the trace element concentration in the liquid *before* continuous melting starts, which is seen from eq. [2] and [4] to be

$$c_o^l = \frac{c_o}{D_o + f_p(1-P)} \qquad [9]$$

If the densities of source and melt are equal, so that $\rho^l = \rho^s$, then $\Psi = f_p$, and eq. [8] becomes

$$\ln\frac{c^l}{c_o^l} = \ln\left(1-\frac{yF}{x}\right)^{\frac{1}{y(1-f_p)}} - \ln(1-F)^{\frac{1}{1-f_p}} \qquad [10]$$

where

$$x = D_o + f_p(1-D_o); \quad y = P + f_p(1-P)$$

Applications of these equations will be shown below.

The Albarède model

In his formulation, Albarède (1995) showed that the melting of a partially solid source that already contains some previously formed melt can be modeled by treating the melt, *m*, as an extra phase with $K^{m-l} = 1$. This is an analogy (in reverse) to the treatment of intercumulate melt during fractional crystallization.

The bulk partition coefficient for the solid phases, D, in the initial modal melting is constant. The mass fraction of melt when permeability is established is f_p, and so the bulk partition coefficient becomes

$$D_c = D(1-f_p) + f_pK^{m-l} = D(1-f_p) + f_p$$

which is Albarède's eq. [9.3.21].

Fractional melting now follows the Rayleigh modal melting equation,

$$c^s = c_o[1-F]^{\left(\frac{1}{D}-1\right)}$$

whose exponent changes to

$$\left[\frac{1}{D_c}-1\right], \text{ that is, } \frac{(1-f_p)(1-D)}{(1-f_p)D+f_p} \qquad [11]$$

Albarède's formulation does not discuss the source of the melt already present.

To carry his treatment over to the non-modal case, it is necessary to modify eq. [11] using eq. [2], that is

$$D = \frac{D_o - PF}{1-F}$$

and to assume the initial trapped melt and solid to be in equilibrium, with the mass ratio melt/solid (α), where

$$\alpha = \frac{f_p}{1-f_p},$$

which leads to the following expression for the trace element concentration

$$\frac{c^l}{c_o^l} = \left[\frac{D_o + \alpha - F(P+\alpha)}{D_o + \alpha - f_p(P+\alpha)}\right]^{\frac{1-P}{P+\alpha}} \qquad [12]$$

or the equivalent equation using f_p in place of α,

$$\frac{c^l}{c_o^l} = \left[\frac{D_o(1-f_p) + f_p - F\left[P(1-f_p)+f_p\right]}{D_o(1-f_p) + f_p - f_p\left[P(1-f_p)+f_p\right]}\right]^{\frac{(1-P)(1-f_p)}{f_p+P(1-f_p)}}$$

where

$$c_o^l = c_o \cdot \frac{1}{D_o + f_p(1-P)} \qquad [13]$$

and

$$f_p < F < 1$$

The Sobolev & Shimizu model

Sobolev & Shimizu (1993) described what they call a "*critical continuous melting* (*CCM*)" model, which is based on "... non-modal equilibrium melting ... of mantle material ... in a closed system and (which) cannot lose melt until the degree of melting attains a certain critical level ... (p. 183), where the restite (*i.e.*, the residue) always retains the critical content of melt (the critical level has the same meaning as in the two previous models). Their basic differential equation (their eq. [1]) reflects the mass conservation of a trace element:

$$c^l dm_s = d(Kc^l m_s) + d(\alpha c^l m_s)$$

where m_s is the mass of solid material in the mantle source, α is the ratio of the mass of the melt m_l to that of the solid m_s in the mantle material, K is the bulk partition coefficient, and c^l is the trace element concentration in the melt. Expressed in the terms used here, this equation becomes

$$c^l dW_s = d\left(c^s W_s\right) + d\left(c^l L\right)$$

After batch melting below the critical melt level, the subsequent concentrations of trace elements are expressed in their eq. [3], as follows:

$$\frac{c^l}{c_o} = \frac{1}{D_o + \dfrac{\alpha(1-P)}{(1+\alpha)}} \cdot \left[\frac{D_o + \alpha - F(P+\alpha)}{D_o + \alpha - \dfrac{\alpha(P+\alpha)}{1+\alpha}} \right]^{\frac{1-P}{P+\alpha}} \qquad [14]$$

but details of the integration are not given. This equation is identical with the combination of eq. [12] and [13] in the previous section.

The Zou model

Some of the difficulties encountered in the models just discussed have been treated by Zou (1998). After re-examining modal melting along the lines used by McKenzie (1984), he adopted the Albarède approach to derive expressions for trace element variation during non-modal melting, and then for open-system melting. His expression for c^l, the concentration in the melt after expelling a melt fraction, X, is as follows, substituting f_p where he uses ϕ, and modifying slightly his eq. [33]:

$$c^l = c_o^l \cdot \left[1 - \frac{X\left[P + f_p(1-P)\right]}{D_o + f_p(1-P)} \right]^{\left[\frac{1}{P+f_p(1-P)} - 1\right]} \qquad [15]$$

$$c_o^l = \frac{c_o}{D_o + f_p(1-P)}$$

These equations give the same results as eq. [12], [13], and [14], but Zou (1998) recommended his formulation, because he finds that ϕ (or f_p) is more useful than α.

"CLOSED-SYSTEM" MODEL

Theory

Before considering an open-system model, it will be useful to lay out the summary theory, in the light of the four models reviewed above, for continuous non-modal melting with trapped melt.

Using the sketch in Figure 5, a solid source of mass W_o containing a trace element concentration c_o begins batch melting, which continues to the critical level of

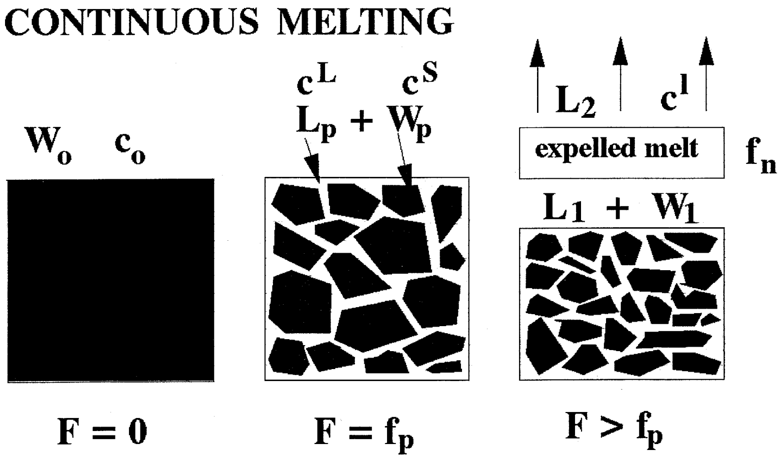

FIG. 5. Continuous melting in a *"closed" system*. Initial batch melting proceeds until there is a mass L_p of melt and W_p of residual solid, at which point the source becomes permeable and *continuous melting* ensues; the mass ratio liquid/solid (α) remains constant, and liquid L_2 is expelled, carrying the trace element concentration c^l. The fraction of *new* melt is $f_n = L_2/[L_p + W_p] = F(1 + \alpha) - \alpha$, where F is the *total* melt fraction, including L_1.

permeability. At this point, there is a melt mass, L_p, and solid mass, W_p, such that $L_p/W_p = \alpha$, and the degree of melting is

$$f_p = \frac{L_p}{L_p + W_p} = \frac{\alpha}{1+\alpha}$$

The trace element concentrations are c^L and c^S, and the bulk concentration remains c_o. Batch melting may be treated as a mixing of equilibrated liquid and solid, so mass balance for the trace element is expressed by

$$w^L + w^S = w_o \text{ or } Lc^L + Wc^s = c_o W_o$$

so that

$$c^L = \frac{c_o}{D + D(1-F)} \qquad [16]$$

where D is as given in eq. [2]. It follows that, for non-modal melting in the range $0 < F < f_p$,

$$\frac{c^L}{c_o} = \frac{1}{D_o + F(1-P)}$$

and

$$\frac{c^s}{c_o} = \frac{D_o - FP}{1-F} \cdot \frac{c^L}{c_o}$$

The terminal values, where $F = f_p$, are $c_p{}^L$, $c_p{}^S$ and D_p.

With further melting (Fig. 5), a melt mass L_2 with concentration c^l is expelled, leaving a residue ($L_1 + W_1$), with mass ratio α, which henceforth remains constant. If the total melt fraction is F, then the additional degree of melting, after melt expulsion begins, is f_n, defined by L_2/W_o, and so

$$F = \frac{L_2 + L_1}{W_o} = f_n + \frac{\alpha W_1}{W_o}$$

but

$$W_1 = W_o(1-F)$$

and so

$$f_n = F(1+\alpha) - \alpha = \frac{F - f_p}{1 - f_p}.$$

The trace element mass balance is given by

$$w_o = w^{L_2} + w^{L_1} + w^{W_1} = w^{L_2} + w^r$$

so, taking differentials

$$dw^{L_2} + dw^r = 0 \qquad [17]$$

i.e., the increase in the trace element mass in the liquid plus the increase in its mass in the residue equals zero.

The bulk concentration in the source material residue now is c^r, and the concentrations in the melt and the solid are c^l and c^s, respectively. This melting process embodies the principle expressed in eq. [7], that the melting of a small portion of the source, containing the trace element, creates a momentary concentration c^l in the liquid, which is the ratio of the mass of the trace element dw^{L2} added to the liquid while its mass increases by dL_2, *i.e.*,

$$c^l = \frac{dw^{L_2}}{dL_2},$$

and consequently from eq. [17]

$$c^l dL_2 + dw^r = 0$$

But

$$w^r = L_1 c^1 + W_1 c^s$$

and so

$$dw^r = L_1 dc^l + c^l dL_1 + W_1 dc^s + c^s dW_1,$$

whence

$$c^l dL_2 + L_1 dc^l + c^l dL_1 + W_1 dc^s + c^s dW_1 = 0 \qquad [18]$$

The participating masses are

$$\begin{aligned}
L_2 + L_1 &= FW_o \\
W_1 &= (1-F)W_o \\
L_1 &= \alpha W_1 = \alpha(1-F)W_o \\
L_2 &= f_n W_o = W_o\left[F(1+\alpha) - \alpha\right] \\
W_r &= L_1 + W_1 = W_o(1-F)(1+\alpha)
\end{aligned}$$

so it follows that

$$\begin{aligned} dW_1 &= -W_o dF \\ dL_1 &= -W_o \alpha dF \\ dL_2 &= +W_o(1+\alpha)dF \\ dW_r &= -W_o(1+\alpha)dF \end{aligned}$$

and eq. 18 becomes

$$\begin{aligned} &c^l(1+\alpha)dF + \alpha(1-F)dc^l \\ &-\alpha c^l dF + (1-F)dc^s - c^s dF = 0 \end{aligned} \qquad [19]$$

We also know that

$$c^s = c^l D, \quad D = \frac{D_o - PF}{1-F},$$

and note that D is not constant, so

$$dc^s = c^l dD + D dc^l, \quad dD = \frac{D_o - P}{(1-F)^2} \cdot dF$$

and by substituting these in eq. 19, we obtain

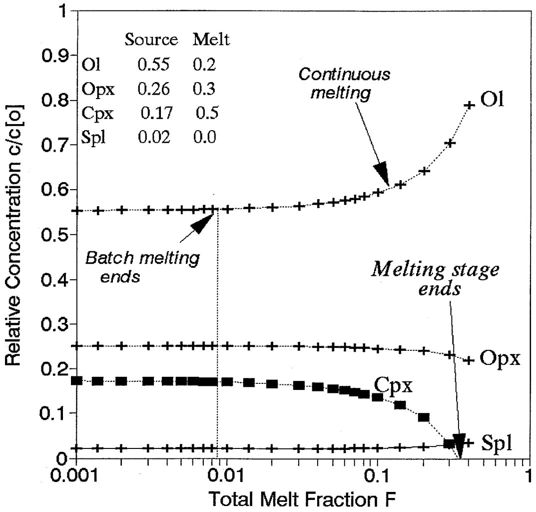

FIG. 6. Changes in mineral proportions as a consequence of non-modal continuous closed-system melting. Any mineral *i* is totally consumed when $F = X_o^i/p^i$, at which point the melting step is at an end. This occurs for clinopyroxene when F = 0.34, so points to the right of this value would be meaningless.

$$\frac{dc^l}{c^l} = -\frac{(1-P)dF}{D_o + \alpha - F(P+\alpha)}$$

Integration for the range $f_p < F < 1$, gives:

$$\frac{c^l}{c^l_o} = \left[\frac{D_o + \alpha - F(P+\alpha)}{D_o + \alpha - f_p(P+\alpha)}\right]^{\frac{1-P}{P+\alpha}} \qquad [20]$$

where

$$c^l_o = \frac{c_o}{D_o + f_p(1-P)}$$

and

$$c^s = Dc^l.$$

We note that eq. [20] is, of course, the same as eq. [12]. An alternative derivation of eq. [20] is given in the Appendix.

It may be noted that, if the source is permeable from the beginning of melting, then $f_p = \alpha = 0$, and eq. [20] becomes

$$\frac{c^l}{c_o} = \frac{1}{D_o}\left[1 - \frac{PF}{D_o}\right]^{\left(\frac{1}{P}-1\right)}$$

which is Rayleigh fractionation in a simple non-modal melting system.

The concentration, c^l, of the trace element varies as the melt is expelled but, after the total degree of melting is F, the accumulated concentration is $\bar{c}$. The mass balance of the trace element may now be written

$$w^l = w_o - w^s,$$

so

$$\bar{c}(L_1 + L_2) = c_o W_o - c^s W_1,$$

whence

$$\frac{\bar{c}}{c_o} = \frac{1}{F}\left[1 - (1-F)\frac{c^s}{c_o}\right] \qquad [21]$$

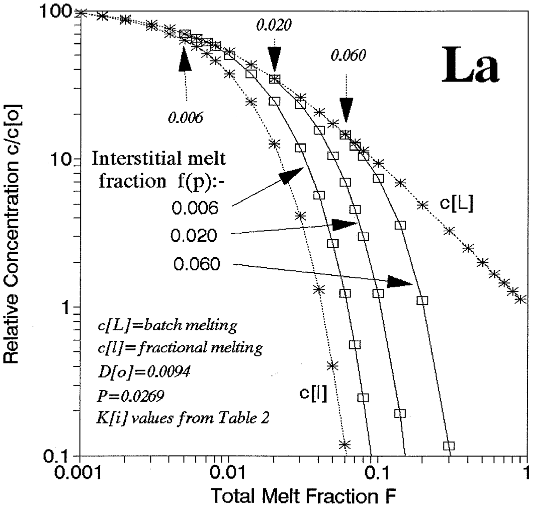

FIG. 7. Behavior of an incompatible element during peridotite non-modal melting, and its dependence on the critical melt fraction for the onset of permeability. Continuous melting leads to essentially the same concentration trend as fractional melting, but beginning at the point where the interstitial batch-melt fraction (f_p) begins to be expelled, as seen here for three chosen values.

DISCUSSION

The effect of non-modal melting on mineral proportions is shown in Figure 6. When any mineral has been totally consumed, the melting step ends; this takes place for clinopyroxene, in this case, when F = 0.34. If melting calculations are to continue, a new set of mineral proportions is needed.

An example of calculations following eq. [20] for an incompatible element is shown in Figure 7. The limits of concentration behavior are shown by the batch-melting and the fractional-melting curves. Continuous-melting curves resemble fractional melting in their dependence on F, but are offset according to the magnitude of the critical melt fraction f_p for the onset of permeability. Each of the three continuous melting processes starts with batch melting, then the retention of a constant fraction of melt leads to a degree of fractionation intermediate between batch and fractional melting. The onset of permeability for a mantle peridotite probably corresponds to a small value of f_p, and so the concentration behavior would be close to that for fractional melting; lesser permeability would lead to lesser fractionation.

A similar calculation for all the rare-earth elements shows (Fig. 8a) the difference in behavior of the heavier *REE*, which are less incompatible in common peridotite-forming minerals than the *LREE*. The *HREE* thus show much less divergence in the different melting mechanisms, as seen already in step-melting theory (Fig. 3).

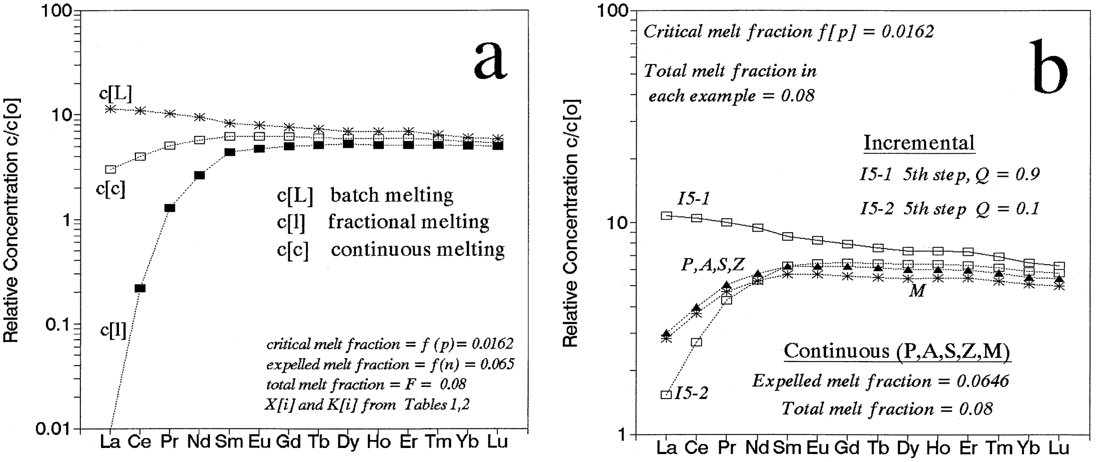

FIG. 8. a. Calculated relative concentrations of the *REE* for three types of non-modal melting, for a critical melting fraction (f_p) of 1.62%, expelled melt fraction of 6.5% and total degree of melting of 8%. Continuous melting gives concentrations intermediate between batch and fractional melting, but the differences are very small for the heavier, more compatible elements. b. *REE* behavior in two continuous melting processes (P, present formulation in eq. [20]; S: Sobolev & Shimizu (1993), A: Albarède (1995), Z: Zou (1998), M: McKenzie (1984), as expressed in eq. [8]), and in two incremental batch-melting processes, both on the fifth step but with the fraction of melt retention, Q, equal to 0.9 for I5–1 and 0.1 for I5–2. In all cases, the critical melt fraction is 0.0162, and the total degree of melting is about 8%.

It was seen above that the trace element concentrations in the liquid obtained by following the McKenzie approach (eq. [8]) differ from those using the present approach and those of Albarède, Sobolev & Shimizu, Zou (eq. [20], [12], [14], [15]). This is seen to be the case in Figure 8b; the differences are quite insignificant, but do not depend on D_0, which is the same in each case. Also, a comparison between concentrations expected in continuous melting and incremental batch melting shows that, for the latter, the degree of melt retention, Q, strongly influences the behavior of the more incompatible *LREE*, even where the total melt fraction is the same (about 8%) in each of the cases illustrated.

OPEN-SYSTEM MODEL

Theory

As mentioned above, a number of authors have addressed melting in an open system, where source and melt may move independently at different velocities (Iwamori 1993, O'Hara 1995a, b, Ozawa & Shimizu 1995, Navon & Stolper 1987, Spiegelman 1996, Vernières *et al.* 1997, Zou 1998). The simplest and essential feature of an open system in the present context is that solid material melts while matter is being added and melt is being released. A one-dimensional version is shown in Figure 9. The added mass at some degree of evolution (Fig. 9b) is denoted by W_{aa}, and the analysis is simplified if the rate of addition is assumed to be constant (see Ozawa & Shimizu 1995) and equal to β per unit of melting F, so that

$$W_{aa} = \beta F_a W_o \qquad [22]$$

It has not yet been specified whether the matter added is solid or liquid; Ozawa & Shimizu (1995) referred to fluid. The choice is significant, and at least three options are open. The first is addition of fluid, which dissolves in available melt, but is not directly added to the solid phases. The second is addition of fluid, part of which dissolves in the melt, and part is added to the solids: this addition most likely takes place by metasomatism. The third is addition of both fluid and solid, both being incorporated into the melt, the latter by the process usually referred to as *assimilation*. Evidence exists for all three processes, but perhaps the most relevant to simple mantle melting is the first option. So, for the present, it will be taken that the added material is *fluid*, that it contributes only to the melt, and that there is momentary equilibration between added matter, melt and solid. The added material is at the ambient temperature and pressure, and the enthalpy of mixing will not be considered.

The degree of melting, F, is defined in terms of the mass of melt produced from the original solid source, the remaining solid being

$$W = W_o\left(1 - F\right)$$

In this context, F is the degree of melt from the solid source, but it is NOT the total melt fraction, where melt is being expelled.

During the first phase of batch melting (Figs. 9a, b), the degree of melting is F_a, and the matter added contributes to the melt, so the mass balance is

$$L_a + W_a = W_o + W_{aa}$$

and from eq. [22]

$$L_a + W_a = W_o\left(1 + \beta F_a\right)$$

So

$$W_a = \left(1 - F_a\right)W_o$$
$$L_a = W_o F_a\left(1 + \beta\right)$$

At this stage the melt fraction, ϕ", and melt/solid ratio, α", are given by

$$\phi'' = \frac{\text{melt}}{\text{total}} = \frac{F_a\left(1+\beta\right)}{1+\beta F_a}$$
$$\alpha'' = \frac{\text{melt}}{\text{solid}} = \frac{F_a\left(1+\beta\right)}{1-F_a}$$

The mass balance for mineral i may be written (see eq. [1]) as

$$W_a X^i = W_o X_o^i - p^i L_a ,$$

so, using eq. [2]

$$D = \sum X^i K^{i-1} = \frac{W_o}{W_a}\sum X_o^i K^{i-1} - \frac{L_a}{W_a}\sum p^i K^{i-1}$$

whence,

$$D = \frac{D_o - F_a P\left(1+\beta\right)}{1 - F_a} \qquad [23]$$

The partition coefficient is thus affected by the material added, and becomes a variable even for modal melting; for, if $P = D_o$, then

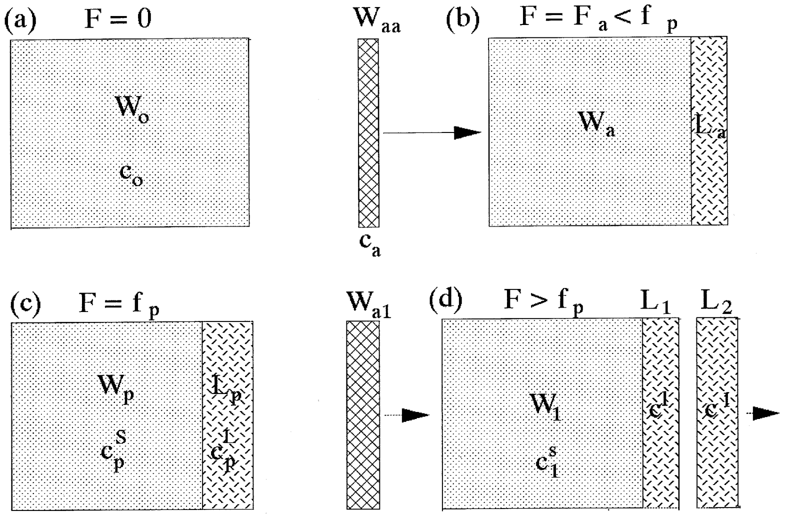

FIG. 9. In the case of *open-system melting* pictured here, fluid mass W_{aa} carries a concentration c_a of the trace element, and is being added to the system at a constant rate, which is proportional (β) to the degree of melting F. The fluid W_{aa} is assimilated into the melt only. Batch melting begins (a) in the solid system, before any fluid is incorporated. (b) At any degree of melting (F_a) less than f_p, the material added amounts to $\beta F_a W_o$, and the melt mass is L_a. (c) After the critical melt fraction, f_p, has been reached, batch melting ends. (d) At melting degree $F > f_p$, a mass of melt L_2 is expelled, with trace element concentration c^l, as in the residual trapped melt L_1. At this point, a fluid mass $W_{aa} + W_{a1}$ has been added to the system [note that $W_{a1} = \beta W_o(F - f_p)$].

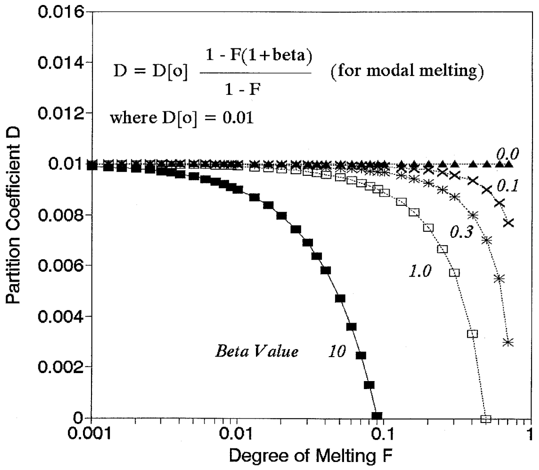

FIG. 10. In an open system where material is continually added to the melt, the partition coefficient (see eq. [24]) is no longer constant, even if the melting is modal (as in the case shown here), *whether or not the material added contains the trace element*. This effect takes place because the influx of material affects the mass balance of each mineral in the system, and so the value of D is largely controlled by the parameter β. In non-modal melting, the magnitude of P is also affected by β.

$$D = D_o \cdot \frac{1 - F_a(1+\beta)}{1 - F_a} \qquad [24]$$

This relationship is illustrated in Figure 10, which shows how dependent D is on β. For example, at 10% melting, the effective partition coefficient has decreased by about 10% if the rate of fluid addition equals the degree of melting (*i.e.*, $\beta = 1.0$). The curve for $ß = 10$ is included to show this dependence, even though it is difficult to envisage an influx of this magnitude (which would more closely resemble the behavior of a fumarole). Note that D is variable *whether or not the material added contains the trace element,* because the material influx affects the mass balance of each mineral in the system; the value of D_o thus is controlled mainly by the parameter β.

Taking f_p again as the batch melting limit before melt is expelled, then for the range $0 < F < f_p$, the trace element's behavior is governed by

$$c^L L_a + c^S W_a = c_o W_o + c_a W_{aa}$$

so

$$c^L = \frac{c_o W_o + c_a W_{aa}}{L_a + DW_a} = \frac{c_o + c_a \beta F}{D_o + F(1+\beta)(1-P)}$$

where c_a is the trace element's concentration in the material being added. At the limit, when $F = f_p$, the melt fraction is

$$\phi = \frac{f_p(1+\beta)}{1+\beta f_p}$$

and

$$\begin{aligned} \alpha &= \frac{f_p(1+\beta)}{1-f_p} \\ c_p^L &= \frac{c_o + c_a\beta f_p}{D_o + f_p(1+\beta)(1-P)} \\ c_p^S &= Dc_p^L = \frac{D_o - f_p(1+\beta)}{1-f_p}\cdot\frac{c_o + c_a\beta f_p}{D_o + f_p(1+\beta)(1-P)} \end{aligned} \qquad [25]$$

Continued melting (*i.e.*, where $F > f_p$) leads to expulsion of melt L_2 (Figs. 9c, d), leaving the residue, W_r, consisting of L_1 and W_1, whose ratio, α, remains constant as melting and melt expulsion continue. The total mass of melt comprises melted source plus the matter added, which is

$$W_{aa} + W_{al} = \beta F W_o.$$

So, the mass balance becomes

$$L_1 + L_2 + W_1 = W_o + W_{aa} + W_{al} = W_o(1+\beta F) \quad [26]$$

But, since

$$W_1 = W_o(1-F), \text{ consequently } L_1 + L_2 = W_o F(1+\beta)$$

and

$$\begin{aligned} L_1 &= \alpha W_1 = W_o\alpha(1-F) \\ L_2 &= W_o\left[-\alpha + F(1+\alpha+\beta)\right] \\ W_r &= L_1 + W_1 = W_o(1+\alpha)(1-F) \end{aligned}$$

If the additional degree of melting (beyond f_p) is f_n where, as above,

$$f_n = \frac{L_2}{W_o},$$

then

$$F = \frac{L_2 + L_1}{W_o} = f_n + \frac{\alpha W_1}{W_o} = f_n + \alpha(1-F),$$

so

$$f_n = F(1+\alpha) - \alpha.$$

The differentials of the various masses are

$$\begin{aligned} dW_{al} &= dFW_o\beta \\ dW_1 &= -\,dFW_o \\ dW_r &= -\,dFW_o(1+\alpha) \\ dL_1 &= -\,dFW_o\alpha \\ dL_2 &= dFW_o(1+\alpha+\beta) \end{aligned}$$

Before continuing, it is useful to survey the relative magnitudes of the masses and their ratios in an open melting system, dependent as they are on the parameters D_o, P and β, and on the variable F. In Figure 11, the influx rate β is equal to 0.10, and when complete melting has occurred, that is when F = 1.0, the influx mass W_{aa} + W_{a1} is now 0.10, and the total mass of the system has thus increased to 1.10. Also, since all of the fluid influx goes into the liquid, the final mass of expelled liquid, L_2, reaches 1.10, and the residual solid mass, W_1, is 0. Here, the idea of complete melting de-

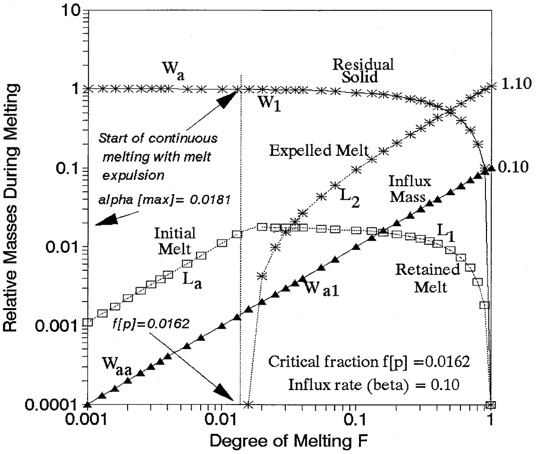

FIG. 11. It is useful to compare the relative masses of different parts of an open system, dependent as they are on the parameters D_o, P, β, f_p and on the variable F. In choosing here that the influx rate β = 0.10, then although "complete melting" has taken place when F = 1.0, the influx mass W_{aa} + W_{a1} is now 0.10, and the total mass of the system has now increased to 1.10. This is the final mass of expelled liquid, L_2, and the residual solid mass, W_1, is 0.00. The idea of complete melting here describes the fate of the original mass of source rocks.

scribes the fate of the original mass of source rock, and this fate will be reflected in the trace element behavior.

The trace element mass balance is, from eq. [26],

$$w^{L_1} + w^{L_2} + w^{W_1} = w^{L_2} + w^{r} = w_o + w^{aa} + w^{al} \quad [27]$$

where

$$\begin{aligned} w^{L_2} &= c^{l} L_2, \\ w^{r} &= c^{r} W_r, \\ w_o &= c_o W_o, \\ w^{aa} + w^{al} &= c_a \beta F W_o \end{aligned}$$

and c^r is the concentration in the residue, so

$$c^{r} = \frac{c^{l} L_1 + c^{s} W_1}{L_1 + W_1} = c^{l} \cdot \frac{\alpha + D}{1 + \alpha}$$

and

$$dc^{r} = \frac{\alpha + D}{a + \alpha}\, dc^{l} + \frac{c^{l}}{1 + \alpha}\, dD$$

To evaluate dD, eq. [23] is used

$$D = \frac{D_o - FP(1+\beta)}{1 - F},$$

so

$$dD = dF \cdot \frac{D_o - P(1+\beta)}{(1 - F)^2}.$$

To express the mass conservation, one must differentiate eq. [27] to obtain

$$dw^{L_2} + dw^{r} = dw^{al} \quad [28]$$

or

$$c^{l} dL_2 + dw^{r} = c_a \beta W_o dF$$

As with the closed system case, one has

$$w^{r} = L_1 c^{l} + W_1 c^{s}$$

and so

$$dw^{r} = L_1 dc^{l} + c^{l} dL_1 + W_1 dc^{s} + c^{s} dW_1,$$

whence

$$\begin{aligned} &c^{l} dl_2 + L_1 dc^{l} + c^{l} dL_1 \\ &+ W_1 dc^{s} + c^{s} dW_1 = c_a \beta W_o dF \end{aligned} \quad [29]$$

With substitution of the differentials, this becomes

$$\frac{dc^{l}}{(1+\beta)(1-P)\, c^{l} - c_a \beta} = - \frac{dF}{D_o + \alpha - F\left[\alpha + P(1+\beta)\right]}$$

and, on integration,

$$\frac{c^{l}}{c_o} = \left[\frac{c^{l}_o}{c_o} - y \cdot \frac{c_a}{c_o}\right] \cdot \left[\frac{1 - zF}{1 - zf_p}\right]^{u} + y \cdot \frac{c_a}{c_o} \quad [30]$$

where,

$$\begin{aligned} y &= \frac{\beta}{(1+\beta)(1-P)} \\ u &= \frac{(1+\beta)(1-P)}{\alpha + P(1-\beta)} \\ z &= \frac{\alpha + P(1-\beta)}{D_o + \alpha} \end{aligned}$$

Also, $c_o^{l} = c_p^{L}$, as given in eq. [25], and $c^{s} = Dc^{l}$. It may be noted that if the proportion of the added matter is vanishingly small (so that $\beta = 0$), then eq. [30] collapses into eq. [20].

The accumulated concentration, $\bar{c}$, in all the melt expelled, is obtained as before by modifying eq. [27] as follows:

$$\begin{aligned} w^{l} &= w_o + w^{aa} + w^{al} - w^{s} \\ \bar{c}(L_1 + L_2) &= c_o W_o + c_a \beta W_o F - c^{s} W_1 \end{aligned}$$

thus obtaining

$$\bar{c} = \frac{c_o + c_a \beta F - c^{s}(1 - F)}{F(1+\beta)} \quad [31]$$

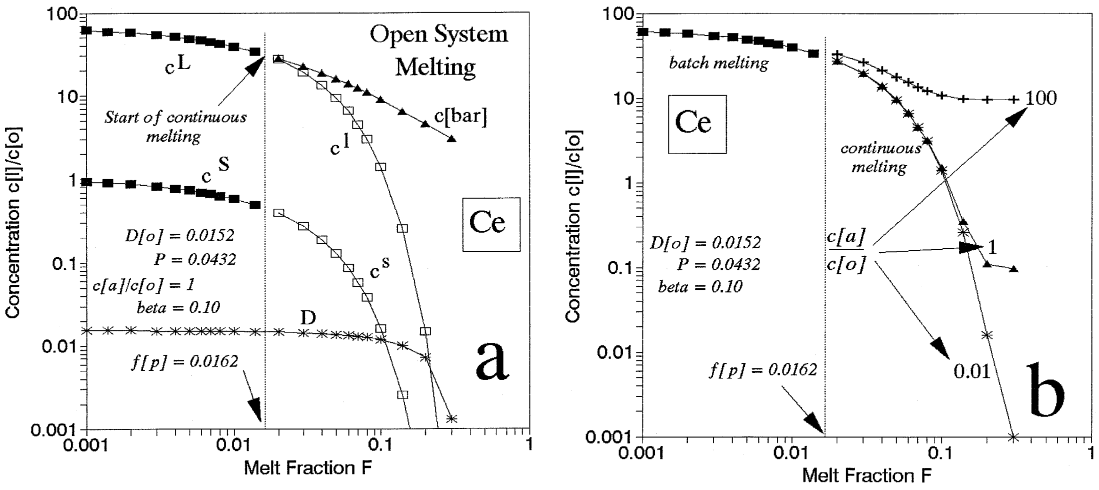

FIG. 12. a. The behavior of Ce, a strongly incompatible element, during peridotite open melting, is similar to the closed system (see Fig.8) at low degrees of melting, but differs when F > 0.1. The melting step ends when F > 0.3, because the clinopyroxene has been exhausted, and D goes negative. b. For a given value of the rate of material addition, β, the most influential parameter on the concentration is seen to be c_a, three values of which are shown. A high value of c_a effectively buffers the concentration.

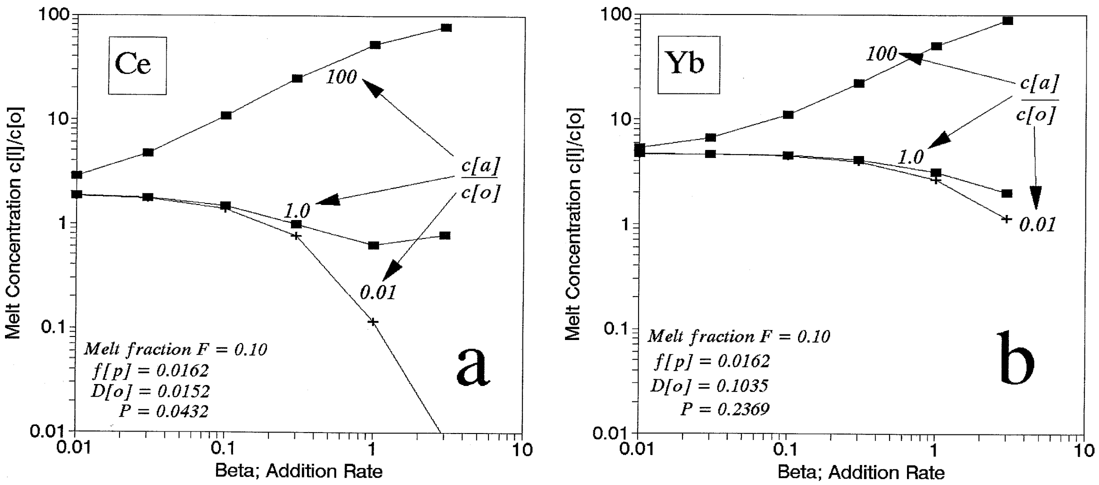

FIG. 13. Both c_a and β influence strongly the behavior of incompatible (a) and compatible (b) elements during open-system melting. Large values of β, of course, correspond to a process that may be considered as magma mixing, and are inappropriate to simple mantle melting.

DISCUSSION

The behavior of Ce, a strongly incompatible element, in open-system melting of peridotite, is shown in Figure 12a for a particular choice of the parameters in eq. [30]. The concentration of Ce in melt is similar to that in a closed system (see Fig. 7) if F is only slightly greater than f[p], but then changes course, depending on the values held by c_a and β. The effect of variation in c_a, when β is held constant is shown in Figure 12b; where the added matter is much richer in the element of interest, it buffers the concentration in the melt.

Increasing the rate of addition, β, reinforces the effect of either high or low c_a, not only for a highly incompatible element like Ce (Fig. 13a), but also for more compatible ones such as Yb (Fig. 13b), so the choice of

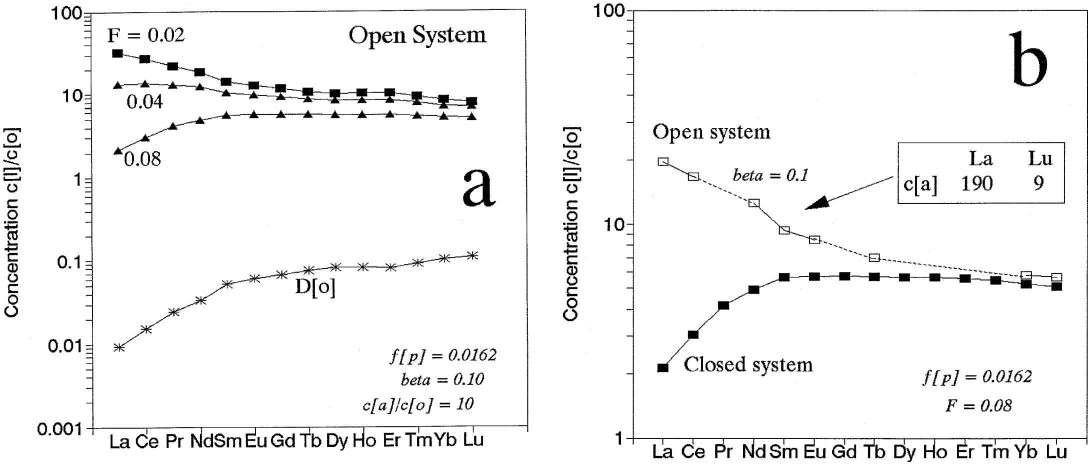

FIG. 14. a. Marked fractionations of the *REE* can occur as a function of differing degrees of open- system melting for a chosen set of parameters. The extent of fractionation is a function of element compatibility, as measured by D_o, from Table 2. To choose c_a at the same value for each element is unrealistic, but thereby avoids obscuring the fractionations. b. The values of c_a chosen here are the alkali basalt values given by Bodinier *et al.* (1990, Fig.10), and illustrate the effect of an infiltrating fluid enriched in the *LREE* relative to the *HREE*. Even with addition at a rate of only 10% shown here, there is an order-of-magnitude increase of the *LREE* in the melt (note the logarithmic scale).

each of these parameters has a powerful effect on open-system modeling. With a more compatible element, the effect of variations in β are minor, except at very large values of c_a. It may be noted that with large values of β, the process will resemble magma mixing.

It may also be noted from Figures 12 and 13 that the behavior of a trace element is more influenced by the influx concentration, c_a, and the rate of influx β, than by the coefficients D_o and P.

When considering groups of related elements such as the *REE*, the comparison of different degrees of melting, for a chosen set of parameters (Fig. 14a), shows the relative fractionations that can occur as a function of compatibility differences. Figure 14a uses the same value of c_a for each element to emphasize the fractionation effects, but this situation is unrealistic.

In Figure 14b, the values of c_a suggest an infiltrating fluid enriched in the *LREE* relative to the *HREE*. These values for alkali basalt were used by Bodinier *et al.* (1990) as a source of metasomatism and melt infiltration reacting on solid peridotite; they are appropriate for the present model. Even with an influx factor of only 10%, these lead to an order-of-magnitude increase for the *LREE* (note the logarithmic scale in Figure 14b).

Other factors

In the previous section, it was assumed that fluid material was added solely to the melt. In the case where the fluid is also added to the solid, it is necessary to specify the mass proportion, (1–γ); the proportion added to the melt is γ. Then, following from eq. [22], the matter added is divided into two portions, liquid and solid. Thus one can write

$$W_{aa} = \text{liquid} + \text{solid} = L_{aa1} + W_{aa1}$$
$$L_{aa1} = \gamma W_{aa}\,; \quad W_{aa1} = (1-\gamma)W_{aa}$$

The mass balance is

$$L_a + W_a = W_o + W_{aa}$$

If the total mass of melt is L_a, then

$$L_a = FW_o + \gamma W_{aa} = FW_o(1+\beta\gamma)$$

where F is the degree of melting. Similarly,

$$W_a = (1-F)W_o + (1-\gamma)W_{aa}$$
$$= W_o\left[1 - F(1-\beta+\beta\gamma)\right]$$

so that

$$L_a + W_a = W_o(1+\beta F),$$

as before.

Further development along these lines will be continued elsewhere. The “solid” component added is really the product of a metasomatic reaction, dominated by element diffusion, which is not within the scope of the present paper and must be so treated (see, for example, Bodinier *et al.* 1990).

CONCLUSIONS

The foregoing pages present a more-or-less complete statement of the formalism needed for modeling the melting of simple closed- and open-mantle systems, and the consequences of choosing values for parameters. The same approach can be readily modified for crustal melting. The principal factors which have *not* been included are 1) variability in partition coefficients, 2) noncotectic mineral proportions, 3) mineral reactions (*i.e.*, incongruent melting), 4) differing times of diffusion through solid and liquid, 5) differing velocity vectors for solids and melts, and 6) directional variation in melting rates.

With such a variety of factors disregarded, it may well be asked whether the exercise was worthwhile. It was embarked upon because of the variety of treatments found in published papers and the difficulty of extracting the assumptions on which most of them were based. It is hoped that it will be useful as a base to build further. No attempt has been made here to apply the analysis to real-world rock series.

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APPENDIX. ALTERNATIVE DERIVATION OF THE CONTINUOUS MELTING EQUATION

An alternative derivation of eq. [20] may be obtained, using the treatment of Rayleigh (1902). Eq. [7] may be adapted to the melting of a mixture of solid and melt and expanded as follows:

$$\frac{dw^r}{dW_r} = c^l = \frac{d\left(c^r W_r\right)}{dW_r} = c^r + W_r \frac{dc^r}{dW_r} .$$

Next, the following expressions are substituted into this equation,

$$W_r = L_1 + W_1 = W_o\left(1 - F\right)\left(1 + \alpha\right)$$
$$dW_r = - W_o\left(1 + \alpha\right)dF$$

$$c^r = \frac{c^l L_1 + c^s W_1}{W_r} = c^l \cdot \frac{\alpha + D}{1 + \alpha}$$
$$dc^r = dc^l \cdot \frac{\alpha + D}{1 + \alpha} + \frac{c^l dD}{1 + \alpha}$$
$$dD = \frac{D_o - P}{\left(1 - F\right)^2} \cdot dF$$

which can be re-arranged to give

$$\frac{dc^l}{c^l} = - \frac{\left(1 - P\right)dF}{D_o + \alpha - F\left(P + \alpha\right)}$$

which gives eq. [20] on integration.