

The crystal structure of syngenite, $K_2Ca(SO_4)_2 \cdot H_2O$

By E. CORAZZA and C. SABELLI

Centro Nazionale di Cristallografia del C.N.R.
Istituto di Mineralogia dell'Università, Firenze, Italia

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Auszug

Syngenit hat die Raumgruppe $P2_1/m$; die Elementarzelle mit $a = 9,77 \text{ \AA}$, $b = 7,15 \text{ \AA}$, $c = 6,25 \text{ \AA}$, $\beta = 104,0^\circ$ enthält $2K_2Ca(SO_4)_2 \cdot H_2O$.

Die Struktur ist aus parallel (100) liegenden Netzen von Sauerstoff-Polyedern um K-Atome aufgebaut. Die Polyeder sind sehr unregelmäßige Würfel, denen eine Ecke fehlt und dafür eine Fläche zentriert ist. Die Ca-Atome sind von neun Sauerstoffatomen umgeben; die Polyeder bilden über gemeinsame Kanten Ketten parallel y . Durch diese Polyeder um Ca und durch die SO_4 -Gruppen werden die K-Netze miteinander verbunden.

Abstract

Syngenite crystallizes in the space group $P2_1/m$, with two molecules in the unit cell of dimensions $a = 9.77$, $b = 7.15$, $c = 6.25 \text{ \AA}$, $\beta = 104.0^\circ$.

Intensities of ten layers were recorded by the precession method from a spherical specimen. Corrections for absorption and secondary extinction were applied.

The structure consists of a two-dimensional net of K polyhedra, each K being surrounded by eight oxygen atoms at the vertices of a very irregular cube lacking one corner, and with one face centered. K polyhedra form a chain parallel to y by the alternate sharing of a quadrangular face and a triangular one. These chains, two per cell, are linked together by edges and corners to form a sheet parallel to (100). The Ca atom binds nine oxygen atoms, and the Ca polyhedra share one edge to form a chain in the y direction. Each Ca polyhedron shares three corners on one side and two on the opposite one with K polyhedra, and in this way bridges the K sheets to each other. Additional bridges among the Ca chains, and among the Ca chains and the K sheets, are provided by the regular SO_4 tetrahedra. The only oxygen not belonging to SO_4 is the water molecule.

Introduction

The crystal structure of syngenite has been investigated as a part of a series of studies on sulphate minerals. ARUJA (1957) determined the lattice constants of syngenite in several ways, both on synthetic and

natural specimens. The parameters measured by us on the Weissenberg and precession photographs agree with ARUJA's ones. In the determination of the present crystal structure the following lattice constants were then assumed:

$$a = 9.77, \quad b = 7.15, \quad c = 6.25 \text{ \AA}, \quad \beta = 104.0^\circ.$$

The space group $P2_1/m$ given by ARUJA was confirmed by the negative result of a piezoelectric test, and the solution of the structure gave no element in favour of the acentric space group $P2_1$.

The specific gravity measured by ARUJA ($2.579 \text{ g} \cdot \text{cm}^{-3}$) agrees well with the calculated one ($2.575 \text{ g} \cdot \text{cm}^{-3}$).

Experimental

The crystals used for this structure investigation come from Kalusz, Galicia. A transparent and colorless fragment was ground to the spherical shape (radius 0.52 mm) for a proper absorption correction.

Crystal data:

chemical formula: $K_2Ca(SO_4)_2 \cdot H_2O$

monoclinic, $a = 9.77$, $b = 7.15$, $c = 6.25 \text{ \AA}$, $\beta = 104.0^\circ$

$V = 423.5 \text{ \AA}^3$

$D_m = 2.579 \text{ g} \cdot \text{cm}^{-3}$, $Z = 2$, $D_x = 2.575 \text{ g} \cdot \text{cm}^{-3}$, $F(000) = 328$

space group: $P2_1/m$

absorption coefficient for x-rays [$\lambda(\text{MoK}\alpha) = 0.7107 \text{ \AA}$]:

$$\mu = 21.72 \text{ cm}^{-1}, \quad \mu R = 1.13$$

Intensity data were collected at room temperature, using a zirconium filtered $\text{MoK}\alpha$ source, on an integrating Buerger precession camera, for two sets of reciprocal-lattice levels hkl ($l = 0$ to 3) and hkl ($h = 0$ to 5). Intensities deduced from the optical densities were measured by a microdensitometer for all the ten levels, each recorded with a series of five differently exposed films.

Lorentz-polarization and absorption corrections were made by a program written for the I.B.M. 1620 computer by CATANI and ZANAZZI (1965). The $|F_o|^2$'s were converted to an appropriate relative scale by the correlation of the common reflections. 1364 independent reflections were recorded, 343 of which were below the observational limit.

Structure determination

A three-dimensional Patterson function was computed from the corrected intensity data. The general position in $P2_1/m$ being fourfold, Ca atom and the oxygen atom of the water molecule had to lie in special positions; O could lie only on a mirror plane because of the symmetry of the water molecule.

In this Patterson synthesis vectors due to the contribution of K, two sulphur atoms, Ca and four oxygen atoms were recognized, taking into account the main cleavages of syngenite. This could be explained only by admitting the existence of a chain of Ca polyhedra along the y direction.

By means of a three-dimensional Fourier synthesis the remaining atoms were placed. An outstanding feature was that, among 11 non-hydrogen atoms in the asymmetric unit, only K and two oxygen atoms [O(5) and O(7)] lay off the mirror planes. So, the number of atoms in special positions rose from two, as expected before the solution of the structure, to eight. The atomic scattering factors used for the calculations of the structure factors are the ones given by HANSON *et al.* (1964).

The refinement was carried out by two more Fourier syntheses. At this stage, the R index being 0.14, the refinement was continued by the least-squares, block-diagonal, method, using CRUICKSHANK'S weighting formula, and individual thermal parameters, as derived from the difference Fourier map. After 8 cycles of refinement the R index dropped to the value of 0.075 for all the observed reflections, and to 0.096 including the non-observed ones, which were given an intensity one

Table 1. *Fractional atomic coordinates and isotropic thermal parameters with their standard deviations*

| Atom | x | $\sigma(x)$ | y | $\sigma(y)$ | z | $\sigma(z)$ | B | $\sigma(B)$ |
|------|-------|-------------|-------|-------------|-------|-------------|---------------------|--------------------|
| K | .3357 | .0002 | .0068 | .0002 | .1947 | .0002 | 1.35 Å ² | .02 Å ² |
| Ca | .9679 | .0002 | .2500 | | .3327 | .0002 | .47 | .02 |
| S(1) | .9897 | .0002 | .2500 | | .8396 | .0003 | .38 | .02 |
| S(2) | .6373 | .0002 | .2500 | | .2697 | .0003 | .66 | .03 |
| O(1) | .1191 | .0008 | .2500 | | .0167 | .0011 | 1.26 | .10 |
| O(2) | .2244 | .0007 | .2500 | | .5015 | .0011 | 1.05 | .09 |
| O(3) | .5322 | .0007 | .2500 | | .0586 | .0011 | 1.12 | .09 |
| O(4) | .5650 | .0008 | .2500 | | .4493 | .0012 | 1.49 | .11 |
| O(5) | .7296 | .0005 | .0842 | .0007 | .2837 | .0008 | 1.47 | .07 |
| O(6) | .8687 | .0008 | .2500 | | .9421 | .0011 | 1.20 | .10 |
| O(7) | .9813 | .0005 | .0840 | .0006 | .6959 | .0007 | .89 | .06 |
| H | .243 | | .354 | | .597 | | | |

Table 2. Observed and calculated structure factors ($\times 10$)
The calculations were made with the atomic coordinates given in Table 1

| h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c | | |
|--------|----------------|----------------|--------|----------------|----------------|---------|----------------|----------------|---------|----------------|----------------|--------|----------------|----------------|------|------|
| 0 2 0 | 589 | -658 | 11 4 0 | -18 | -18 | -5 3 1 | 255 | 246 | -10 3 1 | 187 | -172 | 2 9 2 | 208 | 218 | | |
| 4 | 1057 | 1245 | 12 0 0 | 5 | 5 | 1 | 119 | 120 | 1 | 312 | 290 | -3 9 2 | 228 | 240 | | |
| 6 | 569 | -600 | 1 | 273 | 276 | 0 | 835 | -802 | 0 | 412 | 378 | 8 | 219 | -220 | | |
| 8 | 637 | 634 | 2 | 311 | 290 | 5 0 1 | 106 | 128 | 10 0 1 | 233 | -256 | 7 | 287 | -296 | | |
| 1 0 0 | 318 | 326 | 3 | 134 | -140 | 1 | 63 | 48 | 1 | 176 | 196 | 6 | 131 | 142 | | |
| 1 | 431 | 374 | 4 | 79 | 10 | 2 | 392 | 412 | 2 | 238 | -176 | 5 | 263 | 250 | | |
| 2 | 523 | -548 | 5 | 241 | 244 | 3 | 182 | 188 | 3 | 121 | -138 | 4 | 366 | -392 | | |
| 3 | 57 | -32 | 6 | 88 | 84 | 4 | 84 | 86 | 4 | 100 | -64 | 3 | 551 | -594 | | |
| 4 | -10 | -10 | 7 | 238 | 124 | 5 | 46 | 46 | 5 | 135 | 174 | 2 | 162 | -160 | | |
| 5 | 169 | 150 | 8 | 424 | -408 | 6 | 243 | 234 | 6 | 86 | -90 | 1 | 354 | 350 | | |
| 6 | 468 | -456 | 9 | 73 | 70 | 7 | 256 | 256 | 7 | 122 | -128 | 2 | 385 | 372 | | |
| 7 | 231 | -214 | 10 | 241 | 244 | 8 | 50 | 50 | 8 | 8 | -28 | 3 | 303 | -310 | | |
| 8 | -8 | -8 | 11 | 215 | 212 | 9 | -18 | -18 | -11 8 1 | 92 | -72 | 4 | 752 | -806 | | |
| 2 0 0 | 290 | 302 | 12 | 298 | -282 | -6 9 1 | 63 | -72 | 7 | 7 | -12 | 5 | 171 | 190 | | |
| 1 | 303 | -282 | 13 | 118 | 124 | 8 | 8 | -42 | 6 | 223 | -216 | 6 | 293 | 308 | | |
| 2 | 736 | -812 | 14 | 92 | 86 | 9 | 152 | -134 | 5 | 6 | 6 | 7 | 213 | -206 | | |
| 3 | 260 | 254 | 15 | 88 | 84 | 10 | 127 | 112 | 4 | 159 | -124 | 8 | 452 | -460 | | |
| 4 | 223 | 224 | 16 | 161 | -182 | 11 | 60 | 58 | 3 | 3 | -18 | 9 | 88 | 104 | | |
| 5 | 80 | -70 | -1 9 1 | 42 | 42 | 12 | 4 | -46 | 2 | 250 | -226 | -4 9 2 | 169 | 174 | | |
| 6 | 443 | -440 | 17 | 94 | 88 | 13 | 78 | 80 | 1 | 0 | -10 | 8 | 293 | 296 | | |
| 7 | 150 | 79 | 18 | 79 | -76 | 14 | 345 | 340 | 0 | 97 | -84 | 7 | 279 | -292 | | |
| 8 | 145 | 144 | 19 | 338 | 334 | 15 | 160 | 138 | 11 0 1 | 137 | 154 | 6 | 221 | 218 | | |
| 9 | -12 | -12 | 20 | 222 | 220 | 16 | 0 | 215 | 214 | 2 | 26 | 5 | 297 | 330 | | |
| 3 0 0 | 1367 | 1248 | 21 | 293 | 276 | 17 | 6 0 1 | 296 | 302 | 1 | 6 | 4 | 499 | 548 | | |
| 1 | 560 | 552 | 22 | 207 | 188 | 18 | 1 | 26 | 3 | 5 | -68 | 3 | 320 | -328 | | |
| 2 | 389 | 336 | 23 | 290 | 296 | 19 | 324 | -356 | 4 | 192 | 224 | 2 | 423 | 420 | | |
| 3 | 562 | -558 | 24 | 344 | 374 | 20 | 3 | 126 | 128 | 5 | 40 | 4 0 2 | 479 | -456 | | |
| 4 | 596 | 556 | 25 | 419 | 428 | 21 | 4 | 376 | 436 | 6 | 68 | 64 | 1 | 131 | -108 | |
| 5 | 320 | 304 | 26 | 727 | -826 | 22 | 5 | -32 | 5 | 7 | -32 | 2 | 71 | 64 | | |
| 6 | 120 | -120 | 27 | 51 | -26 | 23 | 6 | 92 | -110 | -12 7 1 | 255 | -246 | 3 | 51 | -54 | |
| 7 | 214 | -214 | 28 | 200 | -194 | 24 | 7 | 66 | -60 | 6 | 120 | 80 | 4 | 136 | -142 | |
| 8 | 376 | 368 | 29 | 185 | 186 | 25 | 8 | 224 | 266 | 5 | 249 | 232 | 5 | 5 | 10 | |
| 9 | 204 | 200 | 30 | 492 | -470 | 26 | 9 | 120 | -120 | 4 | 111 | 32 | 6 | 207 | 198 | |
| 4 0 0 | 192 | 242 | 31 | 117 | -116 | -7 9 1 | 167 | 174 | 3 | 301 | -260 | 7 | 7 | 80 | | |
| 1 | 260 | 254 | 32 | 76 | -60 | 8 | 101 | 84 | 2 | 158 | 128 | 8 | 100 | -96 | | |
| 2 | 244 | -240 | 33 | 292 | 292 | 9 | 144 | -140 | 1 | 333 | 300 | 9 | 126 | 114 | | |
| 3 | 68 | -62 | -2 9 1 | 111 | -96 | 10 | 6 | 28 | 0 | 44 | 44 | -5 9 2 | 158 | 162 | | |
| 4 | 217 | -188 | 34 | 54 | -54 | 11 | 5 | 249 | 256 | 12 0 1 | 138 | 134 | 8 | 8 | -78 | |
| 5 | 152 | 134 | 35 | 34 | -34 | 12 | 4 | 154 | 136 | 1 | 75 | 92 | 7 | 93 | -102 | |
| 6 | 382 | -374 | 36 | 458 | -462 | 13 | 3 | 227 | -200 | 2 | 86 | -102 | 6 | 6 | 30 | |
| 7 | 220 | -212 | 37 | 196 | 184 | 14 | 2 | 196 | 184 | 3 | 133 | -142 | 5 | 5 | 236 | |
| 8 | 114 | -106 | 38 | 169 | -164 | 15 | 1 | 293 | 276 | 4 | 122 | 138 | 4 | 163 | -150 | |
| 9 | 20 | 20 | 39 | 432 | 462 | 16 | 0 | 353 | 334 | 5 | 79 | 64 | 3 | 123 | -122 | |
| 5 0 0 | 165 | 178 | 40 | 776 | -890 | 17 | 7 0 1 | 151 | -222 | 6 | 81 | -72 | 2 | 2 | 297 | |
| 1 | 293 | 280 | 41 | 186 | 138 | 18 | 1 | 199 | 198 | -13 6 1 | 125 | -120 | 1 | 297 | 286 | |
| 2 | 478 | -460 | 42 | 212 | 218 | 19 | 2 | 308 | -372 | 5 | 170 | 138 | 0 | 257 | -244 | |
| 3 | 109 | 82 | 43 | 366 | 400 | 20 | 3 | 205 | -198 | 4 | 214 | 180 | 5 0 2 | 0 | -32 | |
| 4 | 114 | 116 | 44 | 394 | 388 | 21 | 4 | 36 | 34 | 3 | 3 | -64 | 1 | 237 | 210 | |
| 5 | 211 | 208 | 45 | 215 | 196 | 22 | 5 | 154 | 178 | 2 | 2 | -60 | 2 | 599 | 606 | |
| 6 | 363 | -344 | 46 | 133 | 136 | 23 | 6 | 219 | -238 | 1 | 176 | 168 | 3 | 544 | -506 | |
| 7 | 280 | -260 | 47 | 281 | 292 | 24 | 7 | 129 | -120 | 0 | 0 | 211 | 286 | 4 | 126 | -114 |
| 8 | 71 | 56 | 48 | 95 | 80 | 25 | 8 | 32 | 32 | 0 0 2 | 2 | 735 | 5 | 168 | 182 | |
| 6 0 0 | 638 | 630 | 49 | 112 | -94 | -8 9 1 | 9 | 104 | 122 | 1 | 481 | 514 | 6 | 403 | 420 | |
| 1 | 268 | 250 | 50 | 38 | -38 | 10 | 8 | 44 | 44 | 2 | 561 | -558 | 7 | 145 | -174 | |
| 2 | 181 | 120 | -3 9 1 | 112 | -94 | 11 | 7 | -42 | 3 | 535 | -540 | 8 | 58 | -62 | | |
| 3 | 254 | -246 | 51 | 100 | -88 | 12 | 6 | 279 | -276 | 4 | 234 | -222 | 9 | 203 | 210 | |
| 4 | 496 | 484 | 52 | 71 | 68 | 13 | 5 | 126 | 116 | 5 | 307 | 300 | -6 9 2 | 227 | 274 | |
| 5 | 238 | 220 | 53 | 231 | 230 | 14 | 4 | 88 | -74 | 6 | 6 | 42 | 8 | 67 | -74 | |
| 6 | 4 | 4 | 54 | 46 | -36 | 15 | 3 | 64 | -68 | 7 | 311 | -310 | 7 | 305 | -366 | |
| 7 | 207 | -198 | 55 | 203 | -204 | 16 | 2 | 472 | -454 | 8 | 184 | -192 | 6 | 112 | -110 | |
| 8 | 323 | 302 | 56 | 137 | 130 | 17 | 1 | 169 | 166 | 9 | 199 | 198 | 5 | 335 | 360 | |
| 7 0 0 | 62 | 62 | 57 | 603 | 676 | 18 | 0 | 156 | -136 | -1 9 2 | 247 | 258 | 4 | 176 | -156 | |
| 1 | 338 | 334 | 58 | 113 | 96 | 19 | 8 0 1 | 229 | 308 | 8 | 188 | 194 | 3 | 565 | -634 | |
| 2 | 191 | -190 | 59 | 8 | -8 | 20 | 7 | 400 | -392 | 7 | 400 | -392 | 2 | 263 | -276 | |
| 3 | 318 | -308 | 60 | 176 | -134 | 21 | 6 | 234 | 236 | 6 | 234 | 236 | 1 | 522 | 522 | |
| 4 | 150 | -134 | 61 | 355 | -376 | 22 | 5 | 479 | 448 | 5 | 479 | 448 | 0 | 422 | -366 | |
| 5 | 193 | 214 | 62 | 88 | -76 | 23 | 4 | 421 | 416 | 4 | 421 | 416 | 6 0 2 | 537 | -590 | |
| 6 | 252 | -254 | 63 | 367 | 370 | 24 | 3 | 556 | -580 | 3 | 556 | -580 | 1 | 217 | -198 | |
| 7 | 238 | -244 | 64 | 291 | 278 | 25 | 2 | 351 | 370 | 2 | 351 | 370 | 2 | 194 | 192 | |
| 8 | 103 | -68 | 65 | 272 | -260 | 26 | 1 | 707 | 802 | 1 | 707 | 802 | 3 | 50 | 50 | |
| 8 0 0 | 210 | 280 | 66 | 137 | 130 | 27 | 0 | 505 | 582 | 0 | 505 | 582 | 4 | 549 | -564 | |
| 1 | 281 | 278 | 67 | 79 | 88 | 28 | 8 | 140 | 178 | 1 0 2 | 646 | -634 | 5 | 151 | -150 | |
| 2 | 351 | -330 | 68 | 158 | 156 | 29 | 7 | 88 | 74 | 1 | 49 | 24 | 6 | 89 | 106 | |
| 3 | 110 | 116 | 69 | 218 | -212 | -9 9 1 | 6 | 6 | 6 | 2 | 121 | 102 | 7 | 7 | 66 | |
| 4 | 237 | 244 | -4 9 1 | 137 | 160 | 30 | 5 | 380 | -372 | 3 | 380 | -372 | 8 | 332 | -364 | |
| 5 | 340 | -332 | 7 | 127 | -134 | 31 | 4 | 191 | -188 | 4 | 191 | -188 | 9 | 57 | -50 | |
| 6 | 131 | -128 | 8 | 6 | 149 | 134 | 32 | 3 | 120 | 114 | 3 | 120 | 114 | -7 9 2 | 8 | 94 |
| 7 | 93 | 98 | 9 | 5 | 123 | 106 | 33 | 2 | 112 | 106 | 2 | 112 | 106 | 8 | 290 | 330 |
| 8 | 365 | 404 | 10 | 4 | 4 | 26 | 3 | 113 | -116 | 3 | 113 | -116 | 7 | 117 | -132 | |
| 9 0 0 | 435 | 428 | 11 | 41 | 42 | 34 | 2 | 261 | 248 | 4 | 261 | 248 | 6 | 112 | 110 | |
| 1 | 261 | -272 | 12 | 182 | -166 | 35 | 1 | 175 | 168 | 5 | 209 | 218 | 5 | 116 | 126 | |
| 2 | 359 | 360 | 13 | 144 | -118 | 36 | 0 | 32 | 32 | -2 9 2 | 238 | 246 | 4 | 494 | 552 | |
| 3 | 338 | 340 | 14 | 1 | 264 | 254 | 37 | 7 | 82 | -84 | 2 | 82 | -84 | 3 | 182 | -172 |
| 4 | 65 | 40 | 15 | 151 | 160 | 38 | 6 | 256 | 250 | 3 | 256 | 250 | 2 | 67 | -66 | |
| 5 | 287 | -300 | 16 | 188 | -204 | 39 | 5 | 100 | -90 | 4 | 100 | -90 | 1 | 165 | 158 | |
| 6 | 211 | -196 | 17 | 65 | -50 | 40 | 4 | 446 | -448 | 5 | 446 | -448 | 0 | 390 | 370 | |
| 7 | 84 | 82 | 18 | 261 | -252 | 41 | 3 | 123 | -92 | 6 | 123 | -92 | 7 0 2 | 0 | -116 | |
| 8 | 52 | -52 | 19 | 60 | 52 | 42 | 2 | 123 | -92 | 7 | 123 | -92 | 1 | 178 | 158 | |
| 9 | 100 | -90 | 20 | 232 | -230 | 43 | 1 | 309 | 328 | 8 | 309 | 328 | 2 | 24 | 24 | |
| 10 0 0 | 211 | -196 | 21 | 7 | -28 | 44 | 0 | 235 | 238 | 9 | 235 | 238 | 3 | 122 | -120 | |
| 1 | 84 | 82 | 22 | 8 | 111 | -100 | 1 | 253 | 272 | 2 0 2 | 253 | 272 | 4 | 146 | 142 | |
| 2 | 52 | -52 | 23 | 9 | 55 | 50 | 2 | 253 | 272 | 3 | 253 | 272 | 5 | 146 | 142 | |
| 3 | 100 | -90 | 24 | 85 | -66 | -10 9 1 | 143 | 154 | 4 | 278 | 718 | 6 | 7 | 72 | | |
| 4 | 170 | -178 | 25 | 8 | 213 | -202 | 5 | 469 | -482 | 5 | 469 | -482 | 7 | 58 | -58 | |
| 5 | 62 | -46 | 26 | 7 | 166 | -166 | 6 | 88 | 76 | 6 | 88 | 76 | 8 | 58 | -58 | |
| 6 | 161 | 182 | 27 | 6 | 166 | -166 | 7 | 237 | 212 | 7 | 237 | 212 | 9 | 108 | 120 | |
| 7 | 107 | -100 | 28 | 6 | 109 | -108 | 8 | 372 | 372 | -8 9 2 | 372 | 372 | 8 | 79 | 92 | |
| 8 | 84 | -76 | 29 | 5 | 261 | 262 | 9 | 223 | -226 | 9 | 223 | -226 | 7 | 74 | -84 | |
| 9 | 84 | -76 | 30 | 4 | 205 | 180 | 10 | 8 | 10 | 10 | 8 | 10 | 10 | 6 | 74 | -84 |

Table 2. (Continued)

| h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c |
|---------|----------------|----------------|---------|----------------|----------------|---------|----------------|----------------|---------|----------------|----------------|--------|----------------|----------------|
| -8 6 2 | 67 | -70 | -13 2 2 | 1 | 42 | 5 3 3 | 179 | -172 | 10 5 3 | -6 | 2 2 4 | 47 | -52 | |
| 5 | 186 | 196 | 1 | 59 | -60 | 4 | 54 | 76 | 6 | 180 | 160 | 3 | 604 | 616 |
| 4 | 67 | 78 | 0 | 269 | 252 | 5 | 76 | -76 | 7 | 18 | 18 | 4 | 433 | -433 |
| 3 | -4 | -4 | 0 0 3 | 20 | 20 | 6 | 77 | -88 | 8 | -30 | -30 | 5 | 405 | -368 |
| 2 | 218 | -248 | 1 | 134 | 156 | 7 | 156 | -150 | 9 | 4 | 4 | 6 | 72 | 56 |
| 1 | 210 | 200 | 2 | 383 | -388 | 8 | 36 | 36 | -11 9 3 | 44 | 44 | 7 | 282 | 280 |
| 0 | 77 | -54 | 3 | 62 | -48 | 9 | 40 | 40 | 8 | 0 | 0 | 8 | 300 | -272 |
| 8 0 2 | 131 | -148 | 4 | 258 | -248 | -6 9 3 | 4 | 40 | 7 | 8 | 8 | 9 | 373 | -342 |
| 1 | 50 | 50 | 5 | 141 | 136 | 8 | -48 | -48 | 6 | 163 | 180 | -3 9 4 | 195 | -182 |
| 2 | 471 | 472 | 6 | 366 | -378 | 7 | 121 | 126 | 5 | 38 | 38 | 8 | 164 | -152 |
| 3 | 206 | -214 | 7 | 192 | -180 | 6 | 204 | -234 | 4 | 30 | 30 | 7 | 257 | 232 |
| 4 | -80 | -80 | 8 | 113 | -114 | 5 | 97 | -108 | 3 | -28 | -28 | 6 | 117 | 110 |
| 5 | 389 | 400 | 9 | 91 | 94 | 4 | 90 | -84 | 2 | 233 | 240 | 5 | 207 | -188 |
| 7 | 55 | -58 | -1 9 3 | 57 | -54 | 3 | 16 | 16 | 1 | 24 | 24 | 4 | 227 | -228 |
| 8 | 70 | -86 | 8 | 58 | 58 | 2 | 498 | -468 | 0 | 73 | 58 | 3 | 483 | 496 |
| 9 | 103 | 90 | 7 | 44 | 44 | 6 0 3 | 264 | -242 | 11 0 3 | 78 | -76 | 2 | 345 | 360 |
| -9 9 2 | 110 | 126 | 6 | 256 | -234 | 1 | 259 | 260 | 1 | 107 | 110 | 1 | 284 | -282 |
| 8 | 112 | 112 | 5 | 180 | -180 | 2 | 36 | 36 | 2 | 108 | 118 | 0 | 0 | -6 |
| 7 | 223 | -246 | 4 | 100 | 98 | 3 | 299 | -298 | 3 | 5 | -48 | 3 0 4 | 20 | -20 |
| 6 | 317 | -326 | 3 | 179 | -170 | 4 | 315 | -330 | 4 | 145 | -150 | 1 | 259 | -254 |
| 5 | 213 | 226 | 2 | 132 | -112 | 5 | 206 | 226 | 5 | 70 | 70 | 2 | 108 | 64 |
| 4 | 123 | 136 | 1 | 193 | -188 | 6 | 99 | -92 | 6 | 36 | 36 | 3 | 314 | 304 |
| 3 | 264 | -272 | 0 | 514 | 0 | 7 | 147 | -140 | 7 | 112 | -114 | 4 | 67 | 54 |
| 2 | 458 | -472 | 1 0 3 | 353 | 332 | 8 | 208 | -210 | 8 | 111 | -110 | 5 | 185 | -172 |
| 1 | 311 | 306 | 1 | 248 | 220 | 9 | 181 | 194 | -12 9 3 | 3 | 6 | 6 | 92 | 74 |
| 0 | 175 | 134 | 2 | 443 | 428 | -7 9 3 | 147 | -156 | 8 | 185 | -182 | 7 | 237 | 232 |
| 9 0 2 | 263 | -248 | 3 | 149 | -136 | 8 | 113 | -110 | 7 | 56 | 56 | 8 | 40 | 40 |
| 1 | 189 | -178 | 4 | 183 | 184 | 7 | 184 | 105 | 4 | 4 | 4 | 9 | 131 | -130 |
| 2 | -22 | -22 | 5 | 128 | 140 | 6 | 144 | 156 | 5 | -42 | -42 | -4 9 4 | 131 | -114 |
| 3 | 277 | -282 | 6 | 270 | 266 | 5 | 196 | -230 | 4 | 274 | -268 | 8 | 423 | -436 |
| 4 | 168 | -156 | 7 | 46 | 46 | 4 | 157 | -162 | 3 | 98 | 98 | 7 | 98 | 96 |
| 5 | 97 | 98 | 8 | 68 | 68 | 3 | 170 | 174 | 2 | -2 | -2 | 6 | 114 | 116 |
| 8 | 188 | -196 | 9 | 82 | 82 | 2 | 256 | 236 | 1 | -50 | -50 | 5 | 170 | -170 |
| 9 | -40 | -40 | -2 9 3 | 52 | 52 | 1 | 274 | -262 | 0 | 291 | -288 | 4 | 667 | -698 |
| 8 | 280 | 294 | 8 | 129 | 128 | 0 | 190 | -164 | 12 0 3 | 359 | -328 | 3 | 170 | 154 |
| 7 | -52 | -52 | 7 | 119 | 116 | 7 0 3 | 334 | 342 | 1 | -18 | -18 | 2 | 265 | 238 |
| 6 | -14 | -14 | 6 | 178 | 180 | 1 | 26 | 26 | 2 | 147 | 140 | 1 | 233 | -222 |
| 5 | 42 | 42 | 5 | 59 | 48 | 2 | 265 | 26 | 3 | 94 | 100 | 0 | 747 | -734 |
| 4 | 420 | 444 | 4 | 321 | 326 | 3 | 40 | 40 | 4 | 302 | -294 | 4 0 4 | 164 | 156 |
| 3 | 91 | -92 | 3 | 79 | -48 | 4 | 112 | 102 | 5 | -2 | -2 | 1 | 246 | -230 |
| 2 | 87 | -86 | 2 | 229 | 240 | 5 | 30 | 30 | 6 | 132 | 118 | 2 | 316 | 300 |
| 1 | 52 | 52 | 1 | 264 | -258 | 6 | 86 | 78 | 7 | 26 | 26 | 3 | 271 | 234 |
| 10 0 2 | 449 | 416 | 0 | 477 | 526 | 7 | 84 | 2 | -13 8 3 | 169 | -162 | 4 | 57 | 50 |
| 1 | 104 | -110 | 2 0 3 | 187 | 176 | 8 | 50 | 50 | 5 | 175 | -202 | 5 | 175 | -202 |
| 2 | 151 | 164 | 1 | 236 | -218 | 9 | 10 | 10 | 6 | 275 | 274 | 6 | 176 | 166 |
| 3 | 152 | -158 | 2 | 64 | -68 | -8 9 3 | 111 | 126 | 5 | -26 | -26 | 7 | 133 | 112 |
| 4 | 82 | -82 | 3 | 113 | -106 | 8 | 36 | 36 | 4 | 224 | -208 | 8 | 8 | 32 |
| 5 | 82 | -114 | 4 | 59 | 48 | 7 | -30 | -30 | 3 | 122 | -106 | 9 | 177 | -158 |
| 6 | 82 | -114 | 5 | 152 | -158 | 6 | 126 | 140 | 2 | 283 | 274 | -5 9 4 | 177 | -158 |
| 7 | 82 | -114 | 6 | 131 | -130 | 5 | 88 | 102 | 1 | -12 | -12 | 8 | 74 | -62 |
| 8 | 82 | -114 | 7 | 22 | 22 | 4 | 75 | 90 | 0 | 311 | -298 | 7 | 74 | -62 |
| 9 | 82 | -114 | 8 | 42 | 42 | 3 | 195 | -202 | -14 7 3 | 203 | -44 | 6 | 367 | 342 |
| -11 9 2 | 88 | -88 | 9 | 271 | -288 | 2 | 283 | 290 | 5 | 120 | 216 | 5 | 16 | 16 |
| 6 | 160 | 148 | -3 9 3 | 93 | 88 | 1 | 124 | 110 | 5 | 103 | 88 | 4 | -18 | -18 |
| 5 | -4 | -4 | 8 | 102 | -96 | 0 | 289 | 270 | 4 | -26 | -26 | 3 | 3 | 6 |
| 4 | 91 | -110 | 7 | 56 | -56 | 8 0 3 | 2 | 2 | 3 | 103 | -90 | 2 | 531 | 492 |
| 3 | 152 | -152 | 6 | 273 | -272 | 1 | 192 | 188 | 2 | 221 | 204 | 5 0 4 | 212 | -188 |
| 2 | 160 | 148 | 5 | 136 | 148 | 2 | 105 | -102 | 0 | 115 | 92 | 1 | 302 | -288 |
| 1 | 160 | 148 | 4 | 228 | -212 | 3 | 105 | -102 | 2 | -90 | -90 | 2 | 284 | -290 |
| 11 0 2 | 122 | -120 | 3 | 152 | -146 | 4 | -56 | -56 | 0 0 4 | 214 | 266 | 3 | 404 | 458 |
| 1 | 367 | 364 | 2 | 408 | -428 | 5 | 112 | 126 | 1 | 322 | -332 | 4 | 137 | -142 |
| 2 | 95 | -90 | 1 | 205 | 216 | 6 | -34 | -34 | 2 | 387 | 394 | 5 | 247 | -268 |
| 3 | 292 | 296 | 0 | 271 | -288 | 7 | 180 | -176 | 3 | 76 | 64 | 6 | 172 | -184 |
| 4 | 70 | -88 | 3 0 3 | 112 | -94 | 8 | -44 | -44 | 4 | 223 | -220 | 7 | 166 | 210 |
| 5 | 131 | -132 | 1 | 177 | 178 | 9 | 30 | 30 | 5 | 223 | -220 | 8 | 72 | -72 |
| 6 | 209 | -212 | 2 | 124 | -132 | -9 9 3 | -20 | -20 | 6 | -46 | -46 | 9 | 257 | -286 |
| 7 | 116 | 104 | 3 | 215 | -204 | 8 | 117 | -128 | 7 | 293 | 278 | 0 0 5 | 57 | -50 |
| 8 | 91 | 92 | 4 | 363 | -352 | 7 | 167 | 176 | 8 | 8 | 24 | 1 | 180 | -192 |
| 9 | 315 | -304 | 5 | 182 | 174 | 6 | 120 | -108 | -1 9 4 | 196 | -180 | 2 | 305 | 330 |
| 12 0 2 | 279 | -252 | 6 | 269 | -260 | 5 | 152 | -174 | 8 | 350 | -326 | 3 | 117 | 112 |
| 1 | 172 | -174 | 7 | 93 | -96 | 4 | 180 | -194 | 7 | 129 | 130 | 4 | 156 | 164 |
| 2 | 60 | 60 | 8 | 200 | -192 | 3 | 48 | 48 | 6 | 67 | 62 | 5 | 133 | -132 |
| 3 | 227 | -228 | -4 9 3 | 168 | -160 | 2 | 195 | -198 | 5 | 202 | -190 | 6 | 390 | 384 |
| 4 | 178 | -160 | 7 | 133 | 134 | 1 | 268 | -250 | 4 | 486 | -498 | 7 | 137 | 122 |
| 5 | 101 | 114 | 8 | -52 | -52 | 0 | 263 | -260 | 3 | 233 | 242 | 8 | 108 | 76 |
| 6 | 156 | -154 | 9 | 44 | 44 | 9 0 3 | 289 | -392 | 2 | 54 | -66 | -1 9 5 | 114 | 124 |
| 7 | 165 | 158 | 6 | 297 | -300 | 1 | 111 | 110 | 1 | 196 | -202 | 8 | 80 | -72 |
| 8 | 36 | 36 | 5 | 112 | -106 | 2 | 93 | 93 | 0 | 589 | -704 | 7 | 127 | -130 |
| 9 | 14 | 14 | 4 | 112 | -106 | 3 | -82 | -82 | 1 0 4 | 110 | 118 | 6 | 6 | -24 |
| 13 8 2 | 165 | 158 | 3 | 179 | 172 | 4 | 335 | -350 | 1 | 154 | -128 | 5 | 237 | 244 |
| 1 | 315 | -304 | 4 0 3 | 116 | 124 | 5 | 112 | 112 | 2 | 514 | 496 | 4 | 121 | -108 |
| 2 | 164 | 148 | 1 | 48 | -34 | 6 | 85 | 88 | 3 | 383 | 346 | 3 | 146 | -130 |
| 3 | 58 | 58 | 2 | 652 | 616 | 7 | -64 | -64 | 4 | -24 | -24 | 2 | 219 | -234 |
| 4 | 279 | -252 | 3 | 191 | 176 | 8 | 231 | -238 | 5 | 145 | -136 | 1 | 297 | 310 |
| 5 | 172 | -174 | 4 | 64 | -58 | 9 | 82 | 84 | 6 | 297 | 260 | 0 | 212 | -338 |
| 6 | 60 | 60 | 5 | -12 | -12 | -10 9 3 | -60 | -60 | 7 | 85 | 50 | 1 0 5 | 69 | -68 |
| 7 | 227 | -228 | 6 | 280 | 288 | 8 | 102 | -112 | 8 | -32 | -32 | 1 | 46 | 46 |
| 8 | 178 | -160 | 7 | 44 | 44 | 7 | -32 | -32 | 9 | 255 | -210 | 2 | 212 | -186 |
| 9 | 101 | 114 | 8 | -38 | -38 | 6 | 210 | 228 | -2 9 4 | 160 | -140 | 3 | 209 | -192 |
| -13 8 2 | 165 | 158 | -5 9 3 | 88 | 88 | 5 | 110 | -110 | 8 | -10 | -10 | 4 | 6 | 6 |
| 1 | 36 | 36 | 7 | 96 | 94 | 4 | 135 | -136 | 7 | -44 | -44 | 5 | 0 | 0 |
| 2 | 14 | 14 | 8 | -92 | -92 | 3 | 0 | 0 | 6 | 308 | 302 | 6 | 99 | -96 |
| 3 | 212 | 222 | 9 | 395 | 388 | 1 | | | | | | | | |

Table 2. (Continued)

| h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c | h k l | F _o | F _c |
|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|
| -2 4 5 | 177 | 166 | 5 9 5 | 36 | 36 | -4 2 6 | 230 | -210 | -3 7 7 | 120 | -110 | -2 5 8 | 286 | 304 |
| 3 | 69 | -72 | 0 0 6 | 312 | 328 | 1 | 301 | -300 | 6 | 205 | -218 | 4 | 109 | 116 |
| 2 | 353 | -376 | 1 | 394 | 394 | 0 | 323 | 448 | 5 | 73 | 74 | 3 | 208 | -216 |
| 1 | 85 | -78 | 2 | 98 | -78 | 4 0 6 | 113 | 98 | 4 | | -28 | 2 | 85 | 98 |
| 0 | 126 | 184 | 3 | 89 | -86 | 1 | 20 | 20 | 3 | | -30 | 1 | 243 | 344 |
| -2 0 5 | 418 | -426 | 4 | 259 | 266 | 2 | 454 | -432 | 2 | 170 | -194 | 0 | 62 | 98 |
| 1 | 12 | -12 | 5 | 12 | 12 | 3 | 5 | -16 | 1 | 101 | 126 | 2 0 8 | 294 | -284 |
| 2 | 70 | -12 | 6 | 126 | -94 | 4 | 72 | 80 | 0 | 74 | 68 | 1 | 128 | 110 |
| 3 | 72 | 70 | 7 | 68 | -70 | 5 | 5 | 20 | 3 0 7 | | 54 | 2 | 254 | 240 |
| 4 | 323 | -314 | -1 8 6 | 330 | 334 | 6 | 331 | -346 | 1 | | 24 | 3 | 83 | -76 |
| 5 | 22 | 22 | 7 | 44 | 44 | 7 | 7 | -16 | 2 | 207 | -212 | 4 | 214 | -190 |
| 6 | 95 | 80 | 6 | 356 | -344 | 8 | 8 | 80 | 3 | | -22 | 5 | 113 | 84 |
| 7 | 38 | 5 | 5 | 28 | 28 | -5 9 6 | | -86 | 4 | | 6 | 6 | 279 | 254 |
| 8 | 209 | -198 | 4 | 447 | 478 | 8 | 8 | -62 | 5 | | 2 | -3 9 8 | 96 | 106 |
| 9 | 24 | 3 | 3 | 66 | -38 | 7 | 188 | 172 | 6 | 209 | -202 | 8 | 8 | -46 |
| -3 9 5 | 60 | 60 | 2 | 320 | -318 | 6 | 262 | -240 | 7 | | -40 | 7 | 139 | -124 |
| 3 | 120 | -130 | 3 | 89 | 84 | 0 | 5 | -202 | 8 | | 30 | 6 | 115 | -132 |
| 4 | 90 | 1 | 1 | 14 | 14 | 5 | 130 | -140 | 9 | | -26 | 5 | 167 | 162 |
| 7 | 0 | -12 | 0 | 593 | 708 | 4 | 4 | -140 | 9 | | -40 | 4 | 4 | -84 |
| 6 | 291 | 302 | 1 0 6 | 26 | 26 | 3 | 108 | 100 | -4 9 7 | | 10 | 3 | 142 | -140 |
| 5 | 46 | 46 | 1 | 227 | -220 | 2 | 259 | -262 | 8 | | 38 | 2 | 165 | -198 |
| 4 | 179 | 170 | 2 | 477 | -474 | 1 | 255 | -242 | 7 | | 38 | 2 | 152 | 178 |
| 3 | 120 | -130 | 3 | 89 | 84 | 0 | 5 | -102 | 6 | 103 | 128 | 1 | 82 | -102 |
| 2 | 289 | 328 | 4 | 67 | 60 | 5 0 6 | 379 | 360 | 5 | 62 | -54 | 0 | 0 | 0 |
| 1 | 61 | 62 | 5 | 210 | -168 | 1 | 212 | 184 | 4 | | 38 | 3 0 8 | 184 | -258 |
| 0 | 98 | 6 | 6 | 391 | -342 | 2 | 134 | -130 | 3 | 62 | 54 | 1 | 68 | 68 |
| 3 0 5 | 51 | 54 | 7 | 133 | 136 | 3 | 242 | -234 | 2 | 279 | 324 | 2 | 2 | -86 |
| 1 | 222 | -214 | 8 | 82 | 70 | 4 | 262 | 80 | 1 | 54 | -60 | 3 | 65 | -66 |
| 2 | 234 | 222 | -2 9 6 | 51 | -40 | 5 | 160 | 180 | 0 | 193 | 206 | 4 | 104 | -128 |
| 3 | 226 | 218 | 8 | 0 | 0 | 6 | 112 | -140 | 4 0 7 | | -46 | 5 | 72 | 72 |
| 4 | 190 | 198 | 7 | 154 | 166 | 7 | 92 | -104 | 1 | | -32 | 6 | 30 | 30 |
| 5 | 176 | -176 | 6 | 311 | -314 | 8 | 171 | 204 | 2 | 24 | 24 | 7 | 77 | -48 |
| 6 | 303 | 280 | 5 | 194 | 190 | 9 | 164 | 180 | 3 | 136 | -151 | -4 8 8 | 8 | 18 |
| 7 | 173 | 152 | 4 | 47 | -44 | 0 0 7 | | 32 | 4 | 84 | -94 | 7 | 208 | -210 |
| 8 | 121 | 100 | 3 | 32 | 32 | 1 | 1 | 18 | 5 | | -16 | 6 | 75 | 84 |
| 9 | 102 | -110 | 2 | 347 | -388 | 2 | 117 | -132 | 6 | | -30 | 5 | 198 | 182 |
| -4 9 5 | 94 | 92 | 1 | 216 | -238 | 3 | 30 | 30 | 7 | | 60 | 4 | 216 | 26 |
| 4 | 4 | 0 | 0 | 46 | -38 | 4 | 83 | -86 | -5 9 7 | 105 | 104 | 3 | 216 | -234 |
| 7 | 67 | -64 | 2 0 6 | 425 | 442 | 5 | 5 | -8 | 8 | 131 | -122 | 2 | 38 | 38 |
| 6 | 83 | -90 | 1 | 28 | 28 | -1 7 7 | 81 | 76 | 7 | | -38 | 1 | 174 | 226 |
| 5 | 144 | 146 | 2 | 124 | -106 | 6 | 85 | 84 | 6 | | 10 | 0 | 0 | -54 |
| 4 | 0 | 0 | 3 | 135 | -136 | 5 | 86 | -100 | 5 | 96 | 110 | -5 9 8 | 157 | 140 |
| 3 | 73 | -70 | 4 | 303 | 296 | 4 | 141 | 132 | 4 | | -178 | 4 | 194 | 180 |
| 2 | 350 | -356 | 5 | 60 | 60 | 3 | 95 | 88 | 3 | 102 | -104 | 7 | 208 | -198 |
| 1 | 154 | 158 | 6 | 206 | -182 | 2 | 172 | 188 | 2 | | 56 | 6 | 6 | -16 |
| 0 | 254 | -280 | 7 | 8 | 8 | 1 | 92 | -110 | 1 | 92 | 96 | 5 | 278 | 266 |
| 4 0 5 | 293 | -270 | 8 | 226 | 216 | 0 | 199 | 256 | 0 | 150 | -160 | 4 | 258 | 268 |
| 1 | 118 | -110 | 9 | 148 | -122 | 1 0 7 | 195 | -104 | 5 7 | 309 | 326 | 3 | 187 | -172 |
| 2 | 2 | -48 | -3 9 6 | | -6 | 1 | 61 | 54 | 1 | | 24 | 2 | 2 | -80 |
| 3 | 60 | -54 | 8 | 247 | 234 | 2 | 2 | 26 | 2 | 84 | -78 | 1 | 236 | 302 |
| 4 | 103 | -104 | 7 | 8 | -8 | 3 | 105 | 102 | 3 | | 78 | 0 | 198 | 226 |
| 5 | 116 | -112 | 6 | 106 | -82 | 4 | 131 | -118 | 4 | 256 | 306 | -2 3 9 | 0 | -26 |
| 6 | 60 | 60 | 5 | 5 | -54 | 5 | 71 | 62 | 5 | | 6 | 2 | 26 | 26 |
| 7 | 30 | 4 | 4 | 342 | 352 | 6 | 60 | -8 | 6 | | -60 | 1 | 64 | 64 |
| 8 | 77 | -68 | 3 | 93 | -90 | -2 9 7 | 31 | 36 | 7 | | -12 | -4 7 9 | 42 | -44 |
| 9 | 22 | -22 | 2 | 184 | -162 | 8 | 62 | -60 | 8 | 164 | 210 | 6 | 78 | -76 |
| -5 9 5 | 171 | -160 | 1 | 36 | 36 | 7 | 7 | -22 | 0 0 8 | 211 | -246 | 5 | 30 | 30 |
| 8 | 2 | -2 | 0 | 299 | 330 | 6 | 141 | -132 | 1 | 115 | 106 | 4 | 4 | -32 |
| 7 | 24 | 24 | 3 0 6 | 254 | 266 | 5 | 82 | 94 | 2 | 87 | -108 | 3 | 76 | -78 |
| 6 | 14 | -14 | 1 | 79 | 66 | 4 | 72 | -82 | 3 | 143 | -120 | 2 | 149 | -166 |
| 5 | 146 | -124 | 2 | 40 | 40 | 3 | 4 | 4 | -1 6 8 | 168 | 150 | 1 | 56 | 56 |
| 4 | 32 | -32 | 3 | 122 | -104 | 2 | 49 | -50 | 5 | 224 | 216 | 0 | 0 | -106 |
| 3 | 237 | 220 | 4 | 124 | 122 | 1 | 74 | -88 | 4 | | -26 | -5 8 9 | 78 | 54 |
| 2 | 92 | -86 | 5 | 38 | 38 | 0 | 88 | -88 | 3 | 235 | -234 | 7 | 126 | 88 |
| 1 | 120 | -106 | 6 | 96 | -78 | 2 0 7 | 219 | 230 | 2 | 78 | 74 | 6 | 86 | 86 |
| 0 | 188 | -174 | 7 | 86 | -90 | 1 | 178 | -166 | 1 | 194 | 276 | 5 | 126 | -126 |
| 5 0 5 | 258 | -246 | 8 | 113 | 100 | 2 | 85 | 80 | 0 | 132 | -138 | 4 | 86 | 86 |
| 1 | 6 | -62 | 9 | 30 | 30 | 3 | 214 | 196 | 1 0 8 | | 40 | 3 | 122 | 102 |
| 2 | 6 | -6 | -4 9 6 | 111 | -118 | 4 | 222 | 216 | 1 | 212 | 200 | 2 | 98 | 120 |
| 3 | 6 | -6 | 8 | 279 | 262 | 5 | 149 | -142 | 2 | 177 | 168 | 1 | 157 | -136 |
| 4 | 187 | -214 | 7 | 242 | 244 | 6 | 64 | 60 | 4 | 3 | 170 | 0 | 0 | 0 |
| 5 | 28 | -28 | 6 | 211 | 194 | 7 | 116 | 110 | 4 | 58 | 42 | 0 | 0 | 0 |
| 6 | 26 | 26 | 5 | 217 | -208 | 8 | 156 | 142 | 5 | 234 | 186 | 0 | 0 | 0 |
| 7 | 78 | 4 | 4 | 376 | 368 | -3 9 7 | | -22 | -2 7 8 | 215 | -230 | 0 | 0 | 0 |
| 8 | 123 | -146 | 3 | 279 | 288 | 8 | 8 | -16 | 6 | 100 | 98 | 0 | 0 | 0 |

half the minimum observable in each level. During the later stages of refinement the correction for secondary extinction was applied, according to HALL and MASLEN's (1965) method:

$$F_o^2 = F_{o1}^2 \exp(\varepsilon \cdot I_c)$$

where F_o and F_{o1} are the corrected and uncorrected observed structure factors and I_c is the calculated intensity; the coefficient of secondary extinction, ε , was 3.3×10^{-5} .

Towards the end of the refinement the hydrogen atom was recognized on a three-dimensional difference Fourier map, and its coordi-

dinates were included in the last two cycles of refinement, but its thermal parameter was kept at the reasonable value of 5 \AA^2 . In Table 1 are reported the final coordinates of the asymmetric unit and the individual thermal parameters, together with their standard deviations. Table 2 lists the observed and calculated structure factors for syngenite after the last cycle of refinement.

Description of the structure

The K atom, in the general position, is surrounded by eight oxygen atoms at the seven corners of a very irregular cube lacking one corner, and on the center of one face. Each K atom is connected to eight more neighbouring K atoms in the following way: by a quadrangular face (K—K distance 3.48 \AA) and a triangular one (K—K distance 3.67 \AA) to its two symmetrical K atoms through the mirror planes; by one edge to each of its two centrosymmetrical ones (shortest K—K distance 4.48 \AA); and by one corner to the four K atoms related by the screw axis.

The projection of the crystal structure along the c axis, with exclusion of Ca atoms, is shown in Fig. 1. In this projection the connec-

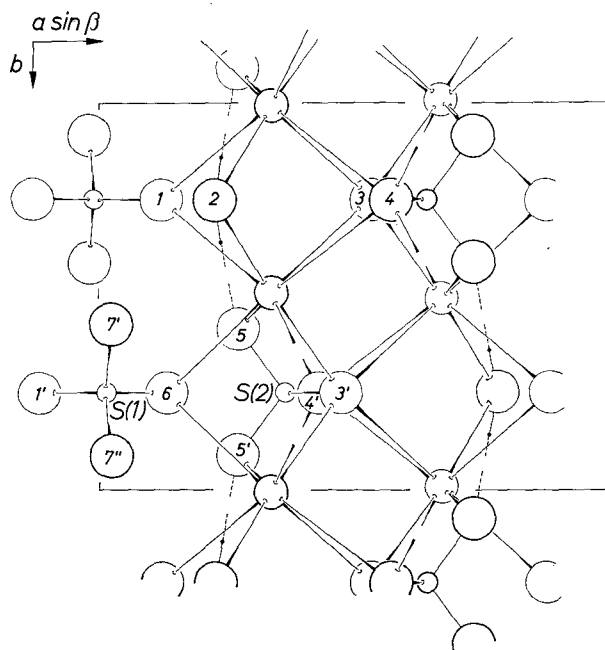


Fig. 1. Projection along the c axis of K and S polyhedra. The small circles are S atoms, the larger ones are O atoms, while the medium-sized are K atoms. The solid dots are H atoms

tion, parallel to the b axis, of K polyhedra, linked to each other by four-sided and three-sided faces alternatively, is evident. These K polyhedra, repeated by mirror planes on which are lying the common oxygens O(1), O(2) (water oxygen), O(3), O(4) and O(3'), O(4'), O(6'), build up a straight chain in the y direction. This chain is connected to the correspondent chains of the adjacent unit cells, *i.e.* at a distance c , by two symmetrical chains, these also running parallel to b at intermediate heights. All these chains (two per unit cell) are linked to each other by edges O(3)—O(3) and O(4)—O(4) to form a sheet parallel to (100), are wavy in the c direction, and have a thickness of about $\frac{3}{4}a$.

The Ca atom, lying on the mirror plane, binds nine oxygen atoms (mean distance Ca—O = 2.512 Å), three of which also lie on the mirror plane, and six more are symmetrically placed on both sides of the same plane. These Ca polyhedra are arranged along the y direction into an infinite zig-zag chain by the sharing of O(7)—O(7) edges among each

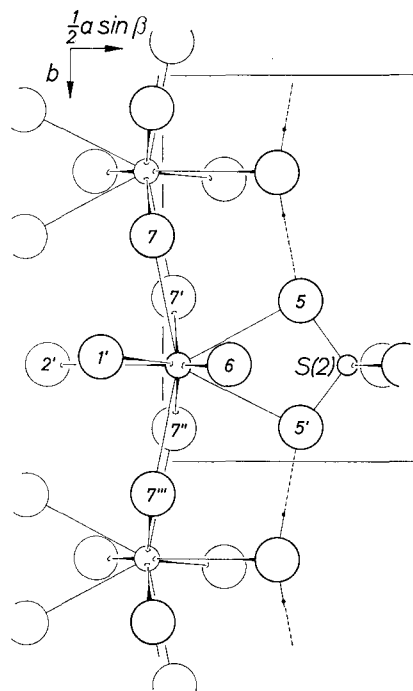


Fig. 2. Projection along the c axis of Ca and S(2) polyhedra. The medium-sized circles are Ca atoms, the remaining ones as in Fig. 1. For a clearer drawing the O(1) atoms are slightly off the mirror planes

polyhedron and its neighbours. Each Ca polyhedron shares three corners of one side and two corners of the opposite side with K polyhedra, bridging the K sheets to each other. These independent Ca chains (one per unit cell) form waves approximately in the *bc* plane. Figure 2 shows the arrangement of Ca polyhedra.

The sulphur atoms, two in special position, instead of one in general position as could be expected, have a regular tetrahedral coordination with a mean S—O distance of 1.473 Å; the bond angles O—S—O range from 107° to 112°.

The S(1) tetrahedron, shown in Fig. 1 only, shares its O(7)—O(7) edge with a Ca polyhedron belonging to a chain and the opposite O(1)—O(6) edge with a Ca polyhedron of another chain. The complex of S(1) tetrahedra, with Ca chains, makes up a sheet parallel to (100); this fills the quarter of cell lacking of K sheets. The strong bonding between S(1) and Ca polyhedra explains the {100} cleavage. Besides, S(1) tetrahedra make additional bridges among the K sheets.

Each S(2) tetrahedron shares one edge with each of its surrounding K polyhedra and the O(5)—O(5) edge with a Ca polyhedron (Figs. 1 and 2) to form further bonds between the K sheets and the Ca chain. Bond lengths and bond angles are given in Tables 3 and 4 respectively.

Cleavages parallel to {110} and {010} are explained by the strong bonds of S(1) tetrahedra with Ca polyhedra in the *z* direction, and by the linkages of S(2) tetrahedra with K polyhedra in the same direction. In fact, both cleavages break the Ca—O—Ca and K—O—K connections.

Table 3. *Interatomic distances and their standard deviations*

| Bond | Length | σ | Bond | Length | σ |
|------------|---------|----------|------------|---------|----------|
| K—O(1) | 2.756 Å | .005 Å | Ca—O(1') | 2.738 Å | .006 Å |
| —O(2) | 2.981 | .004 | —O(2') | 2.472 | .007 |
| —O(3) | 2.867 | .005 | —O(5) | 2.563 | .005 |
| —O(3') | 2.920 | .004 | —O(6) | 2.398 | .005 |
| —O(4) | 2.975 | .006 | —O(7) | 2.453 | .004 |
| —O(4') | 2.866 | .004 | —O(7') | 2.536 | .003 |
| —O(5) | 2.974 | .004 | | | |
| —O(6) | 2.694 | .005 | | | |
| S(1)—O(1') | 1.464 | .006 | S(2)—O(3') | 1.462 | .006 |
| —O(6) | 1.474 | .006 | —O(4') | 1.464 | .006 |
| —O(7') | 1.478 | .004 | —O(5) | 1.478 | .005 |

Table 4. Bond angles and their standard deviations

| Atoms | Angle | σ | Atoms | Angle | σ |
|---------------|-------|----------|------------------|--------|----------|
| O(1)—K—O(2) | 61.8° | .2° | O(1')—Ca—O(5) | 123.2° | .2° |
| —O(3) | 90.1 | .2 | —O(6) | 54.6 | .2 |
| —O(3') | 125.0 | .2 | —O(7) | 78.1 | .1 |
| —O(4) | 104.6 | .1 | —O(7') | 136.5 | .2 |
| —O(4') | 143.1 | .2 | O(2')—Ca—O(1') | 68.9 | .2 |
| —O(5) | 77.0 | .1 | —O(5) | 148.0 | .1 |
| —O(6) | 82.2 | .1 | —O(6) | 123.5 | .2 |
| O(2)—K—O(3) | 102.3 | .1 | —O(7) | 80.2 | .1 |
| —O(3') | 173.1 | .2 | —O(7') | 77.9 | .2 |
| —O(4) | 70.1 | .2 | O(5)—Ca—O(5') | 55.0 | .3 |
| —O(4') | 88.9 | .1 | —O(6) | 75.3 | .2 |
| —O(5) | 138.7 | .2 | O(7)—Ca—O(5) | 74.7 | .2 |
| —O(6) | 104.1 | .2 | —O(5') | 129.4 | .2 |
| O(3)—K—O(3') | 78.7 | .2 | —O(6) | 88.0 | .1 |
| —O(4) | 48.2 | .2 | —O(7') | 69.1 | .2 |
| —O(4') | 119.6 | .2 | —O(7'') | 123.6 | .1 |
| —O(5) | 80.2 | .1 | —O(7''') | 153.3 | .3 |
| —O(6) | 144.8 | .2 | O(7')—Ca—O(5) | 74.9 | .1 |
| O(3')—K—O(4') | 84.7 | .1 | —O(5') | 99.9 | .1 |
| —O(5) | 48.2 | .2 | —O(6) | 146.3 | .1 |
| —O(6) | 77.9 | .2 | —O(7'') | 55.8 | .3 |
| O(4)—K—O(3') | 106.4 | .2 | O(1')—S(1)—O(6) | 107.9 | .3 |
| —O(4') | 83.6 | .3 | —O(7') | 111.7 | .2 |
| —O(5) | 128.0 | .2 | O(7')—S(1)—O(6) | 109.4 | .2 |
| —O(6) | 166.5 | .2 | —O(7'') | 106.7 | .4 |
| O(4')—K—O(5) | 126.2 | .1 | O(3')—S(2)—O(4') | 109.2 | .4 |
| —O(6) | 84.0 | .2 | O(5)—S(2)—O(3') | 109.8 | .2 |
| O(5)—K—O(6) | 64.5 | .2 | —O(4') | 110.8 | .2 |
| | | | —O(5) | 106.5 | .6 |

Table 5. Electrostatic valency table

| Anion | Balancing cations | Charges of cations | Total charges surrounding anion |
|-----------------|-------------------|---------------------------|---------------------------------|
| O(1) | S(1), 2K, Ca | $3/2 + 2/8 + 2/9$ | 1.972 |
| O(2) (H_2O) | 2K, Ca, 2H | $2/8 + 2/9 + 110/72$ | 2.000 |
| O(3) | S(2), 4K | $3/2 + 4/8$ | 2.000 |
| O(4) | S(2), 4K | $3/2 + 4/8$ | 2.000 |
| O(5) | S(2), K, Ca, H | $3/2 + 1/8 + 2/9 + 17/72$ | 2.083 |
| O(6) | S(1), 2K, Ca | $3/2 + 2/8 + 2/9$ | 1.972 |
| O(7) | S(1), 2Ca | $3/2 + 4/9$ | 1.945 |

The electrostatic charge balance is reported in Table 5; it can be seen that the water oxygen is O(2), a result also noted from the differ-

ence map. The hydrogen atom makes a bridge $O(2)\cdots H\cdots O(5)$ of 2.74 Å [distances $O(2)-H = 0.94$ Å and $H-O(5) = 1.80$ Å]. The water angle $H-O(2)-H$ is 104° .

All the calculations were carried out on the I.B.M. 1620 computer. We are indebted to Prof. V. SCATTURIN, Dr. V. ALBANO, Dr. P. L. BELLON and Dr. F. POMPA who supplied us the program for the least-squares refinement.

Note added in proofs. This work was presented at the Seventh International Congress of the Union of Crystallography, Moscow, USSR, July 12–19, 1966.

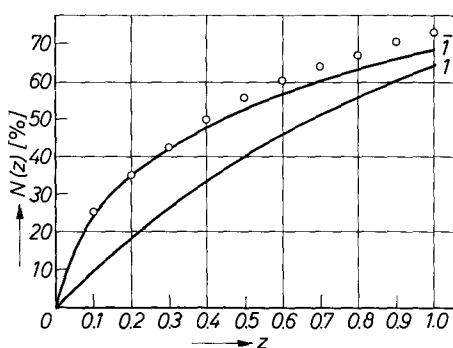


Fig. 3

During the discussion of the paper G. B. BOKIJ informed us that from a crystal structure investigation, made by L. I. GOROGOTZKAYA in 1964 (Doklady Akad. Nauk SSSR, **157**, 1373–1375), syngenite resulted to be acentric. The structure was solved from three-dimensional data, and refined on the xz and xy projections ($R = 18.2$ and 17.9% , respectively), in the $P2_1$ space group.

A statistical test was then carried out by us, with the contribution of all reflections, as a check of centrosymmetry, according the method of HOWELLS, PHILLIPS and ROGERS [Acta Crystallogr. **3** (1950) 210–214]. The test (Fig. 3) confirms the space group assumed in this paper.

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