

The crystal structure of jordanite, $\text{Pb}_{28}\text{As}_{12}\text{S}_{46}$ *

By TETSUZO ITO** and WERNER NOWACKI

Abteilung für Kristallographie und Strukturlehre, Universität Bern

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Auszug

Natürlicher Jordanit aus dem Binnatal (Schweiz) ist monoklin, Raumgruppe $P2_1/m$, mit $a = 8,918(1)$, $b = 31,899(4)$, $c = 8,462(1)$ Å und $\beta = 117,79(1)$ °. Die Elementarzelle ist entlang der a - und der c -Achse ungefähr halbierkt. Von der Viertel-Struktur ausgehend wurde die vollständige Struktur mittels einer Art Minimal-Residual-Methode (der Methode der Hauptverschiebungen) gelöst und durch gewöhnliche Fourier- und Kleinste-Quadrate-Methoden bis zu $R = 7,0\%$ verfeinert.

Die Jordanitstruktur ist ein deformierter PbS-Strukturtyp. Die Elementarzelle enthält 40 Me- und 46 S-Atomlagen. Die 12 unabhängigen Me-Lagen verteilen sich auf drei Schichten $y \approx 0,05$, $\approx 0,15$ und $\approx 0,25$ (die Spiegel-ebene), mit vier Me-Lagen, 3 Pb + As, in jeder Schicht. Zwischen diesen Me-Schichten befinden sich S-Schichten; drei zusätzliche S-Atome liegen in der dritten Me-Schicht, die eine gemischte (Me-S)-Schicht darstellt. Einige der Lagen sind statistisch besetzt; eine Pb-Lage der ersten Schicht ist von 0,50 Pb + 0,50 As, eine andere Pb-Lage in der dritten Schicht von 0,88 Pb besetzt. Dies ergibt eine Formeleinheit von $\text{Pb}_{27,8}\text{As}_{12,0}\text{S}_{45,8}$ pro Elementarzelle, mit der idealen Formel $\text{Pb}_{28}\text{As}_{12}\text{S}_{46}$.

Die voll besetzten Pb-Lagen der ersten, zweiten bzw. dritten Schicht sind von sechs [mittlerer (Pb—S)-Abstand = 3,01 Å], sieben [3,04 Å] bzw. acht [3,08 Å] S-Atomen koordiniert. Die S-Koordination um die As-Atome ist trigonal-pyramidal mit einem mittleren (As—S)-Abstand von 2,25 Å. Die AsS_3 -Pyramiden sind voneinander getrennt (Typus I.c₁ der Klassifikation von Nowacki).

Abstract

Natural jordanite from Binnatal, Switzerland is monoclinic, space group $P2_1/m$, with $a = 8.918(1)$, $b = 31.899(4)$, $c = 8.462(1)$ Å and $\beta = 117.79(1)$ °.

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** Present address: The Institute of Physical and Chemical Research, Rikagakukenkyusho, Wako-shi, Saitama, 351 Japan.

The unit cell is approximately halved along both the a and c axes. Starting from the one-fourth substructure, the complete structure was solved by a kind of minimum residual method (the method of key shifts), and was refined by ordinary Fourier and least-squares methods ($R = 7.0\%$).

The structure of jordanite is a deformed PbS-type structure. The unit cell contains 40 metal and 46 sulfur atom sites. The twelve independent metal sites are distributed on three layers at $y \approx 0.05$, ≈ 0.15 and ≈ 0.25 (the mirror plane), with four metal sites, $3\text{Pb} + \text{As}$, on each layer. These metal layers are interleaved by sulfur layers; three additional S atoms are added to the third metal layer to form a metal-sulfur mixed layer. Some of the sites have statistical nature; one Pb site on the first layer is occupied by $0.50\text{Pb} + 0.50\text{As}$, another Pb site on the third layer is occupied by 0.88 Pb . Thus, the unit-cell content is $\text{Pb}_{27.8}\text{As}_{12.0}\text{S}_{45.8}$, with the ideal formula $\text{Pb}_{28}\text{As}_{12}\text{S}_{46}$.

The fully occupied Pb atoms on the first, second and the third layers are coordinated with six (average $\text{Pb}-\text{S} = 3.01\text{ \AA}$), seven (3.04 \AA) and eight (3.08 \AA) S atoms, respectively. The sulfur coordinations about the As atoms are ordinary trigonal pyramids with an average $\text{As}-\text{S} = 2.25\text{ \AA}$. The AsS_3 pyramids are isolated from each other (type I.c₁ of the classification of NOWACKI).

Introduction

The precise chemical composition of jordanite has been the subject of considerable discussion (SOLLY, 1900; PALACHE, RICHMOND and WINCHELL, 1938; FISHER, 1940; PEACOCK and BERRY, 1940; DOUGLASS, MURPHY and PABST, 1954; WUENSCH and NOWACKI, 1966). The formulae which have been proposed on the basis of the chemical analyses of natural material can be summarized by the range of composition $\text{Pb}_{26-28}\text{As}_{14}\text{S}_{46-49}$. On the other hand, ROLAND (1968) proposed a new formula, $\text{Pb}_{28-x}\text{As}_{12}\text{S}_{46-x}$ [$0.8 < x < 1.4$], from silica-tube quenching experiments and density measurements of synthetic jordanite; moreover, he concluded that jordanite is most probably a high-temperature dimorph of gratonite, $\text{Pb}_{27}\text{As}_{12}\text{S}_{45}$, the structure of which has been established by x-ray work (RIBÁR and NOWACKI, 1969; RÖSCH, 1963). More recently, however, KUTOGLU (1969) proposed an older formula, $\text{Pb}_{27}\text{As}_{14}\text{S}_{48}$, for synthetic jordanite.

Jordanite is a typical superstructure based on the PbS-type substructure. The unit cell is approximately halved along both the a and c axes. The one-fourth substructure (the average atomic arrangement over four closely related subcells) has been determined by WUENSCH and NOWACKI (1966). The present paper deals with the structure determination of the complete structure of natural jordanite. As described below, the obtained unit-cell content is $\text{Pb}_{27.8}\text{As}_{12.0}\text{S}_{45.8}$, with the ideal formula, $\text{Pb}_{28}\text{As}_{12}\text{S}_{46}$.

Crystal data

A specimen of jordanite (sample code Jord. Nr. 1) from Binnatal, Switzerland was used for the present investigation. A fragment was cut out of the specimen with a razor, and it was made into a sphere with a radius of 0.078 mm by BOND's (1951) method. The powder attached to the surface was dissolved away with a hot aqueous HNO_3 solution. Diffraction patterns showed jordanite to be monoclinic. The lattice constants were obtained from three equatorial Weissenberg photographs (about a , b and [201] axes), taken with a back-reflection double-radius camera. Diffraction patterns of Si powder ($a = 5.43074 \text{ \AA}$) were used for calibration. The wave lengths used were 1.54051 and 1.54433 \AA for $\text{Cu}K\alpha_1$ and α_2 radiations, respectively. The results of the least-squares calculations agree fairly well with those given by NOWACKI, ITAKA, BÜRKI and KUNZ (1961) (Table 1)¹.

As was already pointed out by DOUGLASS, MURPHY and PABST (1954), the set of lattice constants given in Table 1 (the reduced cell of jordanite) should carefully be distinguished from another very similar set: $a' = 8.987(1) \text{ \AA}$, $\beta' = 118.61(1)^\circ$, $b' = b$, $c' = c$ and $V' = V$. The transformation is given by: $\mathbf{a}' = \mathbf{a} + \mathbf{c}$, $\mathbf{b}' = -\mathbf{b}$ and $\mathbf{c}' = -\mathbf{c}$.

The observed systematic absences of reflections were only $0k0$ with k odd. Therefore, permitted space groups are $P2_1$ or $P2_1/m$. Since the mineral is not piezoelectric (NOWACKI *et al.*, 1961) and possesses morphology of symmetry $2/m$, the space group was assumed to be $P2_1/m$ ².

Table 1. Lattice constants of jordanite

	Jordanite		Galena*
	Present work	NOWACKI <i>et al.</i> (1961)	
a	8.918(1) \AA	8.96(4) \AA	8.39 \AA
b	31.899(4)	31.92(1)	34.24
c	8.462(1)	8.45(3)	8.39
β	117.79(1)°	117°50(10)'	120°
V	2129.5 \AA^3	2137.1 \AA^3	2085.3 \AA^3

* $a = [1\bar{1}0]_{\text{PBS}}$, $b = (10/3)[111]_{\text{PBS}}$ and $c = [\bar{1}01]_{\text{PBS}}$, where $a_{\text{PBS}} = 5.93 \text{ \AA}$.

¹ Throughout the paper, the estimated standard deviations are given in parentheses in an abbreviated form; for example, 8.918(1) means 8.918 ± 0.001 .

² The results of the present analysis do not rule out the other possible space group $P2_1$. However, even if the correct space group is $P2_1$, deviation from the symmetry of $P2_1/m$ seems to be hardly significant (see also the footnote ⁴).

Since chemical and microprobe analyses on natural jordanite from Binnatal have detected at most negligible amount of Sb (cf. Table 7), it was assumed that the crystal used did not contain Sb.

Intensity measurements

The intensities were measured with an automatic diffractometer of the equi-inclination type (Buerger-Supper-Pace) using Ni-filtered $\text{CuK}\alpha$ radiation. The diffracted beams were detected with a proportional counter. The spherical crystal was rotated in the ω -scan mode about the b (0 to 37th layers) and the a (0 to 9th layers) axes. The scanning speed was varied from 0.5° (for higher angles) to 1.0° per minute (for lower angles). The background was measured before and after each Bragg reflection for the time approximately equal to the scan time of the reflection. About 4000 independent reflections were measured, of which 3200 were considered to be observed [$I > 2.33\sigma(I)$]. They were corrected for Lorentz, polarization and absorption (sphere with $\mu r = 9.7$ for $\text{CuK}\alpha$ radiation) effects.

Structure determination

Approximate structure

Starting from the one-fourth substructure of WUENSCH and NOWACKI (1966), an approximate complete structure of jordanite was solved by a kind of minimum residual method (the method of key shifts; ITO, 1973). Since the procedure of key shifts of jordanite is described in detail in the above reference, only the results of the analysis are given in Table 2. The R value at this stage was 33% for all 3200 observed reflections.

Isotropic refinement

The approximate structure was refined by least-squares (block-diagonal approximation) and Fourier methods. In the least-squares calculations, unit weights were given to all reflections, and the atomic scattering factors for the neutral atoms were used³. Two cycles of isotropic refinement of the twelve metal atoms of Table 2 reduced R from 33 to 28%. The B value of As(11) showed tendency to diverge and that of Pb(12), which had been considered to be As, became exceptionally low (0.5 \AA^2). The corresponding Fourier map also gave a very

³ International tables for x-ray crystallography (1962), Vol. III, pp. 202 (S and As) and 210 (Pb). Birmingham: Kynoch Press.

Table 2. Approximate coordinates of jordanite as deduced by the method of key shifts
The overall temperature factor used was $B = 1.7 \text{ \AA}^2$

Designation of atoms		x/a	y/b	z/c
this paper	Ito (1973)			
Pb(1)	Pb(111)	0.440	0.059	0.346
Pb(2)	Pb(112)	440	048	831
Pb(3)	Pb(121)	912	041	322
As(4)	As(122)	912	046	822
Pb(5)	Pb(211)	266	140	001
As(6)	As(212)	266	140	501
Pb(7)	Pb(221)	794	150	017
Pb(8)	Pb(222)	794	150	517
Pb(9)	Pb(311)	131	250	162
Pb(10)	Pb(312)	116	250	677
As(11)	As(321)	543	250	147
Pb(12)	As(322)	543	250	647

low peak for As(11) and a high peak for Pb(12). At the same time, the map revealed ten reasonable sulfur peaks. Therefore, in the next cycle, As(11) was eliminated, the atomic species of Pb(12) was changed from As to Pb and the ten S atoms, S(1) to S(10), were added (altogether 21 atoms). After additional four cycles, R was 17%. In the Fourier map, As(11) disappeared from the mirror plane; instead, three additional sulfur peaks surrounding the absent As(11) position in a triangular arrangement appeared on the plane. In addition, a peak with approximately the same peak height as that of a sulfur atom appeared about 0.7 Å below (by symmetry also above) the As(11) position; the peak together with the above three sulfur peaks formed a trigonal pyramid typical for the AsS_3 pyramid. Therefore, the peak was assigned as As with half occupancy; As(11) now split into two halves and the three S atoms, S(11) to S(13), were added in the next cycle (altogether 25 atoms). A few cycles of least-squares refinement together with the Fourier maps indicated that S(11) had also to be split into two halves, about 0.4 Å above and below the mirror plane. The R value at this stage was 11%.

Refinement of site occupancy

The results of the least-squares refinement ($R = 11\%$) gave abnormally high B values for Pb(2) and Pb(12) (3.9 and 3.2 \AA^2 , respectively). Both atoms were coordinated with six S atoms. The coordina-

tion about Pb(2) was an intermediate one between typical coordinations of Pb and As; the Pb—S distances were about 2.6 and 3.0 Å for the three shorter and three longer bonds, respectively. On the other hand, the coordination about Pb(12) was a typical one for Pb with the six Pb—S distances of about 3.0 Å. Therefore, it was assumed that the Pb(2) site was statistically occupied by Pb and As, and the Pb(12) site was fractionally occupied by Pb. The occupancies of the two sites were then refined in combination with the isotropic temperature factor of each site by a least-squares method (2×2 block-diagonal approximation). As for the Pb(2) site, preliminary calculations indicated that the distribution was approximately 0.5 Pb + 0.5 As. Therefore, the number of electrons of the site was refined using a unitary atomic scattering factor, $f = \{f(\text{Pb}) + f(\text{As})\}/(82 + 33)$, in a similar way as was applied to binnite by WUENSCH, TAKÉUCHI and NOWACKI (1966).

Table 3. *Atomic coordinates of jordanite with standard deviations*

Atom	<i>x/a</i>	<i>y/b</i>	<i>z/c</i>
Pb(1)	0.4469(2)	0.05466(4)	0.3504(2)
Pb(2)	4392(2)	05194(8)	8422(3)
Pb(3)	9040(2)	04119(4)	3167(2)
As(4)	9194(4)	05256(10)	8299(4)
Pb(5)	2648(2)	14226(5)	-0028(2)
As(6)	2758(5)	14284(12)	5082(5)
Pb(7)	7904(2)	15071(4)	0176(2)
Pb(8)	7925(2)	15021(4)	5149(2)
Pb(9)	1222(3)	25	1714(3)
Pb(10)	1215(3)	25	6896(3)
As(11)	6158(9)	2265(2)	1766(10)
Pb(12)	5504(4)	25	6395(4)
S(1)	3095(9)	0031(3)	0123(11)
S(2)	3067(10)	0051(3)	5504(12)
S(3)	0460(9)	0862(2)	1005(10)
S(4)	0511(9)	0853(2)	6864(11)
S(5)	6520(9)	0969(3)	2028(10)
S(6)	6733(9)	0910(2)	7073(11)
S(7)	4061(10)	1767(3)	3744(12)
S(8)	4021(11)	1774(3)	7699(11)
S(9)	0307(9)	1791(3)	3838(10)
S(10)	9940(10)	1935(2)	8688(11)
S(11)	3783(17)	2617(4)	055(2)
S(12)	7520(15)	25	032(2)
S(13)	7521(15)	25	455(2)

Table 4. Thermal parameters of jordanite with standard deviations

The thermal parameters refer to the expression:

$$T = \exp \{-2\pi^2 (b_{11}h^2a^{*2} + \dots + 2b_{12}hka^*b^* + \dots)\}$$

B_{eq} is the equivalent isotropic temperature factor.

Atom	b_{11}	b_{22}	b_{33}	b_{12}	b_{13}	b_{23}	B_{eg}
Pb(1)	0.0203(6)	0.0191(5)	0.0215(6)	0.0032(5)	0.0082(5)	0.0012(5)	1.65 Å ²
Pb(2)	129(9)	469(15)	153(9)	— 70(8)	54(7)	— 111(9)	2.01
Pb(3)	143(5)	232(5)	206(6)	— 20(5)	43(5)	— 7(5)	1.65
As(4)	68(14)	93(15)	141(16)	— 4(12)	30(13)	— 28(13)	0.85
Pb(5)	197(6)	361(10)	219(7)	— 74(6)	79(5)	— 63(6)	2.10
As(6)	158(17)	.018(2)	.019(2)	— 24(15)	39(14)	— 33(15)	1.54
Pb(7)	192(6)	191(5)	190(6)	— 3(5)	80(5)	— 2(5)	1.54
Pb(8)	180(6)	165(5)	162(6)	— 1(5)	65(5)	— 1(5)	1.38
Pb(9)	225(9)	258(10)	208(9)	— 0	80(8)	— 0	1.88
Pb(10)	233(9)	237(10)	232(9)	— 0	114(8)	— 0	1.83
As(11)	.015(3)	.007(4)	.022(4)	— .001(3)	.005(3)	— .000(3)	1.25
Pb(12)	.0404(14)	.0160(10)	.0388(14)	— 0	.0172(12)	— 0	2.55
S(1)	.003(3)	.022(4)	.018(4)	— .001(3)	— .002(3)	— .011(3)	1.34
S(2)	10(3)	11(4)	29(4)	— 2(3)	12(3)	— 7(3)	1.22
S(3)	7(3)	12(4)	9(3)	— 4(3)	— 2(3)	— 1(3)	0.89
S(4)	9(3)	11(4)	18(4)	— 1(3)	9(3)	— 1(3)	0.88
S(5)	10(3)	16(4)	8(3)	— 7(3)	— 1(3)	— 1(3)	1.04
S(6)	9(3)	9(4)	16(4)	— 2(3)	6(3)	— 2(3)	0.87
S(7)	7(4)	37(5)	22(4)	— 5(4)	8(3)	— 1(4)	1.71
S(8)	21(4)	17(4)	11(4)	— 5(3)	5(3)	— 5(3)	1.38
S(9)	2(3)	16(4)	13(4)	— 0(3)	2(3)	— 1(3)	0.86
S(10)	16(4)	8(4)	15(4)	— 1(3)	11(3)	— 4(3)	0.92
S(11)	6(6)	6(8)	15(7)	— 4(4)	2(5)	— 2(5)	0.79
S(12)	7(5)	38(8)	23(6)	— 0	7(5)	— 0	1.79
S(13)	13(5)	17(6)	26(6)	— 0	6(5)	— 0	1.56

Table 5. Observed and calculated structure factors (0.5)

k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	
0 0 0	17 38 -8	20 118 -112	2 42 -19	32 29 31	16 158 165	34 78 -39	15 29 35														
8 67 -67	20 238 -229	22 99 -96	3 48 -23	33 72 74	17 50 53	35 44 -56	14 48 -45														
10 870 -920	21 259 -236	23 148 -151	5 76 -71	-4 k 1	20 72 73	36 54 -56	13 40 32														
12 110 108	22 178 178	24 37 -17	6 47 44	1 29 34	22 102 -104	3 k 1	17 33 -39														
16 33 43	23 28 17	25 63 -73	8 41 36	3 68 73	23 113 120	0 117 115	19 87 -88														
18 150 -155	27 30 -23	26 53 51	9 41 -28	4 150 -156	24 31 -38	2 80 77	21 70 -74														
20 672 656	28 33 -35	28 45 -42	10 55 -49	5 46 63	25 32 -28	5 132 133	22 48 -46														
22 151 143	29 32 11	29 77 86	11 33 -17	6 131 131	26 131 -151	4 44 40	23 55 -56														
26 53 -52	30 168 175	12 41 -39	7 70 74	27 31 -15	5 111 -98	25 20 -21															
28 123 129	31 260 260	13 67 -67	9 45 49	28 82 -94	7 101 98	26 25 -27															
30 360 -357	32 75 -69	1 100 100	14 31 -39	10 39 40	29 66 -73	9 39 -29	27 30 29														
32 80 56	33 62 64	3 147 -148	15 50 57	11 71 71	30 76 -76	10 50 -35	7 k 1														
34 92 90	34 16 20	16 134 -134	17 45 -43	4 150 -150	18 40 -40	12 14 18	0 106 -113														
36 67 68	35 50 -46	5 228 -220	7 40 40	25 55 -58	13 55 58	12 14 16	1 62 57														
38 93 -110	4 k 0	7 175 -178	19 40 33	14 104 99	33 74 -89	13 159 -156	1 66 -61														
1 k 0	0 84 71	9 88 92	20 29 26	17 99 -101	35 37 -43	15 54 59	3 40 36														
0 36 19	1 178 -167	11 81 -90	21 61 61	18 51 -45	36 106 127	16 122 -118	3 66 -61														
2 99 -92	2 40 -29	12 30 -15	22 41 46	19 70 -71	18 31 -34	6 56 56															
3 94 88	3 224 209	13 95 90	25 38 36	20 52 -50	21 89 -90	0 k 1	19 33 28	7 42 42													
5 103 -98	6 365 -352	14 70 -61	-7 k 1	21 89 -90	3 104 -108	22 79 81	8 49 41														
6 60 61	5 529 489	15 104 102	-7 k 1	22 52 57	4 115 -118	23 132 136	9 72 73														
7 112 120	6 293 276	16 240 235	0 128 132	24 52 -54	5 73 -80	24 47 44	10 61 65														
8 82 88	7 230 224	17 157 157	1 17 -5	25 37 -42	6 40 44	26 118 121	14 65 72														
12 41 40	8 53 49	19 90 -91	2 45 51	7 134 -134	27 39 -40	15 55 -20															
14 129 -148	9 146 -146	20 29 -6	3 63 -77	28 38 -40	28 38 -40	16 23 -19															
15 172 180	10 84 -69	21 68 68	33 33 35	18 50 -58	24 44 -51	27 27 -29															
16 143 -153	11 158 142	22 28 28	5 42 45	31 70 75	10 37 23	30 42 43	19 69 -70														
17 262 -276	13 167 -154	23 34 -42	7 54 -51	32 50 -45	11 64 -62	31 26 -26	20 21 -16														
18 225 -228	14 308 343	24 45 37	8 39 -26	33 46 46	13 66 66	32 28 -28	21 27 -30														
19 96 -88	15 448 -423	25 40 -47	10 72 -77	34 26 17	14 130 140	33 89 -95	22 33 -30														
20 35 17	16 88 -84	26 153 -159	12 70 -65	35 14 14	16 45 -43	34 36 -32	23 21 -13														
21 56 59	17 199 -184	8 k 0	13 84 92	-3 k 1	17 149 155	4 k 1	8 k 1														
23 45 -42	18 81 -84	-3 k 1	14 52 -50	18 48 -45	18 48 -45	14 84 92															
25 65 -66	19 158 149	0 84 -90	18 38 -40	0 55 56	20 33 0	0 130 135	0 35 37														
26 109 115	20 83 80	19 206 -203	19 25 -23	1 78 -65	21 83 91	1 46 45	1 28 -24														
25 179 -191	21 95 -93	2 183 -196	20 32 33	2 93 -89	22 33 34	2 58 51	2 70 -72														
26 155 161	23 96 94	3 128 -135	22 47 49	3 28 25	23 28 -27	3 59 -54	3 59 -53														
27 167 180	24 223 -223	4 142 -142	7 60 -60	5 157 156	24 68 -75	4 54 -54	4 54 -53														
28 95 97	25 223 -223	5 142 -142	7 64 -28	6 45 -47	7 57 -57	5 41 -41	5 41 -40														
29 31 25	26 55 50	6 48 -28	25 23 -15	47 51 -51	27 27 -27	42 46 -46	24 26 -26														
30 36 -33	27 38 41	7 112 -105	26 42 41	8 50 33	27 35 -35	8 43 33	7 39 -41														
31 77 -95	29 170 -166	8 152 152	27 27 19	10 67 -62	28 33 -31	9 74 68	8 48 46														
32 27 13	30 46 -49	9 146 -151	11 45 -46	12 95 87	31 66 -71	11 56 45	10 39 -41														
33 56 67	31 51 54	10 38 38	12 44 51	14 40 -41	15 45 -53	15 23 -29	13 60 46	12 48 -42													
34 71 -66	32 35 36	11 102 106	0 48 -51	14 70 70	32 37 -37	12 62 -71	11 33 36														
35 127 139	33 38 -41	12 44 51	14 40 -41	17 31 31	15 55 60	17 39 40	14 84 92														
2 k 0	5 k 0	14 75 -80	2 63 62	16 170 165	34 14 165	15 41 -26	15 23 -21														
0 91 -96	0 35 -19	16 37 25	4 66 76	19 57 -52	1 18 94	1 94 -94	15 31 -27														
2 27 27	28 37 26	17 64 65	5 51 46	20 118 105	1 18 84	1 84 -77	16 62 67														
2 240 229	2 166 -157	18 153 -158	6 48 47	3 47 -49	0 115 114	0 65 68	9 k 1														
3 462 443	3 240 -317	19 173 186	5 48 -48	21 105 -101	1 107 -100	21 37 -37	9 31 -33														
4 440 429	4 236 -231	20 84 -94	8 47 -47	25 78 -75	4 38 -46	23 35 -35	0 71 -70														
5 87 -80	5 45 35	9 46 -45	10 48 49	26 125 -131	6 49 -43	24 52 50	2 32 -32														
6 442 -437	6 194 187	9 47 -47	11 51 52	22 89 83	7 30 -39	25 42 -42	17 23 -23														
7 584 575	7 157 -147	10 166 183	12 60 -51	28 61 -66	9 241 -241	26 34 38	4 23 -28														
8 472 -456	8 87 -74	11 116 -121	13 49 -53	30 66 -66	10 40 -44	27 32 -32	7 19 -19														
9 151 141	9 119 101	2 49 56	14 128 -120	31 23 23	19 73 -66	.28 75 79	-10 k 2														
10 119 122	11 40 -29	3 73 -80	15 38 35	32 36 45	14 32 -31	29 46 53	0 25 2														
11 36 -34	12 86 82	4 69 80	16 34 -31	33 59 60	16 36 -33	32 33 -33	1 15 57 59														
12 163 -157	13 243 230	5 26 29	17 36 -38	34 43 50	17 38 -25	32 19 -19	1 118 -116														
13 59 -54	14 162 158	6 45 -54	19 29 -15	35 16 24	19 22 -22	33 67 73	2 61 -69														
14 31 -31	15 51 -51	7 46 -45	20 40 -40	36 103 119	19 40 -49	4 26 -26	3 40 -37														
16 200 289	16 120 128	9 47 -47	17 35 -35	28 99 -102	4 104 102	20 52 -52	6 74 -75														
17 500 -289	17 103 90	9 128 -126	22 12 -12	32 37 37	12 53 -53	12 53 -51	0 89 -94														
18 238 232	18 66 57	10 158 -170	24 91 92	0 247 -246	25 27 -27	1 66 -62	3 28 32														
19 49 -21	19 55 -40	11 86 93	26 87 92	3 37 -36	27 46 51	2 52 41	7 33 -33														
20 128 -122	21 111 103	12 18 -19	27																		

Table 5. (Continued)

k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c		
5	398	-398	29	50	46	33	87	-98	2	k	2	5	k	2	-9	k	3	9	45	-33	28	22	20		
6	85	-87	50	51	-58	34	153	-165	7	52	-47	6	52	-47	10	61	64	30	38	-40					
7	49	-55	31	19	13	35	51	-26	0	957	-933	0	325	334	0	56	60	11	39	-34	31	66	-73		
8	48	-27	32	37	37	36	89	95	1	290	282	2	164	-35	1	47	58	12	106	103	32	37	35		
9	88	88	-4	k	2	37	63	-81	2	57	-42	2	55	-54	2	59	37	13	122	-127	34	23	-22		
10	29	0	-4	k	2	3	75	69	4	68	65	3	55	55	14	39	25	35	46	-55					
11	59	-58	0	855	894	-1	k	2	4	56	30	5	53	56	4	65	64	15	49	40	-1	k	3		
12	33	-33	1	340	361	1	140	-153	5	85	-97	6	52	-47	12	16	46	85							
14	103	2	68	9	52	-41	5	85	47	7	46	56	7	57	-56	12	52	43	9	39	31				
15	322	329	5	89	-10	11	309	309	8	47	-54	8	36	-22	7	44	49	18	43	42	2	81	66		
16	31	25	5	76	-83	12	54	36	9	187	187	9	160	-155	9	22	20	40	-37	3	57	64			
17	83	83	7	66	53	17	57	27	10	691	686	10	304	-398	10	37	-28	21	39	-63	5	139	-128		
18	35	-25	9	232	234	19	34	-14	11	194	-202	11	39	-26	11	34	26	22	55	-44	7	79	77		
19	100	-103	10	644	-650	21	339	-347	12	44	39	13	108	119	14	36	-29	23	87	8	63	-57			
21	43	44	11	247	-244	22	25	-26	13	71	-67	14	103	-101	12	21	-15	24	21	-29	12	87	-84		
22	23	23	12	59	-57	23	59	55	14	54	-57	15	48	-55	18	38	42	26	97	-101	13	146	-153		
24	64	-59	13	83	-84	27	28	-35	68	78	17	33	34	19	60	-61	27	45	-9	14	114	-113			
7	k	2	15	56	59	29	31	-24	16	55	-19	19	92	-86	20	34	30	28	58	-64	15	45	38		
9	285	-288	14	39	27	51	503	506	9	181	-174	20	82	-27	20	265	-8	k	3	29	64	-69	16	152	-154
1	26	18	7	69	69	17	51	27	26	460	-457	22	115	-114	0	44	18	51	52	-62	20	95	-95		
2	51	53	181	-180	23	76	80	21	209	209	24	14	1	66	45	32	28	11	22	93	100				
3	44	-38	20	441	452	15	35	-31	22	102	-95	25	57	48	2	31	-30	33	78	-84	23	148	149		
4	72	-70	21	218	212	36	12	-23	15	152	145	27	8	5	57	58	-4	k	3	24	69	70			
5	54	56	22	101	103	37	46	64	24	32	24	28	18	4	76	-86	25	51	-51						
7	31	-36	23	150	159	0	k	2	25	52	-57	6	k	2	5	26	-13	0	46	45	26	109	120		
8	43	25	25	48	-49	26	41	37	6	k	2	6	87	84	1	75	65	27	31	-18					
9	192	-180	26	42	-44	0	147	152	28	89	-94	0	50	11	7	71	69	2	51	-54	28	70	74		
10	356	354	28	84	88	1	89	-91	29	52	49	1	90	85	9	25	12	3	119	-126	29	54	-58		
12	40	-39	29	90	90	29	170	-179	30	251	254	2	23	-22	10	29	-26	4	103	-108	30	52	49		
13	88	99	39	231	-241	5	398	388	31	126	-124	3	36	-33	11	50	42	5	47	-60	31	17	8		
14	107	100	31	128	-129	4	461	-471	53	103	-100	15	312	315	23	27	-17	21	63	66	9	71	76		
15	14	14	29	57	54	35	172	172	34	48	-41	43	57	55	16	60	-61	10	31	-37	34	42	-44		
16	112	102	35	53	54	7	56	56	34	55	55	14	42	44	11	61	-68	0	k	3					
20	244	-241	5	k	2	8	312	314	5	k	2	8	32	11	18	33	-22	13	85	73	0				
21	56	51	-5	k	2	9	97	-102	1	80	80	9	93	94	19	39	-80	14	105	105	0	221	226		
22	139	158	3	229	-218	10	162	-172	2	71	70	11	56	-58	20	45	-33	16	22	-24	1	66	-56		
25	48	48	4	224	231	11	109	105	3	207	-211	12	29	24	21	112	-111	17	120	130	2	48	65		
26	20	19	5	99	-97	12	110	109	4	196	192	15	127	-123	22	45	41	18	32	-18	8	40	27		
27	26	8	6	107	-115	15	352	-358	5	103	-100	15	312	315	23	27	-17	21	63	66	9	71	76		
6	k	2	163	161	6	97	-99	16	71	57	57	24	30	-26	22	32	20	29	10	167	-174				
1	171	-176	12	79	74	16	216	-218	7	172	-183	18	41	40	25	-31	23	36	-22	11	80	84			
2	263	263	15	169	161	21	242	-256	9	54	59	49	21	20	19	28	-80	14	105	105	0	221	226		
3	328	-317	15	159	-152	18	157	162	10	257	241	40	41	0	30	27	26	37	-40	18	85	-87			
4	167	144	5	88	49	19	109	113	5	55	-52	22	24	-20	2	44	-36	27	64	-52	19	47	-47		
5	199	196	16	90	91	20	183	183	12	60	-54	24	88	83	5	63	67	28	-17	20	73	71			
6	140	-151	56	-62	21	68	-67	13	104	87	25	198	-214	6	65	-64	30	17	22	47	44				
7	273	-276	16	163	-162	22	46	-41	14	50	-47	24	57	55	5	72	79	31	39	-34	24	105	89		
8	284	-280	19	115	-112	28	219	213	5	82	-75	19	101	-104	18	42	-42	33	22	-5	25	33	-35		
9	220	-216	21	97	98	24	283	-281	16	303	296	0	84	91	7	84	89	34	31	25	27	27			
10	43	-30	23	189	-190	25	57	-51	17	300	300	1	70	72	9	29	-27	3	k	3	30	35	-23		
11	114	119	25	214	215	25	184	185	19	162	152	2	110	115	11	33	27	-3	k	3	30	35	-23		
12	140	-134	25	159	-160	27	110	108	9	32	-23	5	181	191	12	45	42	0	143	146	31	70	72		
13	19	19	22	27	26	9	26	-29	50	-40	4	101	106	14	72	61	1	64	-68	33	66	68			
14	70	70	27	77	74	29	75	-71	21	36	25	75	-77	15	33	-34	2	50	-31	35	17	-19			
15	173	-174	28	113	113	30	60	-62	22	31	48	6	101	99	16	60	59	6	21	20					
16	110	109	29	112	112	31	55	61	23	7	159	174	12	44	41	8	43	-20	9	26	0	61	-55		
17	187	185	50	49	50	25	-22	25	33	11	8	35	-34	20	57	42	8	26	-24	0	61	-55			
18	225	218	31	67	-73	33	113	-122	25	85	78	9	36	31	22	-31	9	187	-181	1	61	-74			
19	180	177	32	46	-55	34	134	162	26	185	-189	10	47	-58	23	32	-33	10	41	-29	2	96	82		
20	33	32	53	118	126	35	23	-21	27	208	-211	12	107	-111	24	65	-67	12	90	97	4	65	-33		
21	56	-50	34	113	-111	16	74	-75	24	23	-13	19	198	-204	26	98	-98	13	53	-50	5	110	113		
22	54	49	35	78	75	37	67	-83	29	42	40</														

Table 5. (Continued)

k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c				
16	57	58	7	k	3	25	20	-7	6	86	-92	18	183	-185	2	66	60	5	89	-63	18	78	-86	
17	90	-89	0	27	19	28	70	70	8	41	-43	21	51	-25	4	35	-9	7	48	-36	21	28		
18	74	72	0	27	19	28	70	70	10	41	18	23	154	-151	5	84	90	8	42	54	23	34		
19	78	-73	3	26	-54	-6	k	4	11	48	-24	24	189	-186	7	31	-28	9	32	20	70	69		
20	35	18	5	71	-76	-6	k	4	11	48	-24	24	189	-186	7	31	-28	9	32	20	70	69		
21	-40	6	20	17	0	52	-43	12	36	-36	25	50	46	8	24	-27	10	51	-29	26	35	-26		
22	45	-50	7	87	-89	1	135	-146	13	40	34	26	173	170	9	10	-16	11	55	-56	28	81	85	
23	50	-52	8	20	18	3	166	166	14	98	102	27	165	-159	10	296	-295	12	24	30	47	-33		
25	50	-44	10	18	-18	4	258	255	15	155	168	28	141	11	228	-223	13	41	24	31	44	49		
26	60	-61	14	17	10	5	459	472	16	120	120	29	29	-31	12	125	-131	14	109	94	-3	k	5	
27	40	37	-10	k	4	6	219	-224	17	180	180	14	50	21	-14	13	117	121	16	34	-32	8	60	72
29	31	24	0	108	115	9	144	-159	19	46	-39	32	17	9	15	93	-92	18	24	-19	2	44	38	
30	42	-21	0	108	115	9	144	-159	19	46	-39	32	17	9	15	93	-92	18	24	-19	2	44	38	
31	63	71	1	154	-162	10	149	31	20	29	-19	33	92	97	16	38	18	19	36	19	35	-14		
32	44	32	10	154	14	11	122	120	21	45	41	1	k	4	17	28	24	21	25	28	5	34	32	
33	82	85	5	97	-109	13	15	103	33	22	51	45	19	105	105	24	56	-54	6	87	90	90	90	
3	k	3	5	110	103	14	256	-246	24	94	-95	1	99	105	21	243	-237	-7	k	5	7	15	121	
0	195	-203	6	38	53	15	395	-365	25	154	-162	4	42	33	22	67	64	3	52	-52	11	62	63	
1	34	37	7	68	-73	16	67	59	26	151	-157	5	45	43	23	111	-121	4	59	-61	12	75	74	
4	75	-74	8	12	-124	17	16	161	-160	27	130	132	6	22	-23	24	23	6	57	-44	13	71	63	
6	74	74	9	138	-140	16	33	33	28	91	-88	10	360	-370	25	57	67	7	31	-40	14	14	15	
7	33	33	10	74	-78	19	108	116	29	25	14	11	240	-249	26	31	-29	9	55	-54	15	79	73	
8	47	38	11	76	80	20	68	-37	50	70	12	39	-84	15	55	59	5	48	16	111	-112	80		
9	165	168	12	46	-42	21	73	-73	31	73	-84	15	49	49	1	73	-84	13	32	12	19	30	36	
10	82	95	15	120	25	22	56	-52	32	32	35	-24	14	29	-30	142	-142	10	21	-19	25	28	21	
11	41	27	14	62	65	25	59	54	33	42	41	-40	2	36	-30	16	42	-27	20	34	-21	65	-74	
13	40	-31	15	84	-87	24	71	165	34	64	57	20	197	194	3	124	131	18	57	-61	21	65	-74	
14	94	93	-9	k	4	25	210	209	-2	k	4	21	254	244	4	126	-124	19	102	109	22	66	66	
16	36	-56	26	51	-46	26	51	-46	26	51	-46	26	51	-46	26	51	-46	26	51	-46	26	51	-46	
17	31	14	0	27	-17	27	55	61	0	1082	-1098	23	44	-28	6	95	97	21	88	92	26	52	119	
19	147	-140	1	79	76	29	141	-141	1	122	-102	24	28	17	7	191	196	22	23	25	27	41	13	
20	62	-56	2	36	21	50	27	22	4	37	32	27	37	28	8	31	31	23	57	28	77	80		
22	35	-18	3	101	-99	31	39	41	6	22	-11	24	20	10	24	24	24	31	29	49	56	56		
23	54	-48	4	103	-96	-5	k	4	8	47	-57	50	142	-142	14	102	-102	10	39	-26	8	57	66	
24	56	-29	5	166	-162	25	21	209	209	20	20	20	20	20	20	15	15	10	39	-26	8	57	66	
25	44	-26	6	84	86	0	134	141	16	692	716	32	36	32	12	40	33	6	32	40	-58	-58	-58	
26	49	46	7	144	-138	1	100	-79	11	51	-52	12	43	-27	2	4	42	38	-2	k	5	-2	k	5
27	27	-20	8	26	-26	2	57	-67	12	43	-27	6	24	-28	6	95	97	21	88	92	26	52	119	
28	20	-9	9	73	-75	3	77	12	13	40	11	0	79	-86	15	97	-102	3	103	-104	0	41	-58	
29	85	93	13	72	-74	70	42	39	14	53	-52	1	116	126	16	188	-182	127	136	3	43	38		
30	37	24	13	71	69	5	31	-22	16	36	-18	2	73	66	17	164	-167	6	104	-114	4	42	41	
31	83	89	15	102	101	6	37	-50	18	108	106	3	197	-199	18	207	-205	15	21	20	74	66	66	
4	k	3	8	67	-204	9	61	61	19	78	76	4	308	317	19	90	94	1	53	55	31	52	51	
0	57	62	18	23	20	11	188	186	22	104	-102	6	255	-264	21	63	-67	11	64	-59	9	53	51	
2	84	-81	19	88	-88	12	49	56	24	46	50	7	152	-152	14	70	64	2	42	38	-2	k	5	
4	45	-45	20	21	-6	13	38	-39	26	29	29	8	26	-27	5	57	57	6	k	4	13	58	-56	
7	45	43	21	55	58	19	47	-46	27	39	39	9	142	149	0	23	27	14	68	-62	14	99	-83	
8	72	70	-8	k	4	21	182	-182	17	186	-186	17	108	-108	2	139	147	16	39	30	16	66	49	
10	51	51	1	157	-157	3	157	-157	3	157	-157	11	102	-102	13	110	121	17	72	70	17	128	-130	
12	60	64	0	189	201	22	48	-47	22	172	30	279	285	15	150	147	4	25	-15	19	20	22	67	
14	90	91	1	238	245	24	26	7	32	64	-49	14	302	-297	4	25	-15	19	35	40	18	72	-67	
16	36	37	2	92	98	28	22	17	3	106	-105	10	105	-101	15	101	101	15	112	-112	27	11	-27	
21	29	-29	4	52	36	28	22	17	3	106	-105	10	105	-101	15	101	101	15	112	-112	27	11	-27	
24	95	-102	5	73	60	-60	31	29	26	26	26	94	-92	5	250	254	10	64	68	16	121	-113	12	51
26	116	6	49	-35	32	6	28	278	296	18	196	192	30	57	58	2	31	31	33	7	26	27	48	
10	33	38	28	17	29	-25	30	38	31	38	-36	36	38	-36	10	80	87	31	16	8	28	37		
11	46	29	28	24	24	25	32	35	21	36	-32	20	28	-28	4	20	-20	8	44	-40	10	44	-45	
12	54	48	19	142	-140	9	145	22	39	28	-38	17	126	-127	14	41	51	22	54	54	16	55	-48	
13	64	67	20	84	83	10	55	-60	23	49	44	0	35	25	6	22	-22	13	59	-57	9	175	177	
14	29	21	22	220	220	11	58	-51	24	30	30	-73	1	31	31	23	23	-107	10	89	92	92		
16	58	46	22	83	83	12	175	-177	12	172	-172	15	127	5	42	35	29	19	-11	26	28	33		
18	26	18	24	131	131	32	37	-37	10	84	84	25	25	50	47	19	31	50	5	36	-32	3		

Table 5. (Continued)

k	F _a	F _c	k	F _b	F _c	k	F _a	F _c	k	F _a	F _c	k	F _a	F _c	k	F _a	F _c	k	F _a	F _c				
11	55	16	18	35	-35	18	47	49	12	67	-58	1	39	-34	6	26	24	21	38	-59	10	95	-94	
12	44	41	19	69	67	19	54	31	15	71	-45	2	107	-98	7	43	-47	25	63	61	15	55	27	
13	55	-24	20	23	23	29	22	22	19	53	-33	1	167	157	7	27	20	26	92	-92	16	60	-65	
14	86	86	21	58	35	21	23	-55	18	28	-16	5	88	-82	10	23	29	16	36	39	16	36	39	
16	87	85	5	k	5	22	33	-28	21	275	-276	5	88	-131	10	28	-23	-3	k	7	17	21	-13	
17	48	-41				23	31	27	22	57	59	6	105	105	15	57	-61	1	37	-16	18	29	-26	
18	76	66	0	101	106	58	-34	23	18	72	172	173	16	35	-31	2	38	-32	19	88	86			
19	59	62	1	58	-65	26	161	-167	24	34	29	8	91	83	17	15	11	7	64	-62	20	59	58	
21	63	-56	3	24	-14	-6	k	6	25	52	-39	9	30	-26	-8	k	7	4	38	-32	21	28	-26	
23	66	57	4	40	42				27	53	38	10	28	-27		6	68	68	2	k	7			
24	112	-112	6	35	-38	0	462	-476	25	25	11	11	54	30	0	74	74	6	53	33				
26	91	-91	7	25	-22	1	261	-285	29	29	14	12	61	60	1	51	-50	7	81	-82	0	59	-41	
27	20	-17	8	42	-44	2	20	-35	50	19	19	15	52	-61	2	47	-32	8	59	40	1	41	-15	
28	18	15	9	62	-61	64	-69	1	18	37	37	29	31	31	1	49	6	2	47	44				
31	28	51	10	71	-76	4	30	-10	-2	k	6	15	47	46	5	67	-29	17	110	111	3	32	31	
1	k	5	13	36	36	7	40	-44	1	38	56	17	244	-239	8	38	38	15	29	-21	5	18	16	
0	74	-83	14	70	-76	9	184	-193	2	150	158	18	126	-117	9	35	31	16	74	85	8	50	-49	
1	32	-34	15	21	17	10	394	409	3	285	-279	19	19	25	18	10	127	-126	17	36	32	10	36	31
2	82	-75	6	k	5	11	192	194	3	313	314	20	16	55	32	12	45	-38	19	51	-45	12	46	-49
3	127	-128				13	66	68	5	87	-79	21	27	-24	17	66	-74	26	57	57	13	30	13	
4	31	-38	0	39	-42	14	39	-56	5	318	-328	22	51	-56	16	58	-49	21	25	7	14	70	-75	
5	93	85	3	59	70	5	52	-49	7	303	-292	23	36	34	15	31	25	23	66	-45	15	17	9	
55	34	3	43	53	53	17	177	177	8	19	-19	24	34	-29	12	26	-14	93	110	16	75	-75		
7	172	-131	6	-10	k	6	20	248	-299	43	43	44	25	66	-62	18	63	-70	24	75	-73	17	19	-22
12	61	60	10	25	9	21	116	-114	10	113	116	20	99	99	19	19	5	25	41	-46	3	k	7	
13	129	126	0	25	23	22	56	-55	11	58	-53	2	k	6	20	70	67	26	54	-59				
14	48	48	1	59	-63	23	110	-110	12	116	-112	0	70	-27	-7	k	7	0	72	72				
15	60	-58	2	42	-44	24	26	15	13	261	-262	0	70	-27	-7	k	7	0	124	-123	2	67	67	
16	95	95	3	39	38	25	32	32	14	324	-322	1	89	74	0	65	61	2	39	-39	3	79	81	
17	31	31	4	51	41	25	34	-38	15	106	105	2	119	127	0	28	-9	3	29	-22	4	19	20	
19	35	-32	5	311	312	6	16	166	149	3	211	217	1	28	-31	9	48	-55	5	30	-31			
22	67	-66	6	49	52	-5	k	6	17	156	147	4	159	165	5	27	-29	9	48	-55				
23	131	-130	7	25	25	0	35	35	18	123	123	5	79	-78	8	25	22	10	74	74	7	70	77	
24	59	-53	8	65	65	2	87	87	21	21	20	89	-89	19	24	25	9	16	14					
25	23	23	16	60	-59	1	100	97	21	62	28	8	168	-169	8	56	59	18	25	-20	11	22	-15	
27	38	-39	11	46	46	3	100	97	21	62	28	8	169	-169	0	56	56	18	26	36	12	36	-30	
29	23	23	14	37	-36	5	67	61	23	166	-165	10	20	17	10	55	-50	19	35	36	-10	k	8	
2	k	5	-9	k	6	7	46	-45	25	29	-26	12	79	-78	5	58	-54	21	29	26	0	70	-67	
0	91	-92	0	189	189	8	95	-97	26	125	-121	13	147	-141	13	55	52	22	56	-40	1	158	-164	
2	63	-64	1	50	54	9	57	-61	27	91	-93	15	85	-84	16	44	-42	24	62	-57	-9	k	8	
4	40	43	2	39	-36	10	35	-8	28	97	-99	15	118	118	17	50	-48	25	35	28				
5	26	-28	3	36	12	11	31	34	29	58	52	16	146	146	18	53	-53	26	16	10	2	51	-52	
6	50	-55	4	34	34	30	15	116	153	50	61	17	160	-155	20	39	43	-1	k	7	3	100	96	
8	33	-31	11	24	24	-20	186	186	0	181	182	21	27	-34	1	23	22	5	66	-66				
9	71	-75	6	14	-38	15	73	-29	1	56	-51	21	69	68	23	39	-40	1	71	66	6	53	54	
10	35	19	7	53	52	32	16	87	-89	4	56	-51	21	69	68	23	39	-40	1	71	66	6	53	54
11	73	-65	8	52	-54	17	50	48	2	38	-32	21	22	25	-13	-6	24	44	-41	7	25	33		
16	39	42	9	124	125	18	108	103	3	127	125	12	76	78	4	30	-30	8	18	-13				
17	51	32	10	209	-208	19	93	89	4	190	186	3	k	6	0	27	-32	9	51	45	9	37	-30	
18	65	11	25	-28	20	41	41	5	166	157	17	160	-155	20	39	-55	5	72	-69	10	29	-16		
19	80	84	12	61	59	21	45	-52	6	52	-52	0	294	-304	5	75	75	6	63	-62	12	36	37	
20	34	-24	13	50	-24	22	30	-21	8	61	28	8	66	66	7	70	-105	13	108	-105				
21	47	47	14	62	-55	25	123	120	10	44	-26	2	21	4	7	74	73	8	34	-30	16	72	74	
22	36	-36	16	26	22	22	120	120	10	44	-26	5	38	34	15	51	51	20	44	-44	7	39	-57	
23	30	-29	17	27	-14	10	100	100	14	178	178	4	27	-27	16	45	-42	10	59	-59	12	46	-46	
24	47	-42	19	72	-74	27	27	27	27	120	-114	7	24	-27	17	45	-42	16	46	-46	0	23	26	
25	42	-25	30	142	151	21	65	-65	6	24	20	17	94	-94	9	29	19	1	89	89				
26	35	-36	2	28	28	28	28	28	29	105	104	19	42	-36	8	35	-40	0	86	84	0	23	26	
27	26	-25	2	29	29	29	29	29	30	189	-199	-196	9	86	84	0	30	-30	16	64	-59	13	30	-24
7	120	-121	9	171	169	8	97	92	29	109	-109	10	65	-58	16	68	61	0	54	42	15	216	214	
9	30	19	10	76	77	9	101	109	0	6	47	10	65	-67	2	22	-12	2	24	24	5	55	55	
10	50	-30	11	87	-89	10	76	-10	12	86	85	8	22	-11	23	69	-65	11	13	-17	7	25	6	
12	59	56	12	133	132	11	98	-89	9	551	563	1	69	-69	15	40	28	5	60	-61	17	87	89	
13	54	14	13	158	-158	15	181	177	1	168	-153	3	54	56	18	62	64	6	38	30	-7	k	8	
14	53	-																						

Table 5. (Continued)

k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c	k	F _o	F _c				
10	37	9	10	411	-400	9	34	34	10	61	-60	6	33	-16	1	46	46	6	31	-25	3	100	105	
11	60	53	11	59	-55	10	67	68	11	55	58	-8	9	9	2	20	-5	8	25	-22	4	160	160	
12	142	-140	12	50	-54	11	30	-9	12	26	24	3	26	-21	9	22	-22	5	197	196				
13	221	222	13	35	-29	12	55	-57	13	98	-96	0	28	-29	6	65	-61	12	20	-20	6	171	-172	
14	168	-164	14	56	56	13	158	-158	14	178	176	2	24	-21	7	81	-79	14	62	-67	7	112	115	
15	85	-86	16	31	-7	15	212	-207	15	134	-136	3	65	67	8	58	-57	16	49	-50	8	24	-19	
16	129	129	18	39	-35	15	43	-38	16	50	-44	4	75	-79	11	53	-47	-1	k	9	9	65	-67	
17	124	121	19	85	-42	16	208	204	17	56	-57	6	52	50	12	23	18							
18	153	129	20	329	332	17	136	-136	18	70	-40	7	34	34	22	22	0	32	32	-5	k	10		
19	66	-52	21	27	22	18	119	119	1	k	8	8	21	-26	15	19	21	2	43	39	0	66	-53	
20	66	-25	22	45	42	19	24	-17	9	18	-15	16	57	57	5	82	89	1	27	-19				
21	57	-35	-3	56	-3	20	63	-60	0	19	-14	10	20	17	17	64	65	4	41	43	2	27	-24	
22	40	36	-5	k	8	21	42	32	2	47	-50	11	54	58	5	35	-37	7	23	-19				
-5	k	8	1	53	47	23	62	61	3	129	-133	12	20	-14	-4	k	9	6	34	-37	8	18	16	
0	25	23	3	83	78	-1	k	8	4	136	-140	-7	k	9	0	46	45	7	76	79	9	75	-76	
1	31	-17	5	74	-69	0	171	-174	6	109	107	2	21	-18	3	20	-18	9	17	-5	-4	k	10	
2	64	-58	6	73	77	1	55	-50	7	79	-82	4	39	-41	7	56	-58	12	32	-30				
4	24	14	7	124	121	2	32	-21	9	21	-11	5	30	32	8	50	44	13	90	-89	0	57	56	
5	107	102	8	81	76	5	34	-27	10	22	-6	6	32	31	10	21	-16	0	k	9	2	98	-104	
6	97	100	12	37	30	9	47	-42	1	26	-17	7	20	-18	11	27	-27	0	20	17	3	134	133	
7	59	-11	14	27	26	10	228	228	12	16	10	12	17	15	26	26	9	82	80	4	132	130		
9	37	31	14	60	-57	11	139	135	15	112	115	15	40	39	16	51	-20	1	23	12	5	37	23	
12	31	25	15	80	83	12	34	31	14	143	148	14	85	85	17	78	80	2	41	42	6	163	171	
14	83	-79	17	148	-153	15	25	14	2	41	-42	1	24	19	1	29	-22	3	17	14	7	132	130	
15	133	-133	18	118	-111	17	21	16	2	k	8	4	28	-18	4	17	-16	5	32	-30	8	97	100	
16	61	-65	19	59	-58	18	23	17	0	132	-142	-6	k	9	-3	k	9	6	36	41	9	39	-16	
17	98	95	21	20	8	19	24	7	1	129	145	0	85	-80	0	141	146	7	26	25	-3	k	10	
18	56	-55	22	29	-25	20	107	-113	2	41	-42	1	24	19	1	29	-22	1	25	31				
21	22	-16	23	15	-9	0	k	8	5	40	46	3	28	-29	4	30	34	-7	k	10	1	25	26	
22	33	-32	-2	k	8	0	67	68	5	52	-59	5	25	27	6	36	-30	1	47	48	3	41	-42	
-4	k	8	0	88	-86	1	71	-65	6	18	-9	7	32	-36	8	30	34	2	36	-33	4	91	-94	
0	525	525	1	30	-16	2	70	-69	7	18	12	9	28	-28	8	86	-84	4	61	61	5	114	-116	
1	105	66	2	67	59	3	174	172	4	200	-200	-9	k	9	11	28	-30	14	21	-22	6	15	-12	
3	20	20	3	4	244	242	5	192	193	9	17	-18	14	34	37	16	32	30	-6	k	10			
4	51	-30	6	206	-207	7	71	62	2	15	-8	-8	-5	k	9	2	23	20	1	54	-52			
6	27	19	5	90	83	6	109	203	1	18	-12	15	18	-8	-2	k	9	0	68	-58				
7	28	13	6	206	-207	7	71	62	2	15	-8	-8	-2	k	9	2	23	20	1	54	-52			
8	26	25	7	195	191	8	50	50	3	24	28	-5	k	9	2	23	20	1	54	-52				
9	103	101	8	140	-139	9	95	-94	4	27	28	0	34	-33	4	26	29	2	18	12				

The least-squares calculations converged properly after four cycles to give the results: 57.4(1.0) electrons and 88(2)% for Pb(2) and Pb(12), respectively ($R = 9.7\%$). If we assume that the Pb(2) site is fully occupied by Pb and As, the obtained number of electrons corresponds to the distribution, 0.50(2) Pb + 0.50(2) As⁴.

Anisotropic refinement

Anisotropic temperature factors were then introduced to all 25 atoms. After several cycles, the R value converged to 7.0%. The final atomic coordinates and the thermal parameters are given in Tables 3 and 4, respectively. The observed and calculated structure factors are compared in Table 5.

⁴ The almost exact half-and-half distribution of Pb and As may be an indication that the correct space group of jordanite is not $P2_1/m$ but $P2_1$, and the Pb and As atoms of the Pb(2) site occupy crystallographically independent sites. However, three cycles of least-squares refinement with the space group $P2_1$, in which the Pb(2) site was divided into Pb(2) and As(2) (altogether 50 independent atoms), led to essentially the same structure as obtained with $P2_1/m$; although the isotropic temperature factors of Pb(2) and As(2) converged to reasonable values (1.8 and 1.7 Å², respectively), the other atomic parameters of the structure changed very little.

Discussion of the structure

Unit-cell content

The distribution of Pb, As and S atoms in the unit cell of jordanite is given in Table 6. It should be noted that the following five sites on the special positions (the mirror plane) have multiplicity of 2: Pb(9), Pb(10), Pb(12), S(12) and S(13). As is evident from the table, the unit cell of jordanite contains 40 metal and 46 sulfur atom sites; the 40 metal sites are occupied by 27.8 Pb and 12.0 As atoms. If we assume the balance of valency, the number of S atoms should be 45.8, although it was not experimentally confirmed. Thus, the unit-cell content of jordanite is $\text{Pb}_{27.8}\text{As}_{12.0}\text{S}_{45.8}$ with the ideal formula $\text{Pb}_{28}\text{As}_{12}\text{S}_{46}$. These formulae are in fairly good agreement with that of ROLAND (1968), $\text{Pb}_{28-x}\text{As}_{12}\text{S}_{46-x}$ ($0.8 < x < 1.4$), for synthetic jordanite.

The calculated density of jordanite with the formula $\text{Pb}_{27.8}\text{As}_{12.0}\text{S}_{45.8}$ is $6.34 \text{ g} \cdot \text{cm}^{-3}$, in agreement with the observed densities for natural material: 6.38(1) (FISHER, 1940), 6.44 (PEACOCK and BERRY, 1940), 6.32 (PALACHE *et al.*, 1938) and $6.413 \text{ g} \cdot \text{cm}^{-3}$ (SOLLY, 1900). It is, however, significantly higher than $6.15(3) \text{ g} \cdot \text{cm}^{-3}$ reported by ROLAND (1968) for synthetic jordanite. The difference between the higher values and the lower one is probably not caused by the experimental errors,

Table 6. Distribution of atoms in the unit cell of jordanite

Site	Pb	As	Site	S
Pb(1)	4		S(1)	4
Pb(2) ¹	2.0	2.0	S(2)	4
Pb(3)	4		S(3)	4
As(4)		4	S(4)	4
Pb(5)	4		S(5)	4
As(6)		4	S(6)	4
Pb(7)	4		S(7)	4
Pb(8)	4		S(8)	4
Pb(9)	2		S(9)	4
Pb(10)	2		S(10)	4
As(11) ²		2	S(11) ²	2
Pb(12) ³	1.8		S(12)	2
			S(13)	2
Total	27.8	12.0		46

¹ The statistically occupied site.

² The split-atom sites; $0.5 \times 4 = 2$.

³ The fractionally occupied site; $0.88 \times 2 = 1.76$.

Table 7. *Chemical composition* (weight percent) of jordanite
All results are adjusted to 100% total

		Pb	As	S	Sb
I	$\text{Pb}_{27.8}\text{As}_{12.0}\text{S}_{45.8}$ (Present work)	70.87	11.06	18.07	—
II	Chemical analysis (SOLLY, 1900) no. 29	69.22	12.42	18.36	—
III	Chemical analysis (SOLLY, 1900) no. 30	69.03	12.50	18.47	—
IV*	Chemical analysis (NOWACKI and BAHEZRE, 1963)	67.06	12.58	19.10	1.25
V	Microprobe analysis (NOWACKI and BAHEZRE, 1963)	66.8	11.7	21.5	—

* The water content (0.83%) was ignored.

but is an indication of the real difference in the chemical composition of natural and synthetic jordanite.

In Table 7, the calculated composition of formula $\text{Pb}_{27.8}\text{As}_{12.0}\text{S}_{45.8}$ is compared with the results of chemical and microprobe analyses. It should be noted that the calculated composition (I) gives significantly lower As content than the chemically obtained results (II, III and IV); the As content of I is in better agreement with that of the microprobe analysis (V). The above difference in As content is reflected on the number of As atoms in the unit cell; since most of the older formulae have been derived from the results of the chemical analyses (II and III), they contain 14 As atoms, whereas the present formula and that of ROLAND (1968) contain only 12 As atoms.

This point must be clarified by future work. On one hand, the difference may be attributed to the experimental errors in the chemical analyses. On the other hand, it may have been caused by some sys-

Fig. 1. Atomic arrangements in jordanite, viewed along the b axis: (a) the first metal layer ($y \approx 0.05$), (b) the second metal layer ($y \approx 0.15$) and (c) the third metal-sulfur mixed layer ($y \approx 0.25$). The Pb(2) site is statistically occupied by 0.50 Pb + 0.50 As, the Pb(12) site is fractionally occupied by 0.88 Pb, and the As(11) and S(11) sites are split into two halves above and below the mirror plane at $y = 1/4$

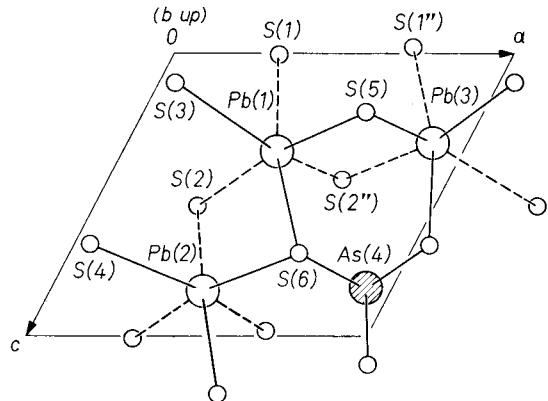


Fig. 1 a

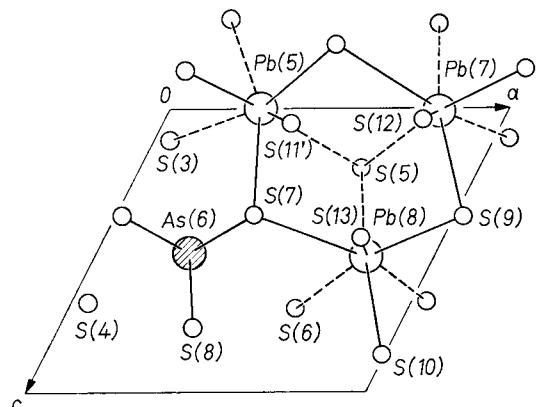


Fig. 1 b

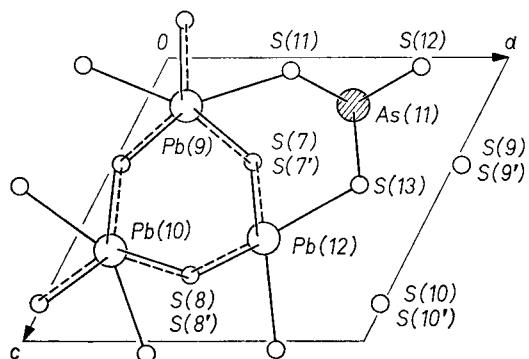


Fig. 1 c

matic errors in the present analysis; for example, it is difficult to conclude definitely that the Pb(12) site is never occupied by As, although the site was assumed to be fractionally occupied by only Pb from the coordination of S atoms.

However, in spite of the possible ambiguities about the exact distribution of Pb and As on the 40 metal sites, the present analysis has clearly shown that most of the older formulae which contain more than 40 metal atoms and/or more than 46 sulfur atoms are improbable.

Description of the structure

The unit cell of jordanite consists of ten metal layers along the *b* axis; the ten metal layers are interleaved by ten sulfur layers. The atomic arrangements in the asymmetric unit ($y = 0-1/4$) are shown in Fig. 1. The twelve independent metal sites are distributed on the three layers at $y \approx 0.05$, ≈ 0.15 and ≈ 0.25 (the mirror plane) (Fig. 1a, b and c, respectively). There are four metal sites, 3 Pb + As, on each layer. Another one-fourth unit cell ($y = 1/4-1/2$) is related to the asymmetric unit by the mirror plane at $y = 1/4$; the remaining half cell ($y = 1/2-1$) is related to the other half by the center of inversion and also by the screw diad along the *b* axis.

Some of the atomic sites have statistical nature (Table 6). The Pb(2) site on the first metal layer is occupied by 0.50 Pb and 0.50 As. The Pb(12) site on the third metal layer is fractionally (88%) occupied by Pb. On the same layer, the As(11) site is split into two halves, 0.75 Å above and below the mirror plane; instead, three additional S atoms, S(11) to S(13), are approximately on the mirror plane and coordinate to the split As atoms⁵. The S(11) site is also split into two halves, 0.37 Å above and below the mirror plane.

As was already pointed out by WUENSCH and NOWACKI (1966), the structure of jordanite within the asymmetric unit ($y = 0-1/4$) can be correlated to the PbS structure by taking the *b* axis of jordanite parallel to the [111] axis of PbS (Table 1). However, the displacements of the atoms from the ideal PbS-type structure, especially those of the S atoms, are so large (Table 8), that the sulfur coordinations about the metal atoms are essentially different from those in PbS.

The displacements of the metal atoms are approximately 0.3 Å. However, that of the split As(11) atom is exceptionally large; the atom is displaced along the *b* axis by 0.75 Å, almost one half (0.80 Å)

⁵ In fact, the third metal layer is a metal-sulfur mixed layer.

Table 8. Displacements of the atoms of jordanite from the ideal PbS-type structure

Layer number	Atom	Ideal coordinates			Displacements (\AA)			
		x/a	y/b	z/c	Δx	Δy	Δz	Δr
1st metal	Pb(1)	5/12	1/20	2/6	+ 0.270	+ 0.149	+ 0.144	0.282
	Pb(2)	5/12	1/20	5/6	+ 0.201	+ 0.062	+ 0.075	0.189
	Pb(3)	11/12	1/20	2/6	- 0.112	- 0.281	- 0.140	0.311
	As(4)	11/12	1/20	5/6	+ 0.025	+ 0.082	- 0.029	0.094
2nd metal	Pb(5)	3/12	3/20	0	+ 0.132	- 0.247	- 0.023	0.286
	As(6)	3/12	3/20	3/6	+ 0.230	- 0.228	+ 0.069	0.308
	Pb(7)	9/12	3/20	0	+ 0.360	+ 0.023	+ 0.149	0.320
	Pb(8)	9/12	3/20	3/6	+ 0.379	+ 0.007	+ 0.126	0.339
3rd metal*	Pb(9)	1/12	5/20	1/6	+ 0.347	0	+ 0.040	0.330
	Pb(10)	1/12	5/20	4/6	+ 0.340	0	+ 0.194	0.303
	As(11)	7/12	5/20	1/6	+ 0.290	- 0.751	+ 0.084	0.795
	As(11')	7/12	5/20	1/6	+ 0.290	+ 0.751	+ 0.084	0.795
	Pb(12)	7/12	5/20	4/6	- 0.294	0	- 0.230	0.276
1st sulfur	S(1)	3/12	0	0	+ 0.531	+ 0.099	+ 0.104	0.501
	S(2)	3/12	0	3/6	+ 0.506	+ 0.161	+ 0.427	0.513
2nd sulfur	S(3)	1/12	2/20	1/6	- 0.333	- 0.440	- 0.560	0.666
	S(4)	1/12	2/20	4/6	- 0.287	- 0.470	+ 0.167	0.613
	S(5)	7/12	2/20	1/6	+ 0.613	- 0.099	- 0.306	0.552
	S(6)	7/12	2/20	4/6	+ 0.802	- 0.286	+ 0.344	0.766
3rd sulfur	S(7)	5/12	4/20	2/6	- 0.094	- 0.744	+ 0.347	0.845
	S(8)	5/12	4/20	5/6	- 0.130	- 0.722	- 0.536	0.872
	S(9)	11/12	4/20	2/6	+ 1.017	- 0.666	+ 0.427	1.120
	S(10)	11/12	4/20	5/6	+ 0.689	- 0.208	+ 0.300	0.644
4th sulfur*	S(11)	3/12	6/20	0	+ 1.144	- 1.220	+ 0.467	1.587
	S(11')	3/12	6/20	0	+ 1.144	- 1.969	+ 0.467	2.215
	S(12)	9/12	6/20	0	+ 0.018	- 1.595	+ 0.266	1.616
	S(13)	9/12	6/20	3/6	+ 0.018	- 1.595	- 0.385	1.643

* 3rd metal and 4th sulfur layers are mixed up to form a metal-sulfur mixed layer.

of the average metal-sulfur interlayer separation. The displacements of the S atoms along the b axis are generally systematic; the magnitudes increase with increasing layer numbers. In fact, the fourth sulfur layer is displaced as much as one interlayer separation (1.59 \AA), and, there-

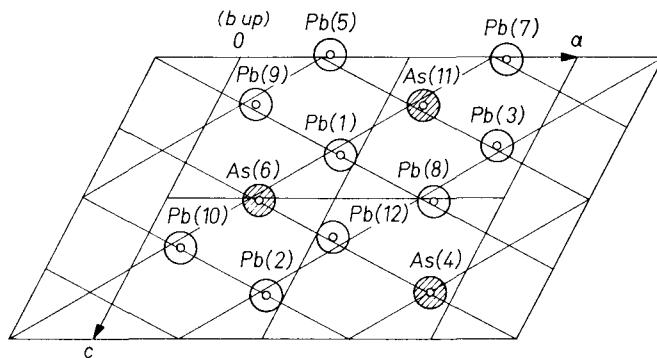


Fig. 2a

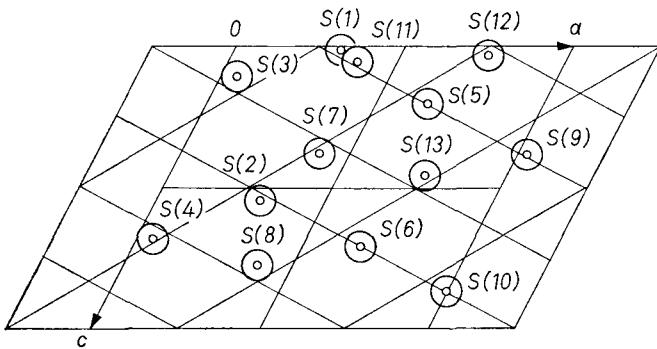


Fig. 2b

Fig. 2. Displacements of (a) the metal and (b) the sulfur atoms in jordanite from the ideal PbS -type structure

fore, is mixed up with the third metal layer to form the metal-sulfur mixed layer. The displacements of the atoms perpendicular to the b axis can be seen from Fig. 2; most of the atoms are displaced along the [201] direction.

If we denote the idealized atomic arrangements in the first, second and third sulfur layers as a , b and c , and those in the metal layers as B , A and C , respectively (in an analogous way as in the face-centered cubic packing scheme), the stacking sequence along the b axis of jordanite is given by

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	B	c	A	b	C'	b	A	c	B	a	C	b	A	c	B'	c	A	b	C

The two layers, C' and B' , are on the mirror planes of the structure; the primes are added because these layers are not pure metal but metal-sulfur mixed layers. If the above sequence is compared with that in the PbS-type structure, $|aBcAbC|$, it is evident that the correspondence

Table 9. Bond distances in jordanite

Notation of the symmetry operations: single primed: mirror reflection; double primed: inversion. The estimated standard deviations are about 0.008 and 0.010 Å for metal-sulfur and sulfur-sulfur distances, respectively

Pb(1)–S(2'')	2.728 Å	Pb(7)–S(3)	2.903 Å
–S(5)	2.975	–S(5)	2.954
–S(6)	2.976	–S(9)	2.971
–S(2)	2.984	–S(10)	2.979
–S(1)	3.020	–S(6)	3.008
Mean(5)	(2.937)	Mean(5)	(2.963)
–S(3)	3.349	–S(12)	3.194
Mean(6)	(3.005)	–S(8)	3.216
		Mean(7)	(3.032)
Pb(2)–S(2)	2.647	Pb(8)–S(5)	2.889
–S(1'')	2.655	–S(4)	2.932
–S(1)	2.718	–S(9)	2.965
–S(6)	3.072	–S(6)	2.993
–S(5)	3.094	–S(10)	3.014
Mean(5)	(2.837)	Mean(5)	(2.959)
–S(4)	3.255	–S(7)	3.190
Mean(6)	(2.907)	–S(13)	3.217
Pb(3)–S(5)	2.668	Mean(7)	(3.029)
–S(1'')	2.904	Pb(9)–S(10)	2.895
–S(2'')	2.991	–S(10')	2.895
–S(3)	3.028	–S(11)	2.901
–S(4)	3.107	–S(12)	2.942
Mean(5)	(2.940)	Mean(4)	(2.908)
–S(2)	3.393	–S(9)	3.222
Mean(6)	(3.015)	–S(9')	3.222
Pb(5)–S(10)	2.690 Å	–S(7)	3.283
–S(8)	2.941	–S(7')	3.283
–S(4)	3.030	Mean(4)	(3.253)
–S(7)	3.038	Mean(8)	(3.080)
–S(3)	3.059		
Mean(5)	(2.952)		
–S(11')	3.191		
–S(5)	3.383		
Mean(7)	(3.047)		

Table 9. (*Continued*)

Pb(10)—S(11)	2.889 Å	As(6)—S(7)	2.242 Å
—S(10)	2.906	—S(8)	2.250
—S(10')	2.906	—S(9)	2.253
—S(13)	2.949	Mean(3)	(2.248)
Mean(4)	(2.913)	S(7)—S(8)	3.364
—S(8)	3.241	S(7)—S(9)	3.387
—S(8')	3.241	S(8)—S(9)	3.399
—S(9)	3.242	Mean(3)	(3.383)
—S(9')	3.242		
Mean(4)	(3.242)	As(11)—S(11)	2.186
Mean(8)	(3.077)	—S(13)	2.215
		—S(12)	2.222
Pb(12)—S(13)	2.882	Mean(3)	(2.208)
—S(12)	2.945	S(11)—S(12)	3.451
—S(7)	3.077	S(11)—S(13)	3.488
—S(7')	3.077	S(12)—S(13)	3.580
—S(8)	3.114	Mean(3)	(3.506)
—S(8')	3.114	As(11)—As(11')	1.502
Mean(6)	(3.035)	As(11)—S(11')	1.912
—S(10)	3.938		
—S(10')	3.938	Pb(2)—S(2)	2.647
		—S(1'')	2.655
As(4)—S(3)	2.293	—S(1)	2.718
—S(4)	2.297	Mean(3)	(2.673)
—S(6)	2.297	S(1)—S(1'')	3.505
Mean(3)	(2.296)	S(2)—S(1'')	3.696
S(3)—S(6)	3.439	S(1)—S(2)	3.898
S(4)—S(6)	3.461	Mean(3)	(3.700)
S(3)—S(4)	3.526		
Mean(3)	(3.475)		

between the two structures is limited only within the first six layers, because the mirror planes at $y = 1/4$ and $3/4$ in the former are absent in the latter.

Sulfur coordinations about the metal atoms

The sulfur coordinations about the metal atoms in jordanite can be seen from Fig. 1. The bond distances and angles are given in Tables 9 and 10, respectively.

The fully occupied Pb atoms in the first metal layer, Pb(1) and Pb(3), are coordinated with six S atoms in distorted octahedral arrangements with an average Pb—S distance of 3.01 Å. The statistically occupied Pb(2) site (0.50 Pb + 0.50 As) is also coordinated with

six sulfur atoms. However, the coordination is an intermediate one between typical coordination of Pb and As; three Pb—S distances are short (average 2.67 Å) and the other three are long (average 3.14 Å). The large mean-square amplitude of vibration of the site, 0.0469 Å² along the *b* axis (Table 4), is probably an indication of positional disorder of the site; the true coordinations of the 0.50Pb and 0.50As atoms are probably more Pb and As like, respectively, than the averaged coordination given in Tables 9 and 10.

Table 10. *Bond angles in jordanite*

The estimated standard deviations are about 0.3 degrees. For the Pb atoms, only the angles between the four or five shortest Pb—S bonds are given

S(2'')—Pb(1)—S(5)	83.2°	S(10)—Pb(5)—S(8)	94.2°
—S(6)	83.8	—S(4)	86.0
—S(2)	86.9	—S(7)	93.4
—S(1)	79.9	—S(3)	84.2
S(5) —	89.5	S(8) —	—S(4) 86.9
—S(2)	168.6	—S(7)	117.0
—S(1)	84.6	—S(3)	157.7
S(6) —	84.1	S(4) —	—S(7) 156.1
—S(1)	163.2	—S(3)	70.8
S(2) —	—S(1)	S(7) —	—S(3) 85.3
S(2) —Pb(2)—S(1'')	88.4°	S(3) —Pb(7)—S(5)	86.3°
—S(1)	93.2	—S(9)	82.0
—S(6)	88.2	—S(10)	82.1
—S(5)	169.6	—S(6)	71.1
S(1'') —	—S(1)	S(5) —	—S(9) 84.3
—S(6)	82.4	—S(10)	167.9
—S(5)	81.5	—S(6)	92.0
S(1) —	—S(6)	S(9) —	—S(10) 90.9
—S(5)	87.7	—S(6)	153.1
S(6) —	—S(5)	S(10) —	—S(6) 87.3
S(5) —Pb(3)—S(1'')	85.0°	S(5) —Pb(8)—S(4)	86.7°
—S(2'')	83.9	—S(9)	85.5
—S(3)	89.1	—S(6)	90.8
—S(4)	87.2	—S(10)	168.9
S(1'') —	—S(2'')	S(4) —	—S(9) 82.5
—S(3)	85.6	—S(6)	71.5
—S(4)	165.0	—S(10)	82.2
S(2'') —	—S(3)	S(9) —	—S(6) 153.9
—S(4)	83.6	—S(10)	91.7
S(3) —	—S(4)	S(6) —	—S(10) 87.0

Table 10. (*Continued*)

S(10) — Pb(9) — S(10')	77.0°	S(3) — As(4) — S(4)	100.4°
—S(11)	80.2	—S(6)	97.0
—S(12)	74.8	S(4) — —S(6)	97.8
S(10') — —S(11)	70.7	Mean(3)	(98.4)
—S(12)	74.8		
S(11) — —S(12)	140.9	S(7) — As(6) — S(8)	97.0°
S(11) — Pb(10) — S(10)	80.2°	S(8) — —S(9)	97.8
—S(10')	70.7	Mean(3)	(97.6)
—S(13)	143.2		
S(10) — —S(10')	76.7	S(11) — As(11) — S(13)	104.8°
—S(13)	76.5	—S(12)	103.1
S(10') — —S(13)	76.5	S(13) — —S(12)	107.6
		Mean(3)	(105.2)
S(13) — Pb(12) — S(12)	113.8°		
—S(7)	76.1		
—S(7')	76.1		
S(12) — —S(7)	130.5		
—S(7')	130.5		
S(7) — —S(7')	99.0		

The Pb atoms in the second metal layer, Pb(5), Pb(7) and Pb(8), are coordinated with seven S atoms with an average Pb—S distance of 3.04 Å; in addition to the six distorted octahedral S atoms, the seventh S atom, S(11')⁶, S(12) or S(13), respectively, comes into the coordination sphere.

The fully occupied Pb-atom sites in the third metal layer (the metal-sulfur mixed layer on the mirror plane), Pb(9) and Pb(10), are coordinated with eight S atoms with an average Pb—S distance of 3.08 Å. Six of the eight S atoms around each Pb atom occupy the corners of a distorted trigonal prism along the *b* axis, whereas the other two S atoms belonging to the same metal-sulfur mixed layer coordinate to the Pb atom through two sides of the prism. The fractionally occupied Pb(12) atom is coordinated with six S atoms with an average Pb—S distance of 3.04 Å; two other S atoms, S(10) and S(10'), are far apart (3.94 Å) from Pb(12) (Fig. 1e).

The sulfur coordinations about the As atoms are ordinary trigonal pyramidal. The average As—S distances are 2.296, 2.248 and 2.208 Å for As(4), As(6) and As(11)⁷, respectively.

⁶ The other half atom S(11) is much apart (3.915 Å) from Pb(5).

⁷ Between the two possible bonds, As(11)—S(11) or As(11)—S(11'), the former (2.186 Å) was considered to be the real bond, because the latter (1.912 Å) was too short for an As—S bond (see, for example, Nowacki, 1969).

The AsS_3 pyramids are isolated from each other in the structure; even the half-occupied $\text{Pb}(2)\text{S}_3$ pyramid does not share S atoms with the other AsS_3 pyramids (type I.c₁ of the classification of NOWACKI, 1969).

Relation of jordanite and gratonite

From heating experiments on natural gratonite, ROLAND (1968) concluded that gratonite is most probably a low-temperature dimorph of jordanite. On the other hand, the unit cell content of jordanite obtained in this study, $\text{Pb}_{27.8}\text{As}_{12.0}\text{S}_{45.8}$, is appreciably different from that of gratonite ($3\text{Pb}_9\text{As}_4\text{S}_{15} = \text{Pb}_{27}\text{As}_{12}\text{S}_{45}$); jordanite has 0.8 PbS in excess of gratonite. However, because of the statistical nature of the atomic distributions in jordanite, the difference does not seem to be so definite as to rule out the possible dimorphism of the two minerals. For example, if we assume that the $\text{Pb}(12)$ site is occupied by 0.50 Pb in synthetic jordanite (instead of 0.88 Pb as found for natural material in the present work), the unit-cell content will be exactly the same as that of gratonite.

The crystal structure of gratonite has already been established (RIBÁR and NOWACKI, 1969; RÖSCH, 1963). The crystal is rhombohedral, space group $R\bar{3}m$, with $a_{\text{hex}} = 17.758(14)$ and $c_{\text{hex}} = 7.807(6)$ Å. There are no statistical sites in the structure.

It seems hardly possible to find any simple relationship between the atomic arrangements in jordanite and gratonite. As far as the metal atoms are concerned, the structure of gratonite can also be correlated to the PbS structure, when the c axis of gratonite is taken as the [111] axis of PbS⁸. However, the correspondence is only formal.

Table 11
Displacements of the metal atoms of gratonite from the ideal PbS-type structure

Atom	Ideal coordinates			Displacements (RIBÁR and NOWACKI, 1969)			
	x/a	y/b	z/c	Δx	Δy	Δz	Δr
Pb(1)	3/4	3/4	0	+ 0.451 Å	+ 0.114 Å	+ 0.003 Å	0.406 Å
Pb(2)	3/4	1/4	0	+ 0.069	- 0.069	+ 1.234	1.240
As(1)	1/2	1/2	0	+ 0.400	- 0.400	+ 0.920	1.152
As(2)	0	0	0	0	0	- 0.444	0.444

⁸ In fact, the approximate structure of gratonite was first solved by RÖSCH (1963) starting from the ideal PbS-type arrangements of the metal atoms; afterwards, one of the two As positions and all S positions have been corrected and the whole structure has been refined by RIBÁR and NOWACKI (1969).

The ideal structure of gratonite itself is very much deformed from the PbS structure; the average separation of the metal layers ($c/3 = 2.60 \text{ \AA}$) is considerably shorter than those in PbS ($[111]/3 = 3.42 \text{ \AA}$) and jordanite ($b/10 = 3.19 \text{ \AA}$). Moreover, the displacements of the metal atoms from the ideal coordinates are much larger (Table 11) than those in jordanite (Table 8). The sulfur positions in gratonite are completely different from the ideal positions; they can hardly be correlated to those in PbS or jordanite. Thus, it is evident that considerable rearrangements of both the metal and sulfur atoms should take place in the transition from gratonite to jordanite.

Calculations

The main part of the numerical calculations was performed on the Bull-Gamma 30 S computer and the IBM 370/155 computer at the Rechenzentrum der Universität Bern with the program system, "Kristallographische Programme, 1970 and 1972" written by P. ENGEL (Bern). The lattice-parameter refinement, key shifts and site-refinement programs used were written by one of the authors (T.I.).

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