

The crystal structure of, and the bismuth-copper distribution in synthetic cuprobismuthite

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Auszug

Die Kristallstruktur von synthetischem Cuprobismuthit wurde mittels dreidimensionaler Röntgendaten bestimmt. Die Raumgruppe ist $C2/m$ mit $a = 17,520(1)$, $b = 3,926(3)$, $c = 15,261(1)$ Å und $\beta = 100,18(1)^\circ$. Die sich aus der Kristallstrukturanalyse ergebende chemische Formel kann als $(Cu_x, Bi_{\frac{1}{2}(1-x)})(Cu_y, Bi_{1-y})Cu_{xy}, Cu_2Bi_2S_6$ [$x = 0,32$, $y = 0,20$], $Z = 4$ dargestellt werden. Least-squares-Verfeinerungen mit anisotropen Temperaturfaktoren ergaben einen R -Wert von 0,049 für die 997 beobachteten Reflexe (0,054 für alle 1109 Reflexe).

Alle Atome liegen in den Spiegelebenen der Raumgruppe bei $y = 0$ und $\frac{1}{2}$. Die Struktur entspricht wesentlich einer Schicht von Hodrushit, (Pb?, Ag?, Fe, Bi) $Cu_4Bi_5S_{11}$. Zwei Bi-Lagen mit oktaedrischer Koordination gegenüber Schwefel sind teilweise durch Kupfer besetzt. Diese Substitutionen können durch die Valenzregel befriedigend erklärt werden. Die vorliegenden Ergebnisse scheinen nicht darauf hinzuweisen, daß Cuprobismuthit und Emplektit dimorphe Formen sind.

Abstract

The crystal structure of synthetic cuprobismuthite has been determined using three-dimensional x-ray diffraction data. The space group is $C2/m$ with $a = 17.520(1)$, $b = 3.926(3)$, $c = 15.261(1)$ Å and $\beta = 100.18(1)^\circ$. The chemical formula derived from the crystal structure analysis is represented as $(Cu_x, Bi_{\frac{1}{2}(1-x)})(Cu_y, Bi_{1-y})Cu_{xy}, Cu_2Bi_2S_6$ ($x = 0.32$, $y = 0.20$), $Z = 4$. Least-squares refinement with anisotropic temperature factors resulted in a conven-

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tional R index of 0.049 for the observed 997 reflections (0.054 for all the 1109 reflections).

All atoms lie on mirror planes of the space group at $y = 0$ and $\frac{1}{2}$. The structure corresponds essentially to one layer in hodrushite, (Pb?, Ag?, Fe, Bi) $\text{Cu}_4\text{Bi}_5\text{S}_{11}$. Two bismuth sites in octahedral coordination with sulfur are partially substituted by copper atoms. These substitutions can be reasonably explained by the valence rule. The present results do not seem to indicate that cuprobismuthite is dimorphous with emplectite.

Introduction and brief summary of previous studies

The minerals of the system copper—bismuth—sulfur, which have been structurally investigated are wittichenite, $3\text{Cu}_2\text{S} \cdot \text{Bi}_2\text{S}_3$ (MATZAT, 1972; KOCMAN and NUFFIELD, 1973) and emplectite, $\text{Cu}_2\text{S} \cdot \text{Bi}_2\text{S}_3$ (HOFMANN, 1933; KUPČÍK, 1965; PORTHEINE and NOWACKI, 1975). Moreover, the validity of cuprobismuthite as a distinct species has been established by some workers.

HILLEBRAND (1884) reported cuprobismuthite from the Missouri mine, Halls Valley, Park County, Colorado as a probable new mineral and proposed the formula $3(\text{Cu, Ag})_2\text{S} \cdot 4\text{Bi}_2\text{S}_3$. His data were listed in DANA's System of mineralogy (DANA, 1892). Later two mineralogists, SHORT (1931) and PALACHE (1940), much questioned the validity of this mineral based on studies of the specimen from the type locality. NUFFIELD (1952), however, reexamined the type specimen of cuprobismuthite which had been studied by PALACHE, by x-ray diffraction techniques. He found a crystal which gave a new diffraction pattern and confirmed that the crystal had a monoclinic cell with space group $C2/m$. Although his attempts to determine the exact composition of cuprobismuthite were not successful, he could synthesize a nearly homogeneous phase with proportion of $\text{Cu}/\text{Bi} \sim 1$ whose x-ray powder pattern was identical with that of the natural cuprobismuthite specimen. In addition, he could measure the density of his product. The value was almost identical with that of emplectite. After all, he mentioned that it was not unreasonable to suggest that the chemical formula was $\text{Cu}_2\text{S} \cdot \text{Bi}_2\text{S}_3$ and that cuprobismuthite was therefore dimorphous with emplectite, from the intimate relations of the unit cell dimensions, cell volumes, densities and chemical composition between them. The final proof of the composition of cuprobismuthite was, however, left as a subject for future studies.

Recently some experimental investigations on the phase relations of Cu_2S — Bi_2S_3 system have been made. GODOVIKOV and FEDOROVA (1969) analysed emplectite thermographically and obtained the result

that the chemical formula for cuprobismuthite was represented approximately as $3\text{Cu}_2\text{S} \cdot 4\text{Bi}_2\text{S}_3$ or $\text{Cu}_6\text{Bi}_8\text{S}_{15}$ which agreed with the formula proposed by HILLEBRAND (1884). GODOVIKOV and PTITSIN (1969), on the other hand, suggested on the basis of their synthetic studies that the formula for cuprobismuthite was close to $\text{Cu}_{10}\text{Bi}_{12}\text{S}_{23}$. Both formulae proposed by SUGAKI and SHIMA (1970, 1972), $\text{Cu}_{24}\text{Bi}_{26}\text{S}_{51}$, and by BUHLMANN (1971) were also deficient in copper content as compared with emplectite. TAYLOR, RADTKE and CHRIST (1973) reported similar data on the composition of cuprobismuthite from Tunnel Extension Number Two Mine, Ohio mining district, Utah. From the results of their electron microprobe analyses they concluded that the chemical formula for unsubstituted cuprobismuthite is $5\text{Cu}_2\text{S} \cdot 6\text{Bi}_2\text{S}_3$ or $\text{Cu}_{10}\text{Bi}_{12}\text{S}_{23}$ which agreed closely with the experimental results of GODOVIKOV and PTITSIN (1969). In spite of the excellent agreement between the calculated density and measured one of their specimens, a unit-cell content of 23 sulfur atoms was not compatible with the requirements of the possible space groups, $C2/m$, Cm and $C2$, as indicated by them. On the other hand, CHEN and CHANG (1974) described that cuprobismuthite had a solid solution ranging from 52 to 47 mol % Cu_2S and 51 to 48 mol % Cu_2S along the join $\text{Cu}_2\text{S}-\text{Bi}_2\text{S}_3$ at 454° and 300°C, respectively. Their results suggest that cuprobismuthite is stable at higher temperatures and dimorphous with emplectite.

The stability range of cuprobismuthite varies according to authors. For example, the lower temperature limit of stability is 383°C according to GODOVIKOV and FEDOROVA (1969), but 290°C according to BUHLMANN (1971).

The crystal structure of cuprobismuthite was presumed from that of hodrushite, $(\text{Pb}?, \text{Ag}?, \text{Fe}, \text{Bi})\text{Cu}_4\text{Bi}_5\text{S}_{11}$ (KUPČÍK and MAKOVICKÝ, 1968; KODĚRA, KUPČÍK and MAKOVICKÝ, 1970; MAKOVICKÝ and MACLEAN, 1972). The structure of hodrushite consists of two distinct layers, one of which has the unit-cell dimensions of cuprobismuthite. KODĚRA *et al.* (1970) suggested that the layer of hodrushite must correspond to the structure of cuprobismuthite. Present study confirmed the suggestion to be correct in principle. The proposed structure using powder data (MARIOLACOS, KUPČÍK, OHMASA and MIEHE, 1975) was also essentially correct.

The present structure determination was undertaken in order to elucidate the chemical composition and the relationship with emplectite and hodrushite.

Experimental

Synthetic crystals of cuprobismuthite were kindly supplied by Dr. A. A. GODOVIKOV for the present investigation. A zero-level back-reflection Weissenberg photograph about [010] (camera radius 57.3mm) with copper radiation, taken for precise determination of the unit-cell dimensions, was calibrated by the superposition of a powder pattern of silicon. Least-squares methods (T. ITO, unpublished) using 21 measured $h0l$ reflections with θ between 63° and 79° gave the following monoclinic cell dimensions (a , c and β). The b axis was determined with the aid of a Weissenberg diffractometer:

$$a = 17.520(1), \quad b = 3.926(3), \quad c = 15.261(1) \text{ \AA}, \quad \beta = 101.18(1)^\circ.$$

Systematic absences of hkl for $h+k$ odd and $00l$ for l odd indicate that the possible space groups are $C2/m$, Cm and $C2$. Crystals are usually of a slender thin prismatic shape. The longest dimension is along [010], and (001) is the most developed face as described by NUFFIELD (1952). One crystal with dimensions $0.056 \times 0.18 \times 0.020$ mm was selected and mounted for rotation about the [010] axis. A total of 1109 independent reflections were collected on a Supper-Pace single-crystal automatic diffractometer by ω -scan methods using Ni-filtered $\text{CuK}\alpha$ radiation. The intensities were corrected for Lorentz, polarization and absorption effects using the programme of ACACA (WUENSCH and PREWITT, 1965), but not for extinction effect. Based on the counting statistics, a weight was assigned to each reflection.

Table 1. *Crystal data for cuprobismuthite*

(Estimated standard deviations are given in parentheses)

composition $\text{Cu}_{2.58}\text{Bi}_{3.14}\text{S}_6$
 space group $C2/m$, $Z = 4$, $F(000) = 1657$
 $\mu(\text{CuK}\alpha) = 1240 \text{ cm}^{-1}$

| | Missouri mine (NUFFIELD, 1952) | Tunnel Extension Number Two mine (TAYLOR <i>et al.</i> , 1973) | present work |
|----------|-----------------------------------|--|---------------------------------------|
| a | 17.65 \AA | $17.628 \pm 0.01 \text{ \AA}$ | $17.520(1) \text{ \AA}$ |
| b | 3.93 | 3.911 ± 0.002 | $3.926(3)$ |
| c | 15.24 | 15.190 ± 0.01 | $15.261(1)$ |
| β | 100.5° | $100^\circ 16'$ | $100.18(1)^\circ$ |
| ρ_0 | | $6.36 \text{ g} \cdot \text{cm}^{-3}$ | |
| ρ_c | | | $6.53 \text{ g} \cdot \text{cm}^{-3}$ |
| V | 1039 \AA^3 | 1027 \AA^3 | 1031 \AA^3 |

Reflections with I less than $2.33\sigma(I)$ were considered to be unobserved. The final data list contained 102 unobserved reflections. Some crystal data are given in Table 1.

Structure determination and refinement

The statistics of the normalized structure amplitudes, Table 2, suggests the presence of a center of symmetry. Therefore the centric space group $C2/m$ was adopted as the correct one. The intensity distribution of $h0l$ reflections is similar to those of $h2l$ and $h4l$ reflections. This characteristic applies also to the relation between $h1l$ and $h3l$ reflections. These phenomena suggest that almost all atoms should be on planes perpendicular to the b axis with $y = 0$ and $y = \frac{1}{2}$.

The Patterson map was calculated and solved. The assumption that cuprobismuthite might have a part of the structure of hodrushite was very useful for the solution. The approximate coordinates of copper and bismuth atoms except Cu(3), Cu(4) and Cu(5) were thus located. A trial of symbolic addition procedures provided us the same result. A structure-factor and a three-dimensional Fourier synthesis were then calculated with the coordinates of the metal atoms, assuming that all these sites were fully occupied. A block-diagonal least-squares programme and a Fourier programme written by Dr. T. ITO were used. The sulfur atoms were found from the Fourier map. A few cycles of three-dimensional least-squares refinement for positional parameters and isotropic temperature factors reduced the R index to 13%. At this stage, the deficiencies of Bi(1) and Bi(2) sites and an additional peak near Bi(1) site were found on a Fourier map. This peak was assigned to Cu(3), judging from the coordination

Table 2. *Distribution of E values*

| | cuprobismuthite | theoretical* | |
|-----------------------------|-----------------|--------------|----------|
| | | centric | acentric |
| $\langle E^2 \rangle$ | 1.000 | 1.000 | 1.000 |
| $\langle E^2 - 1 \rangle$ | 0.941 | 0.968 | 0.736 |
| $\langle E \rangle$ | 0.816 | 0.798 | 0.886 |
| $ E > 1.0$ | 32.0 % | 32.0 % | 36.8 % |
| $ E > 2.0$ | 4.2 % | 5.0 % | 1.8 % |
| $ E > 3.0$ | 0.1 % | 0.3 % | 0.01 % |

* G. H. STOUT and L. H. JENSEN (1968): *X-ray structure determination*, p. 321. Macmillan, New York.

Table 3. *Atomic coordinates and site occupancies*

(Estimated standard deviations are given in parentheses in terms of the last digit)

| Atom | <i>x</i> | <i>y</i> | <i>z</i> | Site occupancy* |
|-------|------------|----------|------------|-----------------|
| Cu(1) | 0.1598(4) | 0 | 0.1432(3) | |
| Cu(2) | 0.0868(2) | 0 | 0.4772(3) | |
| Cu(3) | 0.0460(9) | 0.5 | 0.0857(10) | 0.34(2) |
| Cu(4) | 0.2962(14) | 0.5 | 0.1532(14) | 0.20(2) |
| Cu(5) | 0.378(6) | 0 | 0.070(4) | 0.07(2) |
| Bi(1) | 0 | 0.5 | 0 | 0.679(8) |
| Bi(2) | 0.3334(7) | 0.5 | 0.09025(7) | 0.803(6) |
| Bi(3) | 0.49227(5) | 0 | 0.28075(5) | |
| Bi(4) | 0.24447(5) | 0.5 | 0.37089(5) | |
| S(1) | 0.1639(3) | 0.5 | 0.0610(3) | |
| S(2) | 0.4873(4) | 0.5 | 0.1245(3) | |
| S(3) | 0.3404(3) | 0 | 0.2175(3) | |
| S(4) | 0.1528(3) | 0 | 0.2887(3) | |
| S(5) | 0.4649(3) | 0.5 | 0.3867(3) | |
| S(6) | 0.1612(3) | 0.5 | 0.4931(3) | |

* All the occupancies were refined independently. Therefore the sum of Cu(3) and Bi(1), and that of Cu(4) and Bi(2) showed a slight deviation from 1.

Table 4. *Thermal parameters in the expression*

$$\exp[-(h^2\beta_{11} + k^2\beta_{22} + l^2\beta_{33} + 2hk\beta_{12} + 2hl\beta_{13} + 2kl\beta_{23})]; \beta_{12} = 0, \beta_{23} = 0$$

(Estimated standard deviations are given in parentheses in terms of the last digit)

| Atom | β_{11} | β_{22} | β_{33} | β_{13} |
|-------|--------------|--------------|--------------|--------------|
| Cu(1) | 0.0055(3) | 0.038(3) | 0.0024(1) | 0.0014(2) |
| Cu(2) | 17(1) | 33(3) | 32(1) | 4(1) |
| Cu(3) | 26(6) | 44(11) | 45(7) | 6(5) |
| Cu(4) | 24(8) | 30(16) | 35(9) | -4(7) |
| Cu(5) | 0.007(5) | -0.01(3) | 0.005(3) | 0.002(3) |
| Bi(1) | 0.0017(1) | 0.038(2) | 0.0017(1) | 0.0000(1) |
| Bi(2) | 12(1) | 33(1) | 21(1) | 1(1) |
| Bi(3) | 14(1) | 20(1) | 17(1) | 3(1) |
| Bi(4) | 11(1) | 19(1) | 15(1) | 3(1) |
| S(1) | 19(2) | 15(3) | 20(2) | 5(2) |
| S(2) | 20(2) | 22(4) | 19(2) | 5(1) |
| S(3) | 12(2) | 19(3) | 20(2) | 3(1) |
| S(4) | 12(2) | 14(3) | 15(2) | 3(1) |
| S(5) | 14(2) | 25(3) | 13(1) | 2(1) |
| S(6) | 12(2) | 27(4) | 15(2) | 2(1) |

Table 5. *Interatomic distances in cuprobismuthite*
 (Estimated standard deviations are given in parentheses in terms of the last digit)

| | | | | | |
|--------------|------------|-----|---------------|------------|-----|
| Cu(1)—S(1) | 2.338(4) Å | 2 × | Bi(3)—S(3) | 2.649(6) Å | |
| —S(4) | 2.246(6) | | —S(5) | 2.645(3) | 2 × |
| —S(2'') | 2.981(9) | | —S(2) | 3.077(4) | 2 × |
| —S(3) | 3.147(8) | | —S(4'') | 3.413(4) | 2 × |
| Cu(2)—S(5'') | 2.308(7) Å | | Bi(4)—S(4) | 2.688(4) Å | 2 × |
| —S(5''') | 2.422(6) | | —S(6) | 2.581(5) | |
| —S(6) | 2.342(4) | 2 × | —S(6'') | 3.092(4) | 2 × |
| Cu(3)—S(1) | 2.17(2) Å | | —S(3) | 3.699(4) | 2 × |
| —S(2'') | 2.35(1) | 2 × | Cu(1)—Cu(3) | 2.82(1) Å | 2 × |
| Cu(4)—S(1) | 2.47(2) Å | | —Cu(4) | 3.07(2) | 2 × |
| —S(3) | 2.26(1) | 2 × | Cu(3)—Cu(3') | 2.79(3) | |
| Cu(5)—S(1'') | 2.00(7) Å | | Cu(4)—Cu(5) | 2.87(6) | 2 × |
| —S(2) | 2.75(7) | 2 × | Cu(5)—Cu(3'') | 2.91(10) | |
| —S(3) | 2.46(7) | | —Cu(3''') | 2.95(7) | |
| Bi(1)—S(1) | 2.842(6) Å | 2 × | | | |
| —S(2'') | 2.771(4) | 4 × | | | |
| Bi(2)—S(2) | 2.644(6) Å | | | | |
| —S(3) | 2.747(4) | 2 × | | | |
| —S(1) | 2.916(6) | | | | |
| —S(1'') | 3.037(4) | 2 × | | | |

and the interatomic distances. The site occupancies of Bi(1) and Bi(2), which were calculated with a programme written by Dr. T. ITO (unpublished), converged to 0.68 and 0.80 respectively. The occupancy of Cu(3) site was assigned to [1 minus occupancy of Bi(1)], *i.e.* 0.32, because the Cu(3) and Bi(1) atoms were presumed to be statistically distributed as observed in the structure of $\text{Cu}_{2+x}\text{Bi}_{6-x}\text{S}_9$ ($x = 1.21$) (OHMASA, 1973). A few additional cycles of least-squares refinement reduced R to 9.0% (OZAWA and NOWACKI, 1975). The subsequent refinement with anisotropic temperature factors using the programme ORFLS (BUSING, MARTIN and LEVY, 1962) and the associated difference Fourier synthesis using the programme GSFFR written by Dr. M. OHMASA revealed the existence of Cu(4) and Cu(5) sites. At the final stage, all the site occupancies of Cu(3), Cu(4), Cu(5), Bi(1) and Bi(2) atoms were independently refined, but they converged to very

Table 6. *Interatomic angles in cuprobismuthite*
 (Estimated standard deviations are given in parentheses in terms of the last digit)

| | | | |
|----------------------|-----------|------------------|----------|
| S(1)—Cu(1)—S(1') | 114.2(3)° | S(3)—Bi(3)—S(5) | 86.3(1)° |
| S(1) S(4) | 122.9(1) | S(5) S(5') | 95.8(2) |
| S(1) S(2'') | 94.8(2) | S(3) S(2) | 80.9(1) |
| S(1) S(3) | 93.4(2) | S(5) S(4'') | 78.3(1) |
| S(4) S(2'') | 81.2(2) | S(5) S(4''') | 130.6(1) |
| S(4) S(3) | 83.6(2) | S(5) S(2) | 91.1(1) |
| | | S(2) S(2') | 79.3(1) |
| S(5'')—Cu(2)—S(5''') | 93.2(2)° | S(2) S(4'') | 63.9(1) |
| S(5'') S(6) | 119.6(1) | S(2) S(4''') | 107.1(1) |
| S(5''') S(6) | 101.8(2) | S(4'') S(4''') | 70.2(1) |
| S(6) S(6') | 113.9(3) | | |
| S(1)—Cu(3)—S(2'') | 121.5(3)° | S(4)—Bi(4)—S(4') | 93.8(2)° |
| S(2'') S(2''') | 113.7(7) | S(4) S(6) | 88.1(1) |
| | | S(4) S(3) | 67.7(1) |
| S(1)—Cu(4)—S(3) | 116.8(6)° | S(4) S(3') | 113.3(1) |
| S(3) S(3') | 120.4(9) | S(4) S(6'') | 92.5(1) |
| | | S(6) S(6'') | 79.5(1) |
| S(1'')—Cu(5)—S(3) | 144(5)° | S(3) S(3') | 64.1(1) |
| S(1'') S(2) | 114(2) | S(3) S(6'') | 80.7(1) |
| S(3) S(2) | 91(2) | S(3) S(6''') | 120.8(1) |
| S(2) S(2') | 91(3) | S(6'') S(6''') | 78.8(1) |
| S(1)—Bi(1)—S(2'') | 89.2(1)° | | |
| S(1) S(2''') | 90.8(1) | | |
| S(2'') S(2''') | 90.2(2) | | |
| S(2'') S(2''') | 89.8(2) | | |
| S(2)—Bi(2)—S(3) | 87.5(1)° | | |
| S(3) S(3') | 91.2(2) | | |
| S(2) S(1'') | 89.1(1) | | |
| S(3) S(1) | 90.7(1) | | |
| S(3) S(1'') | 94.0(1) | | |
| S(1) S(1'') | 92.8(1) | | |
| S(1'') S(1''') | 80.5(1) | | |

reasonable values. The final R index was 0.049 for the observed 997 reflections (0.054 for all the 1109 reflections). Neutral-atom form factors given in *International tables*, Vol. 3, p. 202, 210 and 212 (1968) were used. The effect of anomalous dispersion was considered (CROMER, 1965).

The final atomic coordinates and the anisotropic temperature factors are listed in Tables 3 and 4, respectively. Interatomic distances

Table 7. Observed and calculated structure amplitudes

| h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c |
|-----|---|---|----------------|----------------|-----|---|---|----------------|----------------|-----|---|----|----------------|----------------|-----|----|-----|----------------|----------------|-----|----|----|----------------|----------------|-----|----|-----|----------------|----------------|
| 2 | 0 | 0 | 14* | 10 | -6 | 0 | 5 | 458 | 478 | -2 | 0 | 10 | 146 | 147 | -1 | 0 | 17 | 28 | 21 | -19 | 1 | 5 | 55 | 50 | -5 | 1 | 10 | 92 | 54 |
| 4 | 0 | 0 | 397 | 392 | -4 | 0 | 0 | 327 | 342 | 0 | 0 | 10 | 215 | 221 | -2 | 0 | 17 | 189 | 197 | -17 | 0 | 15 | 149 | 153 | -3 | 0 | 10 | 387 | 407 |
| 6 | 0 | 0 | 510 | 502 | -2 | 0 | 0 | 233 | 238 | 2 | 0 | 10 | 161 | 163 | 0 | 0 | 17 | 65 | 63 | -15 | 0 | 15 | 155 | 154 | -1 | 0 | 10 | 183 | 195 |
| 8 | 0 | 0 | 383 | 366 | 0 | 0 | 0 | 333 | 323 | 4 | 0 | 10 | 34 | 28 | 2 | 0 | 17 | 55 | 56 | -13 | 0 | 15 | 20* | 15 | 1 | 0 | 10 | 312 | 323 |
| 10 | 0 | 0 | 137 | 137 | 2 | 0 | 0 | 145 | 146 | 6 | 0 | 10 | 229 | 222 | 4 | 0 | 17 | 53 | 56 | -11 | 0 | 15 | 42 | 1 | 3 | 0 | 10 | 41 | 34 |
| 12 | 0 | 0 | 619 | 645 | 4 | 0 | 0 | 67 | 75 | 8 | 0 | 10 | 105 | 110 | -10 | 0 | 18 | 44 | 47 | -9 | 0 | 15 | 349 | 352 | 5 | 0 | 10 | 78 | 85 |
| 14 | 0 | 0 | 56 | 34 | 6 | 0 | 0 | 384 | 392 | 10 | 0 | 10 | 20* | 37 | -8 | 0 | 17 | 137 | 146 | -7 | 0 | 15 | 197 | 196 | 7 | 0 | 10 | 18* | 9 |
| 16 | 0 | 0 | 76 | 71 | 8 | 0 | 0 | 173 | 170 | 12 | 0 | 10 | 16* | 8 | -6 | 0 | 17 | 113 | 119 | -5 | 0 | 15 | 380 | 394 | 9 | 0 | 10 | 260 | 252 |
| 18 | 0 | 0 | 119 | 119 | 10 | 0 | 0 | 56 | 46 | 14 | 0 | 10 | 64 | 58 | -4 | 0 | 17 | 47 | 49 | -3 | 0 | 15 | 219 | 232 | 11 | 0 | 10 | 186 | 176 |
| 20 | 0 | 0 | 137 | 142 | 12 | 0 | 0 | 194 | 188 | -18 | 0 | 11 | 44 | 24 | -2 | 0 | 17 | 204 | 218 | -1 | 0 | 15 | 104 | 109 | 13 | 0 | 10 | 102 | 100 |
| -20 | 0 | 1 | 225 | 236 | 14 | 0 | 0 | 23* | 6 | -16 | 0 | 11 | 138 | 134 | 0 | 0 | 17 | 55 | 55 | 1 | 0 | 15 | 150 | 136 | 15 | 0 | 10 | 31 | 30 |
| -18 | 0 | 1 | 162 | 163 | 16 | 0 | 0 | 59 | 52 | -14 | 0 | 11 | 25* | 44 | 3 | 1 | 0 | 477 | 463 | 3 | 0 | 15 | 444 | 460 | -17 | 1 | 11 | 103 | 134 |
| -16 | 0 | 1 | 157 | 164 | 18 | 0 | 0 | 63 | 60 | -12 | 0 | 11 | 224 | 197 | 5 | 0 | 17 | 318 | 284 | 5 | 0 | 15 | 287 | 293 | -15 | 0 | 10 | 58 | 39 |
| -14 | 0 | 1 | 20* | 13 | -20 | 0 | 6 | 107 | 99 | -10 | 0 | 11 | 46 | 45 | 7 | 0 | 17 | 170 | 167 | 7 | 0 | 15 | 215 | 215 | -13 | 0 | 10 | 118 | 104 |
| -12 | 0 | 1 | 181 | 183 | -18 | 0 | 0 | 112 | 108 | -8 | 0 | 11 | 117 | 114 | 9 | 0 | 17 | 414 | 406 | 9 | 0 | 15 | 25* | 11 | -11 | 0 | 10 | 25* | 9 |
| -10 | 0 | 1 | 52 | 50 | -16 | 0 | 0 | 68 | 51 | -6 | 0 | 11 | 35 | 28 | 11 | 0 | 17 | 43 | 38 | -11 | 0 | 15 | 23* | 23 | -9 | 0 | 10 | 459 | 435 |
| -8 | 0 | 1 | 383 | 372 | -14 | 0 | 0 | 209 | 186 | -4 | 0 | 11 | 300 | 308 | 13 | 0 | 17 | 171 | 176 | 13 | 0 | 15 | 241 | 239 | -7 | 0 | 10 | 23* | 8 |
| -6 | 0 | 1 | 429 | 420 | -12 | 0 | 0 | 22* | 17 | -2 | 0 | 11 | 19* | 23 | 15 | 0 | 17 | 159 | 161 | 15 | 0 | 15 | 151 | 155 | -5 | 0 | 10 | 260 | 266 |
| -4 | 0 | 1 | 255 | 247 | -10 | 0 | 0 | 254 | 237 | 0 | 0 | 11 | 374 | 392 | 17 | 0 | 17 | 181 | 195 | 17 | 0 | 15 | 182 | 188 | -3 | 0 | 10 | 232 | 241 |
| -2 | 0 | 1 | 52 | 49 | -8 | 0 | 0 | 86 | 76 | 2 | 0 | 11 | 17* | 5 | 19 | 0 | 17 | 16* | 24 | -19 | 1 | 6 | 51 | 44 | -1 | 0 | 10 | 173 | 177 |
| 0 | 0 | 2 | 86* | 42 | -6 | 0 | 0 | 86 | 85 | 4 | 0 | 11 | 306 | 311 | -19 | 1 | 1 | 76 | 82 | -13 | 0 | 15 | 823 | 834 | 1 | 0 | 10 | 124 | 118 |
| 2 | 0 | 2 | 141 | 141 | -4 | 0 | 0 | 22* | 32 | 6 | 0 | 11 | 121 | 122 | -17 | 0 | 17 | 122 | 127 | -15 | 0 | 15 | 134 | 131 | 3 | 0 | 10 | 423 | 440 |
| 4 | 0 | 2 | 166 | 160 | -2 | 0 | 0 | 95 | 91 | 8 | 0 | 11 | 333 | 324 | -15 | 0 | 17 | 78 | 82 | -13 | 0 | 15 | 265 | 254 | 5 | 0 | 10 | 47 | 44 |
| 6 | 0 | 2 | 579 | 611 | 0 | 0 | 0 | 110 | 111 | 10 | 0 | 11 | 44 | 39 | -13 | 0 | 17 | 19* | 20 | -11 | 0 | 15 | 93 | 85 | 7 | 0 | 10 | 122 | 116 |
| 8 | 0 | 2 | 19* | 24 | 2 | 0 | 0 | 105 | 79 | 12 | 0 | 11 | 315 | 308 | -11 | 0 | 17 | 157 | 162 | -9 | 0 | 15 | 130 | 127 | 9 | 0 | 10 | 232 | 228 |
| 10 | 0 | 2 | 93 | 86 | 4 | 0 | 0 | 75 | 76 | 14 | 0 | 11 | 17 | 18 | -9 | 0 | 17 | 172 | 162 | -7 | 0 | 15 | 77 | 74 | 11 | 0 | 10 | 68 | 58 |
| 12 | 0 | 2 | 215 | 211 | 6 | 0 | 0 | 332 | 332 | -18 | 0 | 12 | 102 | 98 | -7 | 0 | 17 | 232 | 245 | -5 | 0 | 15 | 347 | 337 | 13 | 0 | 10 | 56 | 58 |
| 14 | 0 | 2 | 168 | 165 | 8 | 0 | 0 | 90 | 90 | -16 | 0 | 12 | 166 | 155 | -5 | 0 | 17 | 15 | 21 | -3 | 0 | 15 | 253 | 260 | -17 | 1 | 12 | 159 | 159 |
| 16 | 0 | 2 | 39 | 35 | 10 | 0 | 0 | 74 | 68 | -14 | 0 | 12 | 104 | 99 | -3 | 0 | 17 | 45 | 35 | -1 | 0 | 15 | 301 | 290 | -15 | 0 | 10 | 28 | 22 |
| 18 | 0 | 2 | 237 | 248 | 12 | 0 | 0 | 110 | 101 | -12 | 0 | 12 | 168 | 152 | -1 | 0 | 17 | 191 | 208 | 1 | 0 | 15 | 275 | 266 | -11 | 0 | 10 | 84 | 70 |
| 20 | 0 | 2 | 91 | 97 | 14 | 0 | 0 | 55 | 54 | -10 | 0 | 12 | 157 | 140 | 1 | 0 | 17 | 158 | 164 | 3 | 0 | 15 | 25* | 17 | -13 | 0 | 10 | 215 | 187 |
| -2 | 0 | 2 | 263 | 270 | -6 | 0 | 0 | 396 | 381 | 10 | 0 | 12 | 179 | 167 | 11 | 0 | 17 | 207 | 195 | 13 | 0 | 15 | 131 | 126 | -1 | 0 | 10 | 78 | 49 |
| -4 | 0 | 2 | 48 | 53 | 16 | 0 | 0 | 56 | 58 | -8 | 0 | 12 | 379 | 362 | 3 | 0 | 17 | 488 | 512 | 5 | 0 | 15 | 139 | 164 | -9 | 0 | 10 | 95 | 87 |
| -6 | 0 | 2 | 58 | 57 | 18 | 0 | 0 | 269 | 264 | -6 | 0 | 12 | 169 | 163 | 5 | 0 | 17 | 369 | 363 | 7 | 0 | 15 | 231 | 226 | -7 | 0 | 10 | 95 | 87 |
| -8 | 0 | 2 | 47 | 48 | -20 | 0 | 7 | 139 | 143 | -4 | 0 | 12 | 214 | 211 | 7 | 0 | 17 | 73 | 56 | 9 | 0 | 15 | 340 | 335 | -5 | 0 | 10 | 139 | 140 |
| -10 | 0 | 2 | 264 | 265 | -18 | 0 | 0 | 61 | 59 | -2 | 0 | 12 | 107 | 104 | 9 | 0 | 17 | 67 | 56 | 11 | 0 | 15 | 106 | 105 | -3 | 0 | 10 | 64 | 38 |
| -12 | 0 | 2 | 113 | 107 | -16 | 0 | 0 | 118 | 111 | 0 | 0 | 12 | 179 | 167 | 11 | 0 | 17 | 207 | 195 | 13 | 0 | 15 | 131 | 126 | -1 | 0 | 10 | 49 | 73 |
| -14 | 0 | 2 | 18* | 3 | -14 | 0 | 0 | 466 | 464 | 2 | 0 | 12 | 133 | 138 | 13 | 0 | 17 | 158 | 161 | 15 | 0 | 15 | 72 | 75 | 1 | 0 | 10 | 139 | 139 |
| -16 | 0 | 2 | 180 | 167 | -12 | 0 | 0 | 22* | 21 | 4 | 0 | 12 | 276 | 279 | 15 | 0 | 17 | 314 | 341 | 17 | 0 | 15 | 91 | 84 | 3 | 0 | 10 | 107 | 105 |
| -18 | 0 | 2 | 139 | 123 | -10 | 0 | 0 | 326 | 306 | 6 | 0 | 12 | 128 | 127 | 17 | 0 | 17 | 80 | 79 | -19 | 1 | 7 | 120 | 130 | 5 | 0 | 10 | 59 | 51 |
| -20 | 0 | 2 | 31 | 22 | -8 | 0 | 0 | 394 | 364 | 8 | 0 | 12 | 132 | 133 | 19 | 0 | 17 | 88 | 96 | -17 | 0 | 15 | 88 | 84 | 7 | 0 | 10 | 54 | 49 |
| 0 | 0 | 3 | 49* | 7 | -4 | 0 | 0 | 168 | 158 | 12 | 0 | 12 | 12* | 8 | -17 | 0 | 17 | 165 | 170 | -13 | 0 | 15 | 111 | 108 | 11 | 0 | 10 | 85 | 84 |
| 2 | 0 | 3 | 20* | 3 | -2 | 0 | 0 | 733 | 748 | -18 | 0 | 13 | 16 | 11 | -15 | 0 | 17 | 183 | 174 | -11 | 0 | 15 | 469 | 450 | -17 | 1 | 13 | 89 | 90 |
| 4 | 0 | 3 | 421 | 447 | 0 | 0 | 0 | 47 | 42 | -16 | 0 | 13 | 159 | 159 | -13 | 0 | 17 | 118 | 98 | -9 | 0 | 15 | 127 | 113 | -15 | 0 | 10 | 169 | 164 |
| 6 | 0 | 3 | 136 | 130 | 2 | 0 | 0 | 328 | 322 | -14 | 0 | 13 | 159 | 150 | -11 | 0 | 17 | 65 | 51 | -7 | 0 | 15 | 209 | 205 | -13 | 0 | 10 | 17* | 14 |
| 8 | 0 | 3 | 246 | 240 | 4 | 0 | 0 | 237 | 229 | -12 | 0 | 13 | 180 | -8 | 0 | 17 | 333 | 309 | 5 | 0 | 15 | 97 | 89 | -11 | 0 | 10 | 19* | 14 | |
| 10 | 0 | 3 | 166 | 158 | 6 | 0 | 0 | 331 | 323 | -10 | 0 | 13 | 87 | 54 | -7 | 0 | 17 | 332 | 307 | -3 | 0 | 15 | 394 | 408 | -9 | 0 | 10 | 39 | 47 |
| 12 | 0 | 3 | 82 | 78 | 8 | 0 | 0 | 26* | 5 | -8 | 0 | 13 | 79 | 77 | -5 | 0 | 17 | 438 | 425 | -1 | 0 | 15 | 44 | 24 | -7 | 0 | 10 | 207 | 215 |
| 14 | 0 | 3 | 62 | 55 | 10 | 0 | 0 | 491 | 481 | -6 | 0 | 13 | 18* | 24 | -3 | 0 | 17 | 124 | 120 | 1 | 0 | 15 | 445 | 455 | -5 | 0 | 10 | 289 | 297 |
| 16 | 0 | 3 | 206 | 217 | 12 | 0 | 0 | 142 | 127 | -4 | 0 | 13 | 173 | 175 | -1 | 0 | 17 | 294 | 325 | 3 | 0 | 15 | 316 | 293 | -3 | 0 | 10 | 172 | 172 |
| 18 | 0 | 3 | 61 | 60 | 14 | 0 | 0 | 144 | 180 | -2 | 0 | 13 | 212 | 216 | 1 | 0 | 17 | 95 | 86 | 5 | 0 | 15 | 265 | 255 | -1 | 0 | 10 | 84 | 88 |
| 20 | 0 | 3 | 61 | 64 | 16 | 0 | 0 | 92 | 88 | 0 | 0 | 13 | 335 | 328 | 3 | 0 | 17 | 396 | 433 | 7 | 0 | 15 | 159 | 156 | 1 | 0 | 10 | 65 | 70 |
| -20 | 0 | 3 | 81 | 90 | 18 | 0 | 0 | 100 | 103 | 2 | 0 | 13 | 37 | 33 | 5 | 0 | 17 | 239 | 231 | 9 | 0 | 15 | 253 | 244 | 3 | 0 | 10 | 93 | 92 |
| -18 | 0 | 3 | 79 | 82 | -20 | 0 | 8 | 148 | 151 | 4 | 0 | 13 | 172 | 167 | 7 | 0 | 17 | 424 | 415 | 11 | 0 | 15 | 125 | 121 | 5 | 0 | 10 | 175 | 173 |
| -16 | 0 | 3 | 302 | 312 | -18 | 0 | 0 | 188 | 196 | 6 | 0 | 13 | 60 | 57 | 9 | 0 | 17 | 104 | 89 | 13 | 0 | 15 | 208 | 190 | 7 | 0 | 10 | 281 | 284 |
| -14 | 0 | 3 | 59 | 40 | -16 | 0 | 0 | 37 | 31 | 8 | 0 | 13 | 29 | 23 | 11 | 0 | 17 | 255 | 255 | 15 | 0 | 15 | 94 | 82 | 9 | 0 | 10 | 75 | 76 |
| -12 | 0 | 3 | 154 | 145 | -14 | 0 | 0 | 67 | 55 | 10 | 0 | 13 | 164 | 159 | 13 | 0 | 17 | 72 | 79 | 17 | 0 | 15 | 66 | | | | | | |

Table 7. (Continued)

| h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c | h | k | l | F _o | F _c |
|-----|---|----|----------------|----------------|-----|---|---|----------------|----------------|-----|---|----|----------------|----------------|-----|---|---|----------------|----------------|------|---|----|----------------|----------------|-----|---|----|----------------|----------------|---|---|---|----------------|----------------|
| -7 | 1 | 18 | 201 | 197 | 14 | 2 | 4 | 328 | 350 | 8 | 2 | 9 | 189 | 167 | 3 | 3 | 0 | 283 | 269 | -11 | 3 | 6 | 75 | 69 | -5 | 3 | 13 | 211 | 212 | | | | | |
| -5 | | | 79 | 77 | 16 | | | 16* | 2 | 10 | | | 93 | 85 | 5 | | | 196 | 179 | -9 | | | 104 | 93 | -3 | | | 115 | 116 | | | | | |
| -3 | | | 102 | 99 | -18 | 2 | 5 | 241 | 254 | 12 | | | 47 | 46 | 7 | | | 109 | 108 | -7 | | | 65 | 63 | -1 | | | 74 | 72 | | | | | |
| -1 | | | 22 | 23 | -16 | | | 131 | 135 | 14 | | | 114 | 119 | 9 | | | 256 | 261 | -5 | | | 226 | 234 | 1 | | | 51 | 44 | | | | | |
| 2 | 2 | 0 | 27 | 1 | -14 | | | 191 | 199 | -16 | 2 | 10 | 34 | 34 | 11 | | | 34 | 33 | -3 | | | 180 | 181 | 3 | | | 81 | 82 | | | | | |
| 4 | | | 316 | 308 | -12 | | | 121 | 117 | -14 | | | 31 | 31 | 13 | | | 125 | 132 | -1 | | | 181 | 185 | -5 | 3 | 14 | 209 | 211 | | | | | |
| 6 | | | 370 | 356 | -10 | | | 160 | 156 | -12 | | | 249 | 229 | 15 | | | 97 | 103 | 1 | | | 178 | 177 | -3 | | | 40 | 41 | | | | | |
| 8 | | | 305 | 297 | -8 | | | 37 | 37 | -10 | | | 71 | 55 | -15 | 3 | 1 | 64 | 71 | 3 | | | 22* | 2 | -1 | | | 269 | 274 | | | | | |
| 10 | | | 112 | 111 | -6 | | | 365 | 381 | -8 | | | 147 | 133 | -13 | | | 16* | 15 | 5 | | | 136 | 125 | -2 | 4 | 0 | 13* | 6 | | | | | |
| 12 | | | 486 | 525 | -4 | | | 230 | 241 | -6 | | | 262 | 245 | -11 | | | 118 | 116 | 7 | | | 152 | 142 | 4 | | | 200 | 184 | | | | | |
| 14 | | | 19* | 29 | -2 | | | 191 | 192 | -4 | | | 18* | 21 | -9 | | | 101 | 92 | 9 | | | 259 | 241 | 6 | | | 197 | 193 | | | | | |
| 16 | | | 53 | 63 | 0 | | | 234 | 231 | -2 | | | 142 | 115 | -7 | | | 163 | 157 | 11 | | | 102 | 100 | 10 | | | 176 | 186 | | | | | |
| 18 | | | 95 | 97 | 2 | | | 117 | 121 | 0 | | | 173 | 177 | -5 | | | 16* | 15 | 13 | | | 172 | 176 | -10 | 4 | 1 | 65 | 72 | | | | | |
| -18 | 2 | 1 | 123 | 134 | 4 | | | 44 | 36 | 2 | | | 131 | 137 | -3 | | | 13* | 7 | -15 | 3 | 7 | 172 | 176 | -10 | 4 | 1 | 31 | 31 | | | | | |
| -16 | | | 132 | 143 | 6 | | | 306 | 313 | 4 | | | 17* | 12 | 1 | | | 111 | 119 | -15 | | | 85 | 81 | -8 | | | 180 | 181 | | | | | |
| -14 | | | 19* | 9 | 8 | | | 131 | 123 | 6 | | | 187 | 182 | 3 | | | 292 | 293 | -11 | | | 309 | 304 | -6 | | | 158 | 157 | | | | | |
| -12 | | | 149 | 155 | 10 | | | 37 | 42 | 8 | | | 90 | 88 | 5 | | | 224 | 218 | -9 | | | 90 | 71 | -4 | | | 128 | 122 | | | | | |
| -10 | | | 40 | 44 | 12 | | | 160 | 154 | 10 | | | 30 | 29 | 7 | | | 57 | 54 | -7 | | | 153 | 142 | -2 | | | 13* | 22 | | | | | |
| -8 | | | 302 | 298 | 14 | | | 42 | 8 | 12 | | | 12* | 1 | 9 | | | 19* | 19 | -5 | | | 55 | 55 | 2 | | | 67 | 68 | | | | | |
| -6 | | | 309 | 309 | 16 | | | 42 | 42 | -16 | 2 | 11 | 119 | 114 | 11 | | | 133 | 132 | -3 | | | 290 | 277 | 4 | | | 86 | 78 | | | | | |
| -4 | | | 192 | 188 | -18 | 2 | 6 | 92 | 95 | -14 | | | 36 | 39 | 13 | | | 115 | 116 | -1 | | | 17* | 14 | 6 | | | 244 | 239 | | | | | |
| -2 | | | 36 | 34 | -16 | | | 17 | 42 | -12 | | | 177 | 162 | 15 | | | 217 | 240 | 1 | | | 285 | 292 | 8 | | | 15* | 5 | | | | | |
| 2 | | | 104 | 115 | -14 | | | 154 | 153 | -10 | | | 52 | 38 | -15 | 3 | 2 | 117 | 122 | 3 | | | 195 | 189 | 10 | | | 45 | 55 | | | | | |
| 4 | | | 141 | 141 | -12 | | | 39 | 12 | -8 | | | 112 | 99 | -13 | | | 189 | 177 | -4 | | | 189 | 177 | -4 | 2 | | 14* | 18 | | | | | |
| 6 | | | 451 | 448 | -10 | | | 194 | 185 | -6 | | | 19* | 14 | -11 | | | 18* | 30 | 7 | | | 102 | 98 | -8 | | | 91 | 85 | | | | | |
| 8 | | | 19* | 15 | -8 | | | 61 | 56 | -4 | | | 239 | 252 | -9 | | | 211 | 214 | 9 | | | 202 | 177 | -6 | | | 66 | 63 | | | | | |
| 10 | | | 73 | 75 | -6 | | | 68 | 68 | -2 | | | 16* | 14 | -7 | | | 189 | 190 | 11 | | | 92 | 85 | -4 | | | 15* | 20 | | | | | |
| 12 | | | 163 | 171 | -4 | | | 57 | 26 | 0 | | | 301 | 322 | -5 | | | 301 | 294 | -13 | 3 | 8 | 44 | 40 | -2 | | | 120 | 130 | | | | | |
| 14 | | | 131 | 138 | -2 | | | 90 | 62 | 2 | | | 16* | 2 | -3 | | | 69 | 70 | -1 | | | 35 | 37 | 0 | | | 35 | 34 | | | | | |
| 16 | | | 17* | 29 | 0 | | | 82 | 91 | 4 | | | 254 | 262 | -1 | | | 192 | 210 | -9 | | | 61 | 51 | 2 | | | 14* | 16 | | | | | |
| 18 | | | 195 | 207 | 2 | | | 65 | 49 | 6 | | | 99 | 98 | 1 | | | 56 | 55 | -7 | | | 119 | 113 | 4 | | | 196 | 191 | | | | | |
| -18 | 2 | 2 | 49 | 51 | 4 | | | 62 | 58 | 8 | | | 269 | 274 | 3 | | | 255 | 280 | -5 | | | 113 | 111 | 6 | | | 60 | 65 | | | | | |
| -16 | | | 35 | 40 | 6 | | | 272 | 269 | 10 | | | 35 | 31 | 5 | | | 144 | 140 | -5 | | | 18* | 19 | -8 | | | 115 | 117 | | | | | |
| -14 | | | 216 | 222 | 8 | | | 68 | 71 | 12 | | | 252 | 263 | 7 | | | 292 | 286 | -1 | * | | 125 | 129 | -10 | 4 | 3 | 58 | 60 | | | | | |
| -12 | | | 82 | 82 | 10 | | | 79 | 63 | -16 | 2 | 12 | 133 | 135 | 9 | | | 59 | 51 | 1 | | | 113 | 116 | -8 | | | 123 | 130 | | | | | |
| -10 | | | 17* | 7 | 12 | | | 97 | 88 | -14 | | | 83 | 81 | 11 | | | 172 | 182 | 3 | | | 40 | 37 | -6 | | | 16* | 7 | | | | | |
| -8 | | | 141 | 136 | 14 | | | 67 | 53 | -12 | | | 146 | 136 | 13 | | | 49 | 51 | 5 | | | 18* | 10 | -4 | | | 255 | 261 | | | | | |
| -6 | | | 109 | 99 | 16 | | | 54 | 47 | -10 | | | 125 | 116 | 15 | | | 140 | 146 | 7 | | | 116 | 109 | -2 | | | 15* | 15 | | | | | |
| -4 | | | 30 | 23 | -18 | 2 | 7 | 54 | 50 | -8 | | | 305 | 305 | -15 | 3 | 3 | 53 | 54 | 9 | | | 156 | 145 | 0 | | | 126 | 125 | | | | | |
| -2 | | | 198 | 212 | -16 | | | 95 | 92 | -6 | | | 126 | 128 | -13 | | | 323 | 349 | 11 | | | 63 | 57 | 2 | | | 38 | 35 | | | | | |
| 0 | | | 28* | 20 | -14 | | | 378 | 385 | -4 | | | 179 | 182 | -11 | | | 19* | 16 | -13 | 3 | 9 | 77 | 72 | 4 | | | 161 | 160 | | | | | |
| 2 | | | 27 | 6 | -12 | | | 20* | 16 | -2 | | | 84 | 81 | -9 | | | 366 | 369 | -11 | | | 96 | 86 | 6 | | | 49 | 48 | | | | | |
| 4 | | | 323 | 336 | -4 | | | 275 | 256 | 0 | | | 146 | 151 | -7 | | | 195 | 186 | -9 | | | 69 | 69 | -8 | | | 221 | 217 | | | | | |
| 6 | | | 103 | 100 | -8 | | | 293 | 289 | 2 | | | 108 | 110 | -5 | | | 158 | 160 | -7 | | | 258 | 248 | -10 | 4 | 4 | 108 | 203 | | | | | |
| 8 | | | 199 | 190 | -6 | | | 297 | 302 | 4 | | | 238 | 236 | -3 | | | 41 | 44 | -5 | | | 56 | 55 | -8 | | | 39 | 26 | | | | | |
| 10 | | | 134 | 128 | -4 | | | 124 | 120 | 6 | | | 110 | 103 | -1 | | | 405 | 448 | -5 | | | 187 | 193 | -6 | | | 151 | 142 | | | | | |
| 12 | | | 82 | 75 | -2 | | | 565 | 583 | 8 | | | 115 | 117 | 1 | | | 83 | 83 | -1 | | | 69 | 68 | -4 | | | 44 | 45 | | | | | |
| 14 | | | 40 | 41 | 0 | | | 39 | 34 | 10 | | | 16 | 6 | 3 | | | 177 | 189 | 1 | | | 118 | 120 | -2 | | | 187 | 200 | | | | | |
| 16 | | | 178 | 183 | 2 | | | 267 | 266 | -14 | 2 | 13 | 140 | 129 | 5 | | | 117 | 122 | 3 | | | 86 | 81 | 0 | | | 63 | 57 | | | | | |
| 18 | | | 49 | 48 | 4 | | | 195 | 179 | -12 | | | 161 | 151 | 7 | | | 52 | 56 | 5 | | | 289 | 282 | 2 | | | 261 | 257 | | | | | |
| -18 | 2 | 3 | 64 | 69 | 6 | | | 268 | 260 | -10 | | | 58 | 49 | 9 | | | 21* | 27 | 7 | | | 55 | 56 | 4 | | | 45 | 39 | | | | | |
| -16 | | | 241 | 256 | 8 | | | 23* | 1 | -8 | | | 55 | 60 | 11 | | | 221 | 225 | 9 | | | 183 | 182 | 6 | | | 173 | 167 | | | | | |
| -14 | | | 38 | 35 | 10 | | | 403 | 397 | -6 | | | 37 | 27 | 13 | | | 79 | 83 | -15 | 3 | 10 | 45 | 42 | 8 | | | 27 | 25 | | | | | |
| -12 | | | 125 | 125 | 12 | | | 120 | 108 | -4 | | | 138 | 141 | -15 | 3 | 4 | 14* | 7 | -11 | | | 298 | 261 | -8 | 4 | 5 | 16* | 22 | | | | | |
| -10 | | | 134 | 124 | 14 | | | 164 | 155 | -2 | | | 176 | 183 | -15 | | | 127 | 125 | -9 | | | 19* | 15 | -6 | | | 247 | 234 | | | | | |
| -8 | | | 221 | 219 | -18 | 2 | 8 | 160 | 172 | 0 | | | 100 | 101 | -11 | | | 19* | 25 | -7 | | | 123 | 111 | -4 | | | 113 | 114 | | | | | |
| -6 | | | 17* | 19 | -16 | | | 28 | 22 | 2 | | | 39 | 32 | -9 | | | 70 | 61 | -5 | | | 30 | 25 | -2 | | | 120 | 117 | | | | | |
| -4 | | | 459 | 503 | -14 | | | 70 | 35 | 4 | | | 141 | 139 | -7 | | | 332 | 325 | -5 | | | 265 | 281 | 0 | | | 121 | 122 | | | | | |
| -2 | | | 16* | 11 | -12 | | | 63 | 54 | 6 | | | 56 | 53 | -5 | | | 141 | 147 | -1</ | | | | | | | | | | | | | | |

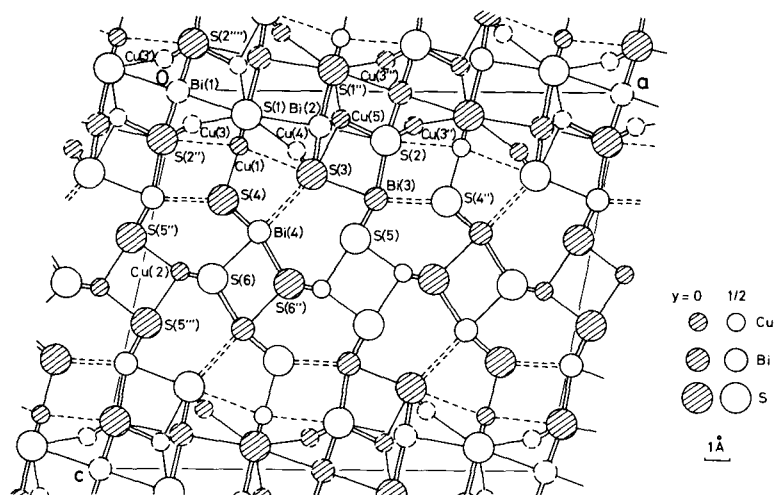


Fig.1. The structure of cuprobismuthite projected along b axis. The broken circles indicate the statistically distributed atoms

Discussion of the structure and composition

There are four independent bismuth atom sites in the structure of cuprobismuthite. Bi(2) has a coordination of three plus three, forming two trigonal pyramids, together a distorted octahedron. Similar coordinations have been also found in the structures of hodrushite (KUPČÍK and MAKOVICKÝ, 1968), $\text{Cu}_4\text{Bi}_4\text{S}_9$ (OZAWA and TAKÉUCHI, 1972), CuBi_5S_8 (OHMASA and NOWACKI, 1973), $\text{Cu}_{2+x}\text{Bi}_{6-x}\text{S}_9$ [$x = 1.21$] (OHMASA, 1973) and cosalite (SRIKRISHNAN and NOWACKI, 1974). On the other hand, three sulfur atoms around Bi(3) form a trigonal pyramid; together with the two next neighbours they complete a square pyramid. The same applies to the coordination of sulfur atoms around Bi(4). The coordination of Bi(1), unusual in bismuth-bearing sulfosalts, has been found for Me(1) and Me(2) atoms in hodrushite and for Bi(1) in CuBi_5S_8 . Each of these bismuth atoms lie on a center of symmetry. Instead of three sulfur neighbours, which approach usually within 2.55–2.8 Å to the associated bismuth atom, forming a trigonal pyramid with the bismuth, these above-mentioned bismuth atoms are coordinated to six sulfur atoms at fairly uniform distance of about 2.8 Å, so that the sulfur atoms may compose an approximately regular octahedron. Indeed the regularity is fairly strict, the bismuth-sulfur bond distances ranging from 2.77 to

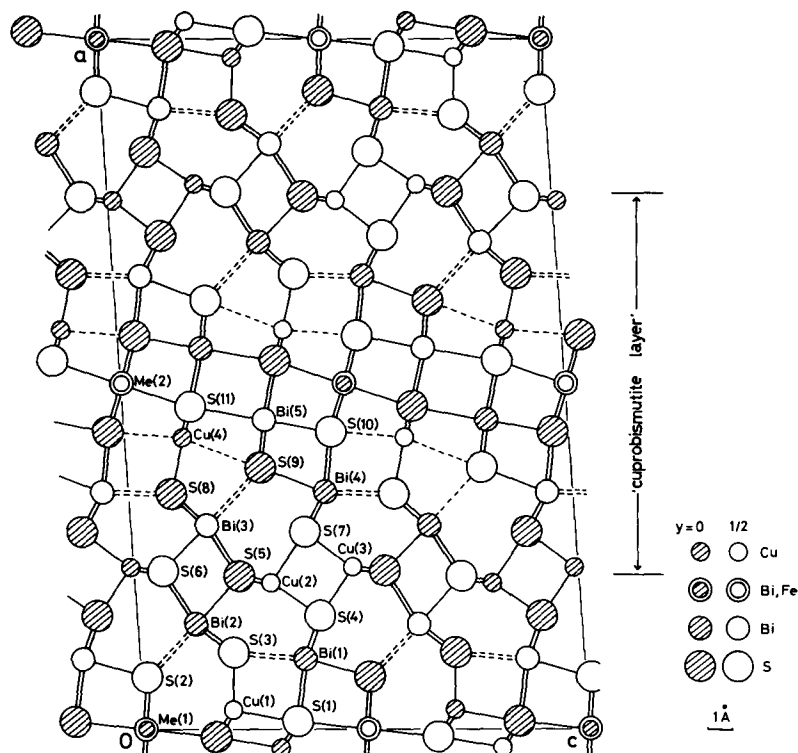


Fig. 2. The structure of hodrushite projected along b axis. Hodrushite has the following unit-cell dimensions (KODĚRA *et al.*, 1970); $a = 27.205$, $b = 3.927$, $c = 17.575$ Å, $\beta = 92^\circ 9'$. "Cuprobismuthite layer", b and c of hodrushite correspond to $c \sin \beta$, b and a of cuprobismuthite, respectively. Me(1) and Me(2) sites are occupied mostly by bismuth atoms (KUPČÍK and MAKOVICKÝ, 1968; KODĚRA *et al.*, 1970)

2.84 Å and the sulfur-bismuth-sulfur bond angles deviating from the ideal 90° value by only $\pm 2^\circ$.

Five independent sites for copper atoms were found in the structure. The coordination of Cu(2) is normal, being surrounded tetrahedrally by four sulfur atoms. Two symmetry-related Cu(2) tetrahedra share an edge in such a way that an endless double-chain along the b axis is formed. A similar double chain of copper has been reported in the structure of $\text{Cu}_4\text{Bi}_4\text{S}_9$, but with a different type of sharing. Cu(1), Cu(3) and Cu(4) are in nearly trigonal planar coordination with sulfur atoms. Such a coordination has been also reported for some copper atoms in hodrushite, $\text{Cu}_4\text{Bi}_4\text{S}_9$, $\text{Cu}_{2+x}\text{Bi}_{6-x}\text{S}_9$ and wittichenite (MATZAT,

1972; KOCMAN and NUFFIELD, 1973). Cu(3) and Cu(4) are substituted for Bi(1) and Bi(2), respectively. These substitutions appear to be very reasonable, judging from the copper-sulfur and copper-copper distances as given in Table 5. On the other hand, the coordination of Cu(5) is strange. Cu(5)—S(1) bond distance is too short. However, the Cu(5)—S, Cu(5) to other copper distances and Cu(5)—Bi(3) distance (3.438 Å) as well appear to indicate that this cavity for Cu(5) would be favourable in general for a copper atom, though it could not be discussed in detail due to the large standard deviations of its positional parameters.

The prediction as to the structure of cuprobismuthite by KODĚRA *et al.* (1970) was correct in principle as is evident from Fig. 2. The comparison of the bond distances between cuprobismuthite and

Table 8. Comparison of the interatomic distances between cuprobismuthite and "cuprobismuthite layer" in hodrushite

| cuprobismuthite | | "cuprobismuthite layer" in hodrushite* | |
|-----------------|------------|--|-----------|
| Cu(1)—S | 2.338 Å 2× | Cu(4)—S | 2.34 Å 2× |
| | 2.246 | | 2.26 |
| | 2.981 | | 2.94 |
| | 3.147 | | 3.26 |
| Cu(2)—S | 2.308 | Cu(3)—S | 2.30 |
| | 2.422 | | 2.35 |
| | 2.342 2× | | 2.36 2× |
| Bi(1)—S | 2.771 4× | Me(2)—S | 2.80 4× |
| | 2.842 2× | | 2.82 2× |
| Bi(2)—S | 2.644 | Bi(5)—S | 2.65 |
| | 2.747 2× | | 2.74 2× |
| | 2.916 | | 2.97 |
| | 3.037 2× | | 3.08 2× |
| Bi(3)—S | 2.649 | Bi(4)—S | 2.73 |
| | 2.645 2× | | 2.67 2× |
| | 3.077 2× | | 3.08 2× |
| | 3.413 2× | | 3.34 2× |
| Bi(4)—S | 2.688 2× | Bi(3)—S | 2.71 2× |
| | 2.581 | | 2.55 |
| | 3.092 2× | | 3.05 2× |
| | 3.699 2× | | 3.67 2× |

* The atom number of hodrushite corresponds to the number in Fig. 2.

“cuprobismuthite layer” in hodrushite is presented in Table 8. The maximum difference is only about 0.1 Å.

From the present results, we may reasonably draw the following conclusion. The crystal structure of cuprobismuthite corresponds essentially to the cuprobismuthite layer in hodrushite (composition $\text{Cu}_4\text{Bi}_7\text{S}_{12}$), but some substitutions of copper for bismuth arise so that the valence rule may be satisfied. As is described above and evident from Fig. 1, Cu(3) is substituted for some part of Bi(1). Although the sum of their calculated occupancies deviates slightly from the ideal value, because they were independently refined, the deviation is only within one σ . Considering the fact that the standard deviation for Bi(1) is much smaller than that for Cu(3), the value for Bi(1) must be more reliable. Then we assign the occupancies of 0.68 and 0.32 for Bi(1) and Cu(3) sites, respectively, in the following discussion. Furthermore, some part of Bi(2) is substituted by Cu(4) and Cu(5). The occupancy of Cu(4) reaches [1 minus occupancy of Bi(2)], while the value for Cu(5) is much smaller, about 1/3 of that for Cu(4). These phenomena can be easily explained. The close approach of Cu(5) to Bi(1), 2.570 Å, does not allow these two sites to be occupied at the same time. The cavity available for Cu(5) fits a copper atom, only when both Bi(1) and Bi(2) sites are vacant. Therefore the occupancy of Cu(5) would be expressed as [1 minus occupancy of Bi(1)] \times [1 minus occupancy of Bi(2)]. The obtained value of about 7% agrees closely with the result expected from this formula.

After all, the chemical formula for cuprobismuthite could be expressed as $(\text{Cu}_x, \text{Bi}_{\frac{1}{2}(1-x)}) (\text{Cu}_y, \text{Bi}_{1-y}) \text{Cu}_{xy} \text{Cu}_2\text{Bi}_2\text{S}_6$ [$x = 0.32$, $y = 0.20$]. This expression leads to the actual formula, $\text{Cu}_{2.53}\text{Bi}_{3.14}\text{S}_6$, in which the ratio of copper to bismuth is approximately 5 to 6, in good accordance with the results from natural specimens by TAYLOR *et al.* (1973).

The present results provide us with another interesting conclusion. Since the discovery by HILLEBRAND (1884), the chemical composition of cuprobismuthite and its relation to emplectite have been the main subjects for studies of this mineral. However, so far as we assume only one type of substitution described above, the existence of cuprobismuthite with the same composition as emplectite which is expressed stoichiometrically as $\text{Cu}_3\text{Bi}_3\text{S}_6$, is rather difficult to be considered. The chemical composition of $\text{Cu}_3\text{Bi}_3\text{S}_6$ can be realized only when both the values of 1 for x and 0 for y in the above-mentioned expression are simultaneously satisfied, in other words, only when Bi(1) is perfectly

substituted by Cu(3), while the Bi(2) site is fully occupied. The range of solid solution for cuprobismuthite by CHEN and CHANG (1974) includes the stoichiometric composition of $\text{Cu}_3\text{Bi}_3\text{S}_6$, but it explains neither the data for the natural specimens by TAYLOR *et al.* nor the present results (45.1 mol % of Cu_2S). We believe that more careful experiments on this point will remove the disagreement between them.

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