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A neutron-diffraction study of hemimorphite

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Abstract

The hydrogen atoms in hemimorphite, $Zn_4Si_2O_7(OH)_2 \cdot H_2O_7$, have been located and its crystal structure refined using 415 three-dimensional singlecrystal neutron-diffraction data. The mineral is orthorhombic, space group Imm2, with a = 8.367(5), b = 10.730(6), c = 5.115(3) Å, and Z = 2. The structure consists of three-membered rings of corner-sharing $Zn(OH)O_3$ (×2) and SiO_4 tetrahedra arranged in compact sheets parallel to (010). Three oxygen atoms in each tetrahedron are bonded to two zinc atoms and one silicon atom, while a fourth oxygen atom forms a bridging bond to an equivalent cation in an adjacent sheet. The water molecules are oriented parallel to (010) inside large cavities between the tetrahedral sheets and are held in place by hydrogen bonds to and from the hydroxyl groups of the Zn - OH - Zn bridging linkages. Mulliken population analyses calculated using constant bond lengths and the observed angles within and between the tetrahedra allow a rationalization of the bond-length variations in the SiO_4 group, but are less successful in the case of $Zn(OH)O_3$. Detailed analysis of observed bond length and calculated overlappopulation variations in a variety of tetrahedral oxyanions suggests that the poorer agreement in the case of the Zn tetrahedron is more a function of the larger overall size of the group than of the relative ionic character of the bonds.

Introduction

The crystal structure of hemimorphite has been of interest following the discovery by ZAMBONINI (1908), and later confirmed by FAUST (1951), Roy and MUMPTON (1956) and TAYLOR (1962), that the mineral

dehydrates in two stages. Crystal-structure analyses by ITO and WEST (1932), BARCLAY and COX (1960) and MCDONALD and CRUICKSHANK (1967) have served to rationalize these dehydration properties by establishing the presence of hydroxyl groups strongly bonded to two zinc atoms, and water molecules located in a series of interconnected cavities parallel to the c axis.

The release of water over the temperature range 400-650 °C (FAUST, 1951), without disruption of the structure, represents the loss of the H₂O molecules, whereas the breakdown of the structure to β Zn₂SiO₄ at about 740 °C is a result of the expulsion of the hydroxyl groups (FAUST, 1951; TAYLOR, 1962).

Although the positions of the non-hydrogen atoms in hemimorphite are known with moderate precision (McDonald and CRUICK-SHANK, 1967), the exact location of the hydroxyl and water oxygen atoms, and of course the protons, remains in some doubt. Indeed, MCDONALD and CRUICKSHANK have suggested that the water molecule may be rather free to move inside the cavity and to form hydrogen bonds to a variety of neighbouring oxygen atoms. Since the channels containing the water (and hydroxyl) are parallel to the polar caxis of the cell, a study of the mechanism of dehydration of this mineral is of particular interest. Moreover, TAYLOR (1962) has indicated that the product of complete dehydration of hemimorphite shows a marked preferred orientation with preservation of the repeat distance in the *c*-axis direction. The present neutron-diffraction study was initiated in order to locate the protons associated with the water molecule and hydroxyl group as a preliminary to a detailed x-ray study of the structure as a function of temperature. We also plan to undertake an analysis and comparison of the experimental chargedeformation distribution with theoretical electron-density maps obtained from accurate molecular-orbital theory.

Experimental

The sample selected for data collection was a 31 mg transparent fragment of hemimorphite from Chihuahua, Mexico, displaying welldeveloped {010} faces, and having approximate dimensions $1.6 \times 2.6 \times$ 2.1 mm. The crystal was oriented with the *c* axis parallel to the φ axis of a fully automated Electronics and Alloys four-circle diffractometer at the CP-5 reactor at Argonne National Laboratory. The unitcell parameters were refined by least-squares methods to give the best fit between calculated and observed angles 2θ , χ and φ (measured¹ at 22 ± 2 °C with $\lambda = 1.142(1)$ Å) for 29 automatically centered reflections in the range $2\theta = 33-63$ °. The resulting cell dimensions (a = 8.367(5), b = 10.730(6), c = 5.115(3) Å) and unit cell volume [459.2(4)Å³] agree well with values reported previously by McDONALD and CRUICKSKANK (1967)

Density measurements were made by the volume displacement method (in toluene) using six crystals in the range 21-58 mg and yielded a value of 3.475(10) g cm⁻³: the calculated density based on the ideal formula $\text{Zn}_4\text{Si}_2\text{O}_7(\text{OH})_2 \cdot \text{H}_2\text{O}$ and Z = 2 is 3.484 g cm⁻³. Several samples of hemimorphite with weights in the range 158-277mg were held at a temperature of $800 \,^{\circ}\text{C}$ for several hours and yielded weight losses between 7.3 and $7.8^{0}/_{0}$, again consistent with the presence of a single H₂O group in the ideal formula (theoretical leight loss = $7.48^{0}/_{0}$).

Intensity data for the structure analysis were collected at 22 \pm 2° C using a neutron wavelength 1.142(1) Å produced by diffraction from the (110) plane of a Be monochromator crystal². Each reflection was step-scanned using the $\theta - 2\theta$ technique at 2θ intervals of 0.1° over a scan range of width $4.2-6.4^{\circ}$ in 2θ . Backgrounds were determined from stationary counts taken at both ends of the scan. A total of 818 reflections of the types hkl and hkl, consistent with a body-centered lattice, were collected to a maximum $(\sin \theta)/\lambda = 0.72$. Two reflections were measured every 50 regular reflections in order to monitor crystal and instrument stability: their integrated intensities were observed to have a maximum random variation of $2.1^{\circ}/_{0}$. The intensity of each reflection was corrected for background, Lorentz, polarization and absorption effects using a μ value of 0.913 cm⁻¹ (the corresponding transmission factors ranged from 0.83 to 0.88). All data were placed on an absolute scale by calibration with a well-characterized NaCl crystal using a method described by PETERSEN et al. (1974). Multiply measured and symmetry-equivalent reflections (consistent with point group mm^2) were averaged (using weights based on counting statistics) to yield a set of 433 unique structure factors, each with a standard deviation estimated from the equation $\sigma = [\sigma_I^2 + 0.03I^2]^{0.5}$ $2I^{0.5}$, where I is the corrected raw intensity and $\sigma_{\rm I}$ is derived from counting and averaging statistics. The "agreement factor" between

¹ Parenthesized figures here, and elsewhere in the text, represent the e.s.d. in terms of the least significant figure quoted.

² Further details are given by ROZIERE and WILLIAMS (1976).

												Table	1. Ob	served	(F_0)	and c	alcula	ted (F	'a)
к ===;	F0	FC	AC	BC		FO	FC	AC	8C	к ====	FU	FC	AC	8C	к =====	F0	FC	AC	BC
H=	ο ι	.= 0			57	109 691	105 675	-105 675	ş	H=	7	L= 1			57	149 249	143 243	-95 229	106 -81
2 4	573	578 141	-578 -141	0	9 H=	397 10 L	407 .= 0	-407	0	24	458 451 343	500 455 337	-498 424 311	49 163 -129	11	372 201 232	389 199 240	-366 -191 233	-131 58 -57
10	1011	41 41	979 -161 41	80	1) 2	597 170	608 177	608 177	0	10	370 119 311	364 112 307	-294 106 245	215 36 -185	н=	6 L	= 2		
12 14	349 439	369 449	369 449	00	468	297 768 203	299 761 197	-299 761 197	000	12 H=	281 8	287 1 = 1	-50	282	24	731 325 260	744 315 257	190 243 72	-201
H= 1	1 L 530	.≃ 0 505	505	G	H≖	11 1	= 0		•	ļ	440	450	-448	-48	10	369 285	372	172	330
357	565	532 495 242	-532 495 242	000	135	118 358 48	123	-355	000	570	248	250	-250	-577	12	127	119	101	-64
11	89 97	115	-115	000	н=	12 L	.= 0	12	Ū	11 	273	- 35 - 1	-81	26	1	256	297	-149	257
15	~60 2	- 40 - 0	² 40	ŭ	0	238	238	238	0	o Q	498	504	323	389	57	472	465	-185	427
0 0	226	213	-213	0		141	.= 1 142	137	-38	46	211	217	-262 17 145	-299	11	138	430 132	-92	95
446	102	127	-127 -101	500	3 5 7	291 543	289 547	-103 -24 465	-288 287	8 H≖	369	367 L= 1	-214	-244	н= 0	272	.= 2 279	-272	59
10 12	103 190 32	136 168 20	-136 -169 -20	6 O C	11 13	598 272 365	607 262 348	-448 260 334	-410 33 95	13	252 436	239 435	201 -329	129 -284	246	128 103 287	119 91 295	117 45 -291	-22 -79 48
14 H=	46 3 L	27 = 0	-27	U	15 H=	509 1 L	536 = 1	-430	-319	57	436 115	433 109	348 3	258 -109	10	137 142	141 143	138 21	-142
13	67 420	41 426	41 426	0	0 2	219 31	215 16	-209	-47	H= .)	209	L= 1 215	-202	73	H= 1	9 L 145	.= 2 142	133	51
579	281 596 43	276 584 27	-276 584 27	Ú Ú Ú	4 6 8	242 148 180	268 148 174	-143 -49 -171	-227 139 -31	Ž 4	174 102	174 109	-105 -6	-138 -109	357	262	266	-196	180 -163 379
11 13	141 313	138 314	$1\overline{38} \\ 314$	ě	10	308	301	-199	-226	H= ~	0	L= 2	904	670	9 4	159	164	-51	~156
H=	4 L	= 0	4.70	~	H=	2 L	,= 1	-940	-107	ž	468	503	323	-385	<u>ہ</u> ۔	489	485	485	-9
244	372	376	376	0.00	1 3	562 378	560 374	-553	-373	10	395	398	-147	-172	4	121 425	119 437	-89 376	-79 222
10	302 336	315 320	315 -320	000		745	736	-727	-113	14	414	421	309	287	н=	11 L	= 2		
14	°77	°77	641	°,	13 15	569 101	578 102	-123 -572 16	-85 -101	H= 1	1 342	L= 2 347	-119	326	13	124	103	-24	-129
H= 1	5 L 177	= 0	167	. 0	H=	3 L	= 1			357	293 474 105	294 483 93	-245 -216 93	-162 432 5	H= 1	0 L 223	.= 3 236	219	-68
357	1207 114 224	1147- 125 220	-1147 125 220	000	C 2 4	432 492 566	432 490 542	32 59 540	431 -486 -49	11 13	157 216 138	161 215 127	-118 -182 105	109 115 72	3 5 7	423 556 193	429 578 198	-250 534 -115	348 -223 162
11 13	591 57 235	689 42 227	-689 -42 227	000	8 10	243 357 589	246 355 601	-211 -98 591	-341 109	H≍	2	L= 2			11	173 205 144	190 210 140	189 210 126	-24 3 61
H≈	6 L	= 0			12 14	369 44	366 58	-334 45	-150 37	0 2 4	369 187 227	351 194 223	-348 183	-46 64 -162	H=	1 1	= 3		
24	1139 195 174	1120 206	1120	6 0	H= 1	4 L	= 1	- 31	186	, é	432	436	-423	107	024	413	400	12 -48	-400
8 10	642 89	625 72	625 -72	ő	35	454	450	-331	-305	12 14	498 147	496 144	-440	229 -67	68	296	321	-176	-269
Ξ	-83	- 75	75	3	11	58	84 89	-83	-13	H= ,	3	L= 2	1.0		îž	241	243	-216	-110
1	233	236	236	ç	H=	5 L	.= 1	-30	10	35	542 212	552 204	469	291 -201	1	680	663	-603	-277
157	411	412	412	000	02	41 350	47 349	-319	-19 -141	11	526 606 179	599 185	572 -39	-181	57	633 514 575	658 494 556	-461 -434	-190 -177 -348
11	454 79	459	459	0 0	4 6 8	460 61 405	464 64 398	-400 63 -373	-236 11 -138	13 H=	314	318 L= 2	36	316	11 13	351 221 453	358 209 447	-73 -369	-196 -252
H≃ 0	8 L 63	.≠ () 44	-44	0	10	550 170	545 169	-475 151	-267 74	02	312 369	317 372	296	113 372	H≖	3 L	= 3		_
246	56 141 105	41 147 100	-147 -107	000	H= 1	6 L 174	.= 1 184	-166	-79	468	55 348 257	79 356 264	-79 165 131	-2 315 230	0 24	778 828 543	778 813 519	-757 811 472	178 -55 217
10	204	57 210	-210	0	357	152 320 285	161 312 281	153 -189 109	-249 -249 259	10 12	180 405	173 407	-142 21	-100 407	10	394 433 373	405 430 369	-404 426 275	-27 -56 245
H= 1	9 L 259	= 0 250	250	n	9 11	370	366	-161 157	-329 40	H= 1	5 45	L= 2	44	5	ÎŽ	ŽII	206	12	-206
â	žóź	197	-197	ŏ	• •	114		20	114	3	652	631	-522	-355	n#	- L			

averaged reflections was 0.021. Of these data only those 415 observations with $I > 2\sigma_{\rm I}$ were included in the subsequent least-squares refinement.

structure factors (\times 100) for hemimorphite

к === -	F0	FC	AC	BC BC	к ====	FC	FC	AC	BC		F0	FC	AC	8C
1357911 13	288 239 285 278 106 156 148	294 287 273 121 138 147	-246 219 -286 -44 -96 -11	-160 -98 -33 -275 74 -115 -147	H≠ 24 68 10	4 L 589 344 227 272 181 205	= 4 575 349 235 273 179 198	574 -334 -146 253 -124 -123	-19 100 185 -103 129 155	02468 =	535 245 119 481 293 6 L	533 236 113 478 282	531 -199 -113 467 -224	-49 -127 -7 -104 -171
024680	400 142 125 327 234	405 142 132 331 219	164 -129 -15 107 -200	-370 -58 -131 -314 -90	H= 13570	5 L 195 588 187 362 527	= 4 -182 -580 -177 -260 -519	128 ~562 ~104 353	-129 143 -144 -71	1 357 H=	137 231 411 293 7 L	135 228 417 304	133 -148 -228 -66	26 -173 257 -297
1ž ∺=	183 6 L	191 = 3	49	-185	11́ н=	189 6 L	195 = 4	-165	-105	024	231 170 423	234 155 418	-58 123 358	227 -95 217
1 3 5 7 9 11	154 237 403 389 352 133	157 241 391 401 336 119	- 34 59 291 -393 335 105	-154 234 -261 -261 -24 -56	02 4 6 10	181 350 330 119 294 434	189 341 335 111 292 435	-132 187 -307 102 182 -430	-135 285 134 -43 229 69	H≕ 1 3 H=	8 L 201 172 0 L	= 5 202 183 = 6	-136 182	-149 18
H⊐ 0	7 L	≠ 3	-264	-272	H= 1	7 L	= 4 160	-165	-7	02	699 472	704 492	703	-35
24.69	196 580 540 46	200 581 525 75	140 580 -508 38	-143 -34 -130 -65	195.79	426 337 178 176	430 337 171 175	-329 125 39	295 -72 117 170	6 8 H=	483 259 1 L	492 254 = 6	-350 -449 -178	-158 201 181
H#	8 L	= 3	0.00	-92	H= 0	8 L	= 4 159	- 79	-138	135	101 487	105 491 160	-470	-24 -142 -151
135	405 503 274	407 510 285	-362 504 -251	-137 -83 -135	246	138 106 159	129 109 178	-81 -85 -129	100 67 -123	ў н≖	213 2 L	206 ≖ 6	82	189
έ 	347	348 - 3	343	-215 -57	H⇔)	9 L 164	= 4	300	-63	2	324	334 264	-160	293 -258
02	343	344	-344	-4	35	663 247	592 255	-585 254	-93 -8	7-6 9	206 234	194	-148	133 -202
44-6	77	62 160	-24	-120 -120	н=	10 L	= 4		_	н=	3 L	= 6		
o H≃	10 L	= 3	124	-144	с Н=	/09 ሳ L	= 5	705	67	35	229 433 460	245 412 469	-140 50 -270	-409 383
135	31 178 105	173 88	38 173 -57	-34 -1 67	135	310 519 560	316 535 591	298 -520 448	$-106 \\ -126 \\ 385$	7 H≂	129 4 L	137 .≭ 6	58	-125
н=	0 L	= 4	• • •		7 9 11	335 396 314	307 401 319	134 -276 312	-277 291 65	02.4	488 240 351	479 239 344	234 -93 -344	-220 12
24	505 551	523 558	192	496 293	H=	1 ι	= 5			ь н=	318 5 L	322 = 6	262	187
10 12	513 431 711 588	523 416 696 602	199 -671 585	366 182 141	12 24 6	401 169 160 300	397 164 154 309	386 -44 117 294	-163 -99 -94	135	207 685 155	211 685 152	-651 -651	-56 -214 -132
H≠	1 L	= 4			10	200 178	189 185	-135 133	-146 129	н=	6 L	= 6		
1357	169 342 361 285	170 346 348 279	-71 -297 -295 278	-154 176 -184	н= 1	2 L 274 346	= 5 277 330	-169	-220	024	381 274 222	377 275 229	371 -215 -127	-70 172 -190
11	378 161	395 151	-380 -137	168 -62	57	283 154 243	272	-35	-270	ਮ≍ 1	0 L	= 7 152	~38	147
H= 0	2 L	= 4	-10	-120	н≃	3 L	= 5	• • •		3	178	165	136	-94
24 6 10	101 138 260 108 151	104 124 270 91 139	-77 81 -210 -58 125	-170 -170 61	02468	417 519 248 120 265	405 523 257 113 270	-258 325 257 -112 123	-313 410 -11 -11 241	0 2 4	300 258 77	319 259 91	-246 257 84	203 -37 35
12 H≃	444 3 L	440 = 4	-404	-176	10 H=	225 4 L	241 = 5	193	-144	H= 1	2 L 217	= 7 209	-209	3
1	122 341	113	-18	111 270	1	175	182	127	-131 187	э н=	422 3 1	429 = 7	422	-76
5 7 11	262 376 506 146	269 377 502 152	207 -376 323 -148	172 21 384 38	57 9 11-	273 251 186	266 232 206	136 200 -183	-229 117 -96	0 2	163 133	163 138	-112 -5	11A 138

Loction of hydrogen atoms and refinement

Refinement of the heavy-atom positions and isotropic temperature factors was initiated in space group Imm2 using the atomic coordinates

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Atom	x	<i>y</i>	z		β ₁₁
\mathbf{Zn}	0.2047(1)	0.1613(1)	0	0.0029(1)
$\mathbf{S}i$	0	1465(2) 0.	5076(5)	23(2)
O(1)	1602(2)	2055(1)	6362(4)	38(1)
O(2)	0	1669(2	2)	1938(4)	27(2)
O(3)	3050(2)	0		410(6)	50(2)
O(4)	0	0		5912(6)	54(3)
O(5)	5000	0		5195(13)	164(10)
H(35)	3740(8)	0		190(2)	154(9)
H(53)	4256(14)	0		643(4)	247(21)
Atom	β_{22}	β33	β_{12}	β13	β_{23}
			-0.0005	-0.0001 (2	0.0003(2)
Zn	0.0020(1)	0.0066(4)	5(1)	0.1(2)	3(2)
Si	13(1)	45(6)	0	0	0(3)
O(1)	26(1)	70(4)	17(1)	9(2)	9(2)
O(2)	33(1)	55(5)	0	0	4(2)
O(3)	18(1)	271(10)	0	-30(4)	0
O(4)	10(2)	124(9)	0	0	0
O(5)	227(12)	221(9)	0	0	0
H(35)	56(4)	692(41)	0	-224(18)	0
H(53)	247(18)	1163(101)	0	102(45)	0

 Table 2. Fractional atomic coordinates and temperature factor coefficients for hemimorphite*

* Parenthesized figures here and in all subsequent tables represent the e.s.d. in terms of the least significant figure to the left.

of McDonald and CRUICKSHANK (1967). As in the previous study, the unit-cell origin was defined by fixing the z coordinate of Zn at 0.0. Minimization of the function $\sum w(F_o - |F_c|)^2$, where F_o and F_c are the observed and calculated structure factors and $w = 1/\sigma^2$, resulted in convergence at a conventional R value of 0.123. The hydrogen atoms were then located from a Fourier difference map and the refinement was continued to convergence at R = 0.081. When the isotropic temperature-factor model was converted to an anisotropic model of the form $\exp\left[-\sum_{i=1}^{3}\sum_{j=1}^{3}\beta_{ij}h_ih_j\right]$ with the symmetry restrictions of

Atom	Axis	Root-	Angle to						
		square displace- ment	+a	+b	+c				
	1	0.090(3) Å	$46(20)^{\circ}$	$52(7)^{\circ}$	$112(37)^{\circ}$				
Zn	2	0.094(3)	113(28)	97(23)	156(34)				
	3	0.116(2)	127(4)	39(4)	81(5)				
	1	0.077(5)	90	89(29)	1(29)				
Si	2	0.086(5)	90	179(29)	89(29)				
	3	0.091(5)	0	90	90				
	1	0.082(3)	43(5)	48(2)	95(14)				
O(1)	2	0.092(3)	103(11)	84(9)	166(6)				
	3	0.151(2)	130(1)	43(1)	77(2)				
	1	0.085(4)	90	95(3)	5(3)				
O(2)	2	0.098(3)	180	90	90				
	3	0.139(3)	90	5(3)	85(3)				
	1	0.104(4)	90	0	90				
O(3)	2	0.125(4)	162(2)	90	108(2)				
	3	0.195(3)	108(2)	90	18(2)				
	1	0.076(6)	90	0	90				
O(4)	2	0.128(5)	90	90	180				
	3	0.138(4)	0	90	90				
	1	0.171(7)	90	90	0				
O(5)	2	0.241(7)	180	90	90				
	3	0.364(9)	90	0	90				
	1	0.146(9)	35(2)	90	55(2)				
H(35)	2	0.181(6)	90	180	90				
	3	0.354(10)	55(2)	90	145(2)				
	1	0.285(13)	17(6)	90	107(6)				
H(53)	2	0.379(14)	90	180	90				
	3	0.401(18)	73(6)	90	17(6)				

Table 3. Magnitudes and orientations of the principal axes of thermal ellipsoids in hemimorphite

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LEVY (1956), the R value dropped to 0.051. Further refinement with the inclusion of an isotropic extinction parameter (g), as defined and scaled by COPPENS and HAMILTON (1970), proceeded smoothly to convergence (parameter shifts in the final cycle were less than one

tenth of the corresponding e.s.d). The final values of R and R_w^3 were 0.028 and 0.035, respectively (0.031 and 0.035 for the entire data set of 433 structure factors), with the error in an observation of unit weight = 1.58. Although nearly one third of the data were affected more than $10^{0}/_{0}$ by extinction [g = 0.61(4)], only $5^{0}/_{0}$ of the reflections needed correction factors between 1.3 and the highest value of 2.2. The final scale factor was 0.989(7), close to the ideal value of unity expected for a data set placed on an absolute scale. The major positive and negative peaks in a Fourier difference synthesis were all located in the immediate environment of the water molecule but had maximum magnitudes somewhat less than $13^{0}/_{0}$ of the density of peaks corresponding to O and H atoms. The residual nuclear density may be a function of a small quantity of excess water present in a slightly different orientation within the channels, but could also be indicative of a minor amount of positional disorder in a single H₂O species.

Values for F_o and F_c (×100) are listed in Table 1. Atomic coordinates and thermal parameters along with their standard deviations estimated from the variance-covariance matrix are given in Table 2. The root-mean-square components of thermal displacement, and thermal-ellipsoid orientations appear in Table 3. The coherent neutron-scattering amplitudes used during least-squares refinement for Zn, Si, O and H were respectively 0.57, 0.42, 0.580 and -0.374, all in units of 10^{-12} cm (BACON, 1972). Programs utilized for solution, refinement and geometry calculations were local modifications of DA-TALIB, DATASORT, FOURIER, ORFLS3, ORFFE3 and ORTEP24.

Discussion of the structure

Hemimorphite crystallizes with the topology displayed in Fig. 1 and the bonding dimensions summarized in Table 4. The framework consists of an assembly of three-membered rings of corner-sharing $Zn(OH)O_3$ (×2) and SiO₄ tetrahedra arranged in compact sheets parallel to (010). Three of the oxygen atoms in each tetrahedron are bonded to one silicon and two zinc atoms, while the fourth oxygen (or, in the case of the zinc polyhedron, the hydroxyl group) forms a bridging bond to an equivalent cation in an adjacent sheet. The crosslinking of the sheets produces additional rings of four, six and eight tetrahedra and forms a series of large cavities connected along the

 $^{^{3}}R_{w} = [\Sigma w(F_{o}|F_{c}|)^{2}/\Sigma wF_{o}^{2}]^{0.5}.$

⁴ All programs are listed in the World list of crystallographic computer programs (3rd ed. and supplements).

SiO ₄ tetrahedron:							
Si - O(2)	1.620(3)Å 0(2) \cdots O(1) ($ imes$	(2) 2.	$662(2) { m \AA}$	$2O(2)$ -Si-O(1)($\times 2$)	$110.4(1)^{\circ}$
${ m O(1)}~(imes 2)$) 1.622(2)	O(4)	2.	709(3)	O(4)	113.0(2)
O(4)	1.629(2) O($(1) \cdots O(1)^i$	2.	381(3)	$O(1) - Si - O(1)^{i}$	111.5(2)
< 8i - 0 >	1.623(2)	${ m O}(4)$ ($ imes$	(2) 2.	591(2)	${ m O}(4)~(imes 2)$	105.7(1)
		<	$0 \cdots 0 >$	2.	649(2)	< 20 - 8i - 0 >	109.5(1)
ZnO ₄ tetrahedron:							
Zn - O(3)	1.935(2	À O	$(3) \cdots O(1)^{ii}$	3.	211(2) Å	$\sqrt{O(3)} - Zn - O(1)^{ii}$	$111.4(1)^{\circ}$
$O(1)^{ii}$	1.951(2)	$O(1)^{iii}$	3.	259(3)	$O(1)^{iii}$	113.7(1)
$O(1)^{iii}$	1.956(2)	O(2)	3.	214(3)	O(2)	110.4(1)
O(2)	1.980(2) O	$O(1)^{ii} \cdots O(1)^{iii}$ O(2)		116(2)	$O(1)^{ii}$ -Zn $-O(1)^{iii}$	105.82(7)
< 2n - 0 >	1.956(1)			169(2)	O(2)	107.47(9)
		0	$(1)^{iii} \cdots O(2)$	3.	179(3)	$O(1)^{iii}$ -Zn— $O(2)$	107.71(8)
		<	$0 \cdots 0 >$	3.	191(2)	$< \Delta O - Zn - O >$	109.4(1)
Framework anions	:						
Zn^{iv} -O(1)-Zn	v 114.01(7) Å	∠Zn—O(2)	—Zn ⁱ 119).8(1)° ∠Zr		$_{pi}$ 126.9(1)° $_{Si}$ (4)-Si ^{vii} 149.5(5
Zn ^{iv} -O(1)-Si	116.7(1)	Zn-O(2)	$-Si(\times 2)$ 119	0.47(6) Zn-	-O(3) - H(3)	5) $(\times 2)$ 110.4(2)	
Zn ^v -O(1)-Si	128.3(1)						
Water molecule an	d hydroxyl gro	up:					
O _d 1	H C) _a	$O_d - H$	$H \cdots O_a$	$O_d \cdots O_d$	$\mathbf{D}_a \qquad \angle \mathbf{O}_a - \mathbf{H} \cdots \mathbf{O}_a \mathbf{H} \cdots \mathbf{H}$	H ∠H−O _d −
O(2)	H(35) C	(5)	0.957(7)	1.986(8)	2 941(6)	175.0(8) —	_
0(3)		(0)	0.001(1)	1.000(0)	2 .011(0)	110.0(0)	

* Symmetry transformations for atoms outside the asymmetric unit:

- symmetry transformations for atoms outside the asymmetric unit: : i. -x, y, z ii. 0.5-x, 0.5-y, -0.5+z iii. x, y, z-1.0 iv. x, y, 1.0+z v. 0.5-x, 0.5-y, 0.5+z vi. x, -y, z vii. -x, -y, z



Fig. 1. Unit-cell diagram of the hemimorphite crystal structure. Thermal ellipsoids for all atoms represent $50^{\circ}/_{\circ}$ probability surfaces. Atoms outside the asymmetric unit are labelled with superscript primes

c axis. The water molecules near the centers of these cavities (in the plane of the eight fold rings), are orientented parallel to (010), and are held in place by hydrogen bonds to the hydroxyl groups (Fig. 2). On heating, it is apparent that the H₂O molecule is able to pass through the 6-membered ring (possibly by a process of proton exchange with the hydroxyl group) into an anjacent (vacated) cavity above or belong it in the c-axis direction without disruption of the structure.

Although the nuclear coordinates determined in the present study (Table 2) for the atoms not involved in bonds to hydrogen are similar to the values derived by x-ray diffraction as reported by McDONALD and CRUICKSHANK (1967), the positions of the oxygen atoms of the hydroxyl group, O(3), and water molecule, O(5), differ by 0.03 and 0.07 Å, respectively. The significance of these differences is, of course, markedly reduced by the large e.s.d.'s associated with the atomic coordinates in the earlier study, but it is interesting to note that in the



Fig. 2. Section through the cavity at y = 0 in the hemimorphite unit cell. Thermal ellipses for all atoms represent $50^{0}/_{0}$ probability surfaces. Hydrogen bonds are indicated by dashed lines

case of O(3) and O(5) the oxygen position in the x-ray study is displaced, relative to the neutron diffraction position, toward the region of space expected to be occupied by the oxygen lone pair electrons. Similar shifts due to asphericity of the electron distribution, as determined in the x-ray study, have been reported in a number of other structures containing terminal O, N and H atoms (see, for example, COPPENS, 1977), but the magnitudes of the nonhydrogen-atom displacements in these instances are only about 0.01 Å. We are at present collecting a set of room-temperature x-ray data in an attempt to verify the apparently larger shift observed for the more isolated watermolecule oxygen atom in hemimorphite.

The O(5) atom lies slightly below the center of a rectangle of nearest-neighbor O(3) atoms in the plane at y = 0 (Fig. 2). Based on the relative sizes of the angles subtended by the O(3) atom pairs above and below O(5), MCDONALD and CRUICKSHANK (1967) inferred that the water-molecule protons are directed towards the O(3) atoms below O(5) in the unit cell but in positions considerably displaced from the O(5) - O(3) lines. In fact, the present study indicates that the H₂O group is oriented in the reverse direction, and although the O(5)-H(53) · · · O(3) angle does deviate significantly from 180° (Table 4),

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the value is only slightly smaller than the mean angle (164°) documented for hydrogen bonds in other structures (BAUR, 1970a). The $O(3) - H(35) \cdots O(5)$ angle, on the other hand, is much wider (175°). Indeed, the orientation of the water molecule appears to be controlled by the arrangement of the Zn = O(3) = Zn linkage which dictates that the H(35) atom is directed in the positive z-axis direction in an attempt to fulfill the requirements of sp^2 hybridization on O(3). The H(53) atoms respond to this situation by positioning themselves on the opposite side of O(5), in which arrangement they are able to form hydrogen bonds to O(3) from below. Since H(35) is bonded to the bridging O(3) atom in a Zn-O-Zn linkage, it is not surprising that its thermal vibration parameters are lower and its hydrogen bonds to O(5) therefore stronger than those involving the H(53) atom. Under these circumstances the observed location of the O(5) atom in a site closer to the O(3) atoms below it in the unit cell is to be anticipated. However, both sets of hydrogen bonds are relatively long (Table 4) and would not be expected to anchor the H₂O group firmly in one place. Nevertheless, the absence of any significant peaks of residual nuclear density suggests that the H_2O group is not disordered over a number of markedly different sites within the cavity as suggested by McDONALD and CRUICKSHANK (1967). Instead, the rather narrow H(53)--O(5)-H(53) valence angle, together with the large thermal parameters for H(53), indicates that the protons are disordered over a few closely spaced sites, each of which violates the overall mm2 symmetry of the cavity but lies within the observed "thermal" ellipsoid of vibration. With this in mind it is interesting to note that the O(3) - H(35) bond distance of 0.957(7) Å is close to the mean value (0.96 Å) in other structures (BAUR, 1970a), whereas the O(5)-H(53) distance of 0.888(17) Å is at the lower limit of the observed range. When these bond lengths are corrected for thermal motion under the "riding" approximation of BUSING and LEVY (1964), the values increase to 1.103(7) and 0.95(2) Å. respectively.

Molecular-orbital study

MCDONALD and CRUICKSHANK (1967) were able to justify the classification of hemimorphite as a framework structure rather than as a pyrosilicate (first proposed by ZOLTAI, 1960) by drawing attention to the approximate equality of the Si—O bond lengths in the pyrosilicate ion, and to the likelihood of sp^2 hybridization of the orbitals on O(1) and O(2) due to the close proximity of their valence angles to

120° (Table 4). Of course, the small range in Si—O bond lengths may also be rationalized in terms of an electrostatic model since all the oxygen atoms are completely charge balanced (BAUR, 1970b). However neither of these models is able to account for the relatively large range in Zn—O bond lengths from 1.935 to 1.980 Å. Therefore, in view of the apparent success of extended Hückel theory (EHT) in accounting for structural trends in a number of moderately complex systems (GIBBS *et al.*, 1972; Tossell and GIBBS, 1977) we were tempted to apply these concepts to the Si and Zn tetrahedral oxyanions in hemimorphite.

All calculations were performed with the program originally written by HOFFMANN (1963) assuming a minimum-valence sp basis for Si, Zn and O, and an s basis for H, with the corresponding valenceorbital ionization energies and Slater orbital exponents listed in Table 5. In an attempt to account for the influence of the surrounding framework on the individual T—O bond lengths within each tetrahedron, the "first coordination sphere" of tetrahedral groups was included in the calculations in both instances: the theoretical basis for the neglect of more distant atoms has been discussed by KIER (1968) In addition, since shorter bonds necessarily tend to produce larger overlap populations, we have removed this induced correlation from consideration by fixing all Si—O and Zn—O bond lengths within each oxyanion cluster to values of 1.623 and 1.956 Å, respectively (the O—H bond was left at 0.957 Å), while maintaining the O—T—O and T—O—T angles at their observed values. During the calculations, the

Atom	Orbital	VOIP*	ξ**
Si	38	$14.83 \mathrm{~eV}$	1.634
	$_{3p}$	7.75	1.428
\mathbf{Zn}	4 <i>s</i>	8.41	1.491
	4p	3.58	1.445
0	2s	32.33	2.246
	2p	15.79	2.227
н	18	13.60	1.200

Table 5. Valence orbital ionization potentials (VOIP) and Slater exponents (ξ)

* VOIP's for Si, O and H are from BASCH et al. (1965); for Zn, from CUSACHS and CORBINGTON (1970).

** Slater exponents for Si, Zn and O are from CLEMENTI and RAIMONDI (1963); for H, from BARTELL, SU and YOW (1970).

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Fig. 3. (a) Observed bond length, d (Si-O), for the central SiO₄ tetrahedron of the Si₂Zn₆O_{22²⁴⁻} cluster in hemimorphite, plotted against the corresponding Mulliken bond-overlap population, n (Si-O). (b) Observed bond length, d(Zn-O for the central ZnO₄ tetrahedron of the Si₃Zn₅O₂₂H²¹⁻ cluster in hemimorphite, plotted against the corresponding Mulliken bond-overlap population, n(Zn-O). In both (a) and (b) the overlap population was calculated assuming observed valence angles, and constant bond lengths of 1.623 and 1.956 Å for Si-O and Zn-O respectively

energy and shape of the molecular-orbital wave functions respond to the angular distortions by establishing different overlap populations for the inidividual bonds.

Figure 3(a) is a plot of the observed bond length, $d(Si-O) \approx n(Si-O)$, the calculated bond overlap population, for the four bonds in the central SiO₄ tetrahedron of the Si₂Zn₆O_{22²⁴⁻} cluster. Despite the very small range in Si-O bond lengths, it is clear that n(Si-O) correlates very well with d(Si-O), shorter bonds tending to involve larger overlap populations. Although the calculated value of n(T-O) cannot be regarded as the actual number of electrons localized in the internuclear region, the general belief that they are closely related (Courson, 1970) is consistent with the above results. Encouraged by the discovery that the correlation between d(Si-O) and n(Si-O) was not as well developed when the calculation was repeated without the influence of the zinc tetrahedra (*i.e.*, on the Si₂O₇⁶⁻ group alone), we were surprised to find that EHT was not able to rationalize the bond-length variations within the ZnO₄ group at the center of the Si₃Zn₅H²¹⁻

cluster (Fig. 3b). With the removal of the proton from consideration, EHT successfully predicts that the Zn-O(3) bond should be the shortest, and also that O(3) is the point of attachment of the proton (by assigning a significantly larger value of calculated electrical charge to that atom), but the lengths of the other bonds remain essentially independent of overlap population (Fig. 3b). In both cases the results were unaffected by the inclusion of the completely filled 3d orbitals on the zinc atoms.

Earlier studies of the isolated tetrahedral oxyanions of a large number of second-, third- and fourth-row elements have indicated that a moderate to very strong correlation exists between n(T-O) and $<O-T-O>_3$, the mean of the three O-T-O angles common to the bond (LOUISNATHAN and GIBBS, 1972; TOSSELL and GIBBS, 1977). The results of these calculations have been used to rationalize similar correlations observed between d(T-O) and $<O-T-O>_3$ in more than 100 precisely determined crystal structures containing the corresponding TO_4^{n-} groups. In both cases the correlation is observed to increase from left to right along each row of the periodic table. In other words, for a given range of $<O-T-O>_3$ the range of observed bond lengths, $\Delta d(T-O)$, and of calculated bond-overlap populations, $\Delta n(T-O)$, is greater for B, S and Se than for T atoms of lower electronegativity in the same period (TOSSELL and GIBBS, 1977).

In the past it has been suggested that these trends are a reflection of a general increase in the directional properties of the bond as the electronegativity difference between the bonded atoms decreases along each period (TOSSELL and GIBBS, 1977). The poor correlations obtained above for the zinc cluster in hemimorphite, and also from an analysis of the relationship between n(Zn-O), d(Zn-O) and $<0-Zn-0>_3$ in a number of other precisely determined crystal structures (HILL and GIBBS, in preparation) are, therefore, consistent with the overall trend observed for the fourth-row elements. However, in spite of the fact that the electronegativity of Si is only slightly larger than that of Zn, EHT was clearly far more successful in rationalizing in spite of the fact that the electronegativity of Si is only slightly larger than that of Zn, EHT was clearly far more successful in rationalizing the bond-length variations for the silicate cluster. Indeed, the intraperiod relationship for $\Delta n(T-0)$ and $\Delta d(T-0)$ with electronegativity consistently breaks down for elements in different periods, even though the bonds may be predicted (PAULING, 1960) to have the same degree of covalent character. For example, small changes in $\langle 0-T-0 \rangle_3$

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Fig. 4. (a) Variation of $\Delta d(T-O)$ as a function of d(T-O) for second-, third, and fourth-row TO_4^{n-} tetrahedral oxyanions. $\Delta d(T-O)$ is defined as $d(T-O) - \langle d(T-O) \rangle$, where $\langle d(T-O) \rangle \rangle$ is the mean d(T-O) value calculated at valence angles of 100° and 120° from published value calculated at valence angles of 100° and 120° from published linear regression equations relating d(T-O)and $\langle O-T-O \rangle_3$ (Tossell and GIBBS, 1977). (b) Variation of $\Delta n(T-O)$ as a function of d(T-O) for the same oxyanions as in (a). $\Delta n(T-O)$ is defined as $n(T-O) - \langle n(T-O) \rangle$, where $\langle n(T-O) \rangle$ is the mean n(T-O) value calculated at valence angles of 100° and 120° assuming a constant d(T-O)value and C_{3v} point symmetry for each ion. The lower curve represents the relationship between $\Delta n(Zn-O)$ and d(Zn-O) for the bond lengths in the range 1.4-2.0 Å; the upper curve gives the equivalent relationship for the SiO₄ group

within the borate group are observed to produce a greater change in the length of the common bond than the same angular changes occurring in the somewhat larger arsenate group, even though B and As have the same electronegativity. On the other hand, tetrahedral oxyanions about As and Be have approximately the same mean bond length and display very similar bond-length changes as a function of $\langle O-T-O \rangle_3$, despite the fact that the cations have significantly different electronegativities. From a detailed analysis of bond-length and valence-angle variations in a wide variety of tetrahedral oxyanions it is now clear that the dependence of bond length on the average of the three valence angles common to the bond is more a function of the mean T-O bond length itself than of the electronegativity difference between the bonded atoms (Fig. 4a).

Despite the fact that EHT is based on a totally covalent-bonding model and would therefore be expected to be strongly influenced by

the electronegativities of the atoms incorporated into the calculations, values of $\Delta n(T-O)$ far all tetrahedral oxyanions studied show a monotonic dependence on d(T-0) which is very similar to the relationship between $\Delta d(T-0)$ and d(T-0) (Fig. 4b). Indeed, this same relationship may be replicated by artificially changing the bond length over the range 1.4 to 2.0 Å for any of the TO_4^{n-} group considered. In Fig. 4b the relationship between $\Delta n(T-0)$ and d(T-0) for T = Znand Si have been plotted over the entire range of d(T-0) values. The displacement of the two curves is a reflection of the different VOIP's and Slater exponents used to characterize the orbitals in the EHT calculations, but their overall shapes are remarkably similar. Therefore, although the observed changes in bond length may indeed be rationalized in terms of changes in bond-overlap population $\left[\Delta d(T-0) \right]$ and $\Delta n(T-0)$ are linearly related with a correlation coefficient of 0.86], the fact that $\Delta n(T-0)$ is essentially independent of the exact EHT parameterization suggests that these changes are really a geo*metric* function of orbital overlap.

Under these circumstances, the success with which EHT is able to simulate changes in bond length within tetrahedral oxyanions is not so much a function of the classical (Pauling) covalent character of the bond, but of the overall size of the tetrahedral oxyanion itself. For large TO_4^{n-} groups, like the zinc tetrahedron in hemimorphite, orbital overlap is little affected by changes in O-T-O angle, and in these cases the observed bond-length distortions probably reflect the influence of other factors, perhaps electrostatic in nature, not modelled in the EHT calculation.

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