Zeitschrift für Kristallographie, Bd. 117, S. 16-36 (1962)

The crystal structures and crystal chemistry of väyrynenite, (Mn, Fe) Be (PO₄) (OH), and euclase, AlBe (SiO₄) (OH)*

By MARY E. MROSE and DANIEL E. APPLEMAN

U.S.Geological Survey, Washington, D.C.

With 2 figures

(Received September 9, 1961)

Auszug

Die Bestimmung und Verfeinerung der Kristallstrukturen von Väyrinenit und Euklas zeigen, daß diese Strukturen ähnlich, aber nicht identisch sind. Väyrinenit weist Ketten von $BeO_2(OH)_2$ - und PO_4 -Tetraedern auf, die durch Mn-O- und Wasserstoffbindungen verknüpft sind; Euklas enthält Ketten von $BeO_3(OH)$ - und SiO_4 -Tetraedern, zusammengehalten durch Al-O-Bindungen. Die Verschiedenheit beider Strukturen beruht hauptsächlich darauf, daß in der einen Struktur, die Mn und P enthält, die elektrostatischen Ladungen anders verteilt sind als in der Struktur mit Al und Si.

Abstract

Determination and refinement of the crystal structures of väyrynenite and euclase show that these minerals have related, but not identical, structures. Väyrynenite contains chains of $BeO_2(OH)_2$ and PO_4 tetrahedra linked by Mn-O and hydrogen bonds; euclase contains chains of $BeO_3(OH)$ and SiO_4 tetrahedra linked by Al-O bonds. The differences between the two structures are due principally to the difference in electrostatic charge distribution between a structure containing Mn and P and a structure containing Al and Si.

During mineralogical investigation of the rare mineral väyrynenite, $(Mn^{+2}, Fe^{+2}) Be(PO_4)(OH)$, a search of *Crystal data* (DONNAY and NOWACKI, 1954) revealed that väyrynenite and euclase, AlBe(SiO₄)(OH), have very close c/b ratios and relatively close a/b ratios. This observation, coupled with the fact that both minerals have the same chemical-formula type, AB (XO₄) Z, similar physical properties, and perfect cleavage parallel to (010), suggested that the two minerals might be isotypic. Although the x-ray powder-diffraction patterns of the two minerals differ considerably, precession photographs of their

^{*} Publication authorized by the Director, U. S. Geological Survey.

hk0, h0l, and 0kl nets are similar (MROSE and VON KNORRING, 1959, Fig. 3). In order to elucidate the relationship of these minerals, we have determined the crystal structure of väyrynenite and re-examined and refined the crystal structure of euclase, originally determined by BISCOE and WARREN (1933).

It is a pleasure for us to acknowledge the cooperation of Dr. GEORGE SWITZER of the U. S. National Museum, who made available the crystal of euclase used in our work. Special thanks are due to our colleague, HOWARD T. EVANS, JR., who gave encouragement, valuable advice, and helpful discussion throughout this study and made the final drawings for Figs. 1 and 2.

Experimental work

Crystal description and crystallographic data

The crystal of väyrynenite used for the structure determination is from the Viitaniemi pegmatite in the parish of Eräjärvi in central Finland (MROSE and VON KNORRING, 1959); it is euhedral (about 0.25 by 0.25 by 0.30 mm), translucent, pale Congo pink according to Ridgway's color standards and nomenclature (1912), prismatic elongated along [001] with $m\{110\}$ dominant over $b\{010\}$ and $c\{001\}$. Data for the redetermination and refinement of the structure of euclase were obtained from a small $\{010\}$ cleavage fragment (about 0.25 by 0.25 by

	Väyrynenite	Euclase
Cell data		
a	$5.411 \pm 0.005\mathrm{\AA}$	$ m 4.763\pm0.005 m \AA$
b	14.49 ± 0.02	14.29 ± 0.02
С	4.730 ± 0.005	4.618 ± 0.005
β	$102^\circ45^\prime\pm05^\prime$	$100^\circ 15^\prime \pm 05^\prime$
a:b:c	0.373:1:0.326	0.333:1:0.323
Space group	$P2_{1}/a-C_{2h}^{5}$	$P2_1/a - C_{2h}^5$
Cell contents	$4[(Mn,Fe)Be(PO_4)(OH)]$	$4[AlBe(SiO_4)(OH)]$
Volume	361.7 A ³	$309.3 A^{3}$
Density, $g \cdot cm^{-3}$		
(calc.)	3.23 (for Mn:Fe = $85:15$)	3.115
(obs.)	3.215 ± 0.005 (microbalance)	3.095 ± 0.005 (microbalance)
. ,	3.22 (suspension method)	

 Table 1. Crystallographic data for väyrynenite and euclase from MROSE and VON KNORRING (1959)

Z. Kristallogr. Bd. 117, 1

0.25 mm), cleaved from a pale emerald-green euclase crystal from Villa Rica, Minas Gerais, Brazil (USNM R3775). Single-crystal x-ray data for both minerals, as well as indexed powder-diffraction data, were reported by MROSE and VON KNORRING (1959); the single-crystal data are repeated in Table 1 for convenience.

Intensity data

Triple-film Weissenberg patterns using Mo/Zr and Cu/Ni radiations were taken of the hk0 and 0kl zones of väyrynenite and euclase. The films, interleaved with 0.0005 inch Ni foil, were given long and short exposures of 72 hours and 9 hours, respectively, with molybdenum radiation at 50 kV and 20 mA. In addition, exposures were made of both minerals with copper radiation in order to obtain intensity data for reflections with small Bragg angles.

For each mineral the intensities were estimated visually by direct comparison with a standard calibrated spot strip, prepared in the Weissenberg camera using a selected reflection from the respective hk0 pattern. Intensity data were collected to the value $(\sin \theta)/\lambda =$ 1.0 Å⁻¹ for the hk0 zone of väyrynenite, and to the value $(\sin \theta)/\lambda =$ 1.4 Å⁻¹ for the hk0 zone of euclase and the 0kl zones of both minerals. Intensity data were estimated for 187 non-equivalent hk0 and 277 non-equivalent 0kl reflections of väyrynenite, and for 262 non-equivalent hk0 reflections and 204 non-equivalent 0kl reflections of euclase. The estimated intensities were converted to F_{hkl}^2 values by applying Lorentz and polarization corrections, calculated with a digital computer for each reciprocal-lattice point as a function of its cylindrical coordinates. The intensity data for each zone, read from the several films, were placed on the same relative scale by the use of appropriate film factors, based on a comparison of intensity readings of equivalent reflections appearing on more than one film. The multiple-film technique gave a range in scaled F_{hkl}^2 of approximately 1 to 1000 for both minerals.

Because small and nearly equant crystals were used for collecting the intensity data, absorption errors were minimized and no corrections were attempted.

Other considerations

During the course of the determination and refinement of the crystal structures, described in the following sections, all Fourier syntheses were carried out on a Burroughs 205 computer using a program devised by H. T. EVANS, JR. (1959). Structure-factor and leastsquares calculations, including the calculation of standard errors in the atomic parameters, were performed on the same computer with a program written by E. MONASTERSKI and D. E. APPLEMAN of the U.S. Geological Survey (written communication); the formulas and procedures followed are described in detail in CLARK and CHRIST (1959). During each structure-factor calculation, the overall isotropic temperature coefficient B and the scale factor relating observed and calculated structure factors were refined by a least-squares method. The following atomic scattering factors were used: Be⁰, O⁰, Al⁺³, and Si⁺⁴ from BERGHIUS and others (1955); P⁰ from Internationale Tabellen (1935); and Mn^o from THOMAS and UMEDA (1957). The effect of 15 atom-percent substitution of Fe⁺² for Mn⁺² on the atomic scatteringfactor curve is slight and has been neglected. Maxima on the Patterson and electron-density projections were calculated by the method of Воотн (1948).

Determination and refinement of the structures

A. Väyrynenite

The determination of the structure of väyrynenite was begun by preparing a Patterson projection on the plane normal to [001]. The resulting Patterson function showed many well-resolved peaks; those associated with the Mn-Mn, P-P, and Mn-P vectors were identified, and the following x and y coordinates were assigned: Mn, x = 0.250 and y = 0.439; P, x = 0.197 and y = 0.102. Structure factors calculated on the basis of these two atoms gave a residual factor R of 0.39 where

$$R = \Sigma ||F_{obs}| - |F_{calc}|| / \Sigma |F_{obs}|$$
 ,

excluding terms for which $|F_{obs}| = 0$. An electron-density projection along [001] was then calculated, using those terms for which signs were determined by the contributions of the Mn and P atoms to the structure factors. All the atoms in the structure (except for the hydrogen) were well resolved and easily identified on this projection.

The x and y coordinates taken from the first electron-density map are given in Table 2, column 1. The trial structure derived from this map consists of chains of $BeO_2(OH)_2$ and PO_4 tetrahedra, held together by octahedrally coordinated Mn. The oxygen atom not bonded to

 2^*

P	ara- eters	St	age of 1	refineme	ent ¹	Standard errors in atomic para- meters (in cy- cles) ²	Individ tropic ten coefficie based o mete colum	ual iso- nperature ents B_i n para- rs of an (4)
			(2)	(3)	(4) (final)	(5)	(6) hk0	(7) 0kl
Mn	x	0.250	0.249	0.248	0.248	$\varepsilon_x 0.0004$		
	y	.439	.439	.439	.4390	ε _y .0001		
	\boldsymbol{z}	.935	.939	.940	.940	ε_z .0003	0.51	0.36
\mathbf{P}	x	.194	.194	.195	.195	ε_x .0006		
	y	.099	.102	.104	.1045	ε_y .0002		
	z	.541	.542	.539	.539	ε_z .0005	.23	.14
O,	x	.388	.376	.366	.366	£0019		
- 1	u	.046	.042	.041	.0401	ε		
	z	.775	.760	.753	.753	ϵ 0014	.59	.40
0	m	471	460	169	162	0.0016		
O_2	2	365	.409 367	.403 386	2662	$\varepsilon_x 0000$		
	9 ~	667	651	.500	.3002	ε_y .0000	90	40
0	~	.007	.001	.004	.004	<i>ez</i> .0015	.09	.40
O_3	x	.354	.353	.346	.346	ε_x .0017		
	y	.190	.190	.193	.1932	ε_y .0006		
	z	.511	.513	.506	.506	ε_z .0015	.46	.37
O_4	\boldsymbol{x}	.115	.109	.106	.106	ε_x .0019		
	y	.060	.059	.060	.0600	ε_y .0006		
	z	.242	.246	.245	.245	ε_z .0014	.59	.40
O_5	x	.206	.210	.212	.212	ε0018		
(OH)y	.276	.276	.276	.2765	ε_{y} .0006		
	z	.964	.964	.968	.969	ε_z .0013	.74	.34
\mathbf{Be}	x	.442	.433	.431	.431	ε		
	y	.268	.268	.264	.2640	ε, .0012		
	z	.801	.803	.784	.783	ε_z .0029	.65	.65
R	hk0	.256	.137	.095	.085	-		
	0kl	.160	.130	.106	.091			
B	hk0	1.00	.49	46	46			
-	0 <i>kl</i>	.44	.33	.33	.33			

 Table 2. Atomic positional parameters, standard errors, and temperature coefficients for väyrynenite

¹ See text for description of stage of refinement.

² For method of calculation see CLARK and CHRIST (1959).

R The residual factor; ΔF omitted in calculation if $F_{obs} = 0$.

B The average isotropic temperature coefficient (in \mathring{A}^2).

 ε_x The standard error in the positional parameter x of an atom.



Fig. 1. Electron-density projections of väyrynenite and euclase: contour interval 10 e/Å², zero contour omitted; (a) väyrynenite, projection along [001]; (b) euclase, projection along [001]; (c) väyrynenite, projection along [100]; (d) euclase, projection along [100]. Atoms are numbered to agree with Tables 7 and 8.

phosphorus is considered to be associated with the hydroxyl group. The R factor based on these coordinates is 0.26.

A second and third electron-density projection along [001] were then computed. In order to resolve the oxygen atoms more clearly, the third electron-density projection was calculated in the following manner. Structure factors consisting only of Mn and P contributions were calculated. A Fourier synthesis was then made using these values, subject to the same termination-of-series conditions imposed in the Fourier syntheses by the observed factors. The resulting electrondensity distribution was subtracted point-by-point from the ordinary electron-density distribution, giving a projection essentially free from the effects of Mn and P. The second electron-density projection along [001], which was the final projection to include the contributions of the Mn and P atoms, is shown in Fig. 1*a*; it led to the *x* and *y* coordinates of Table 2, column 2.

Least-squares refinement of the hk0 zone was carried out in two stages: (1) five cycles, refining the positional parameters, an overall isotropic temperature factor, and the scale factor, yielded the x and y coordinates of Table 2, column 3; (2) eight cycles, fixing the scale factor and refining the positional parameters and individual isotropic temperature coefficients, yielded the x and y coordinates of Table 2, column 4. The final R factor for the hk0 zone is 0.085.

The z coordinates of the atoms in the väyrynenite structure were determined from the data of the 0kl zone, employing Patterson and electron-density projections along [100] and least-squares techniques, in a manner analogous to that of the hk0 refinement described above. The z coordinates of Table 2, columns 1 through 4, represent values at corresponding stages of refinement. The last complete electrondensity projection along [100] is shown in Fig. 1c. A final projection, with the effects of Mn and P removed in the manner described above, yielded the z coordinates of Be and O in Table 2, column 2. Leastsquares refinement was started with the y and z coordinates of Table 2, column 2; the final z coordinates after refinement with an overall temperature coefficient (five cycles) and individual isotropic temperature coefficients (six cycles) are shown in Table 2, columns 3 and 4, respectively. The y coordinates obtained from the two zones of data are virtually identical within the standard errors given in Table 2, column 5. Observed and calculated structure factors for the hk0 and 0kl zones of väyrynenite are listed in Table 3. The final residual factors are $R_{hk0} = 0.085$ and $R_{0kl} = 0.091$.

22

h k	1	"。	•	h k l	۳.	۴.	h k. 1	×.	7 ₆	h k 1	۳.	r.,	b k l	Fo	r _c	h k 1	P_0	P _c
まえる。 脱し コットス・コーロ はん マスクス ひょう ひょういく ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・ス・		900以江江207-314446日 - 7894352153235348292日月9269 - 9692663742737843487988 8264438645 37744611 - 6 - 911368 - 31488 - 31488 - 21177-802 737542139	ー・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・		29110 - 216019 9 - 93478 5 4 - 30 6118 7 51420676 6 - 18 532292401813 18 - 17 - 19 33135 - 29 821982219820 5 20 33339724359 - 10 - 172 500 9119152 - 10 - 12 - 18 - 7 7 1011 - 1512101910 5 90			10 - 7513079-10 - 11 - 18 - 11919811 - 12 4115582373-33725900 - 223 - 9 - 815322576700 - 6248 - 7115109 - 25 - 18 - 778 - 14 - 13 820 - 21 11 - 77. 151025580024433 - 28 - 714 520		22222222222222222222222222222222227)57575757	1919-19-19-10-1910-89841461314-54172455833891450299-25-8122-20-99-89575316239023717-8475162112725-8813-50-20-1710-11791133177-60		00000000000000000000000000000000000000	166~6556928~1682~1686~13~6~~~~~547~250327822372222~34~1113387~9~8~11~11~11~11~11~11~22432321~991523311~7915724~816~19~153945~19~1291109816~				

Table 3. Comparison of observed and calculated structure factors for the hk0 and 0kl zones of väyrynenite

B. Euclase

The crystal structure of euclase was derived by BISCOE and WAR-REN (1933) using trial-and-error methods. Because of the stated inaccuracy in the atomic parameters listed in their paper and the contradictory positions shown for beryllium, we decided to redetermine and refine the euclase structure for comparison with that of väyrynenite. Assuming the two minerals to be approximately isotypic, the

6053542415001106093409219435-049856677

analysis was begun by calculating structure factors for the 0kl zone of euclase, using the y and z coordinates from the corresponding final complete electron-density projection of väyrynenite. Signs from these structure factors were then used to compute the first electron-density projection along [100], which resolved clearly all the atoms except for Be and the hydroxyl oxygen O_5 . Two possible locations for each of these two atoms were indicated, one set corresponding to that in the väyrynenite structure and one set in new positions. The y and z coordinates of Table 4, column 2, include the Be and O_5 positions corresponding to väyrynenite; the R factor for these coordinates is 0.55. Subsequent refinement showed that the alternate positions for Be and O_5 were, in fact, the correct ones; the y and z parameters from the first electron-density projection along [100] (Fig.1d), yielding an R factor of 0.24, are given in Table 4, column 3.

A trial structure of euclase, derived from the electron-density projection along [100], consisted of chains of $BeO_3(OH)$ and SiO_4 tetrahedra, held together by octahedrally coordinated Al. The oxygen atom not bonded to silicon is considered to be associated with the hydroxyl group. Least-squares refinement was then carried out on the data of the 0kl zone; column 4 (Table 4) shows the y and z parameters obtained after five cycles of refinement with an overall temperature coefficient; and column 5 (Table 4) shows the final y and z parameters after twelve cycles of refinement including individual isotropic temperature coefficients.

The differences noted above between the väyrynenite and euclase structures led us to compute a Patterson projection along [001] before attempting refinement of the hk0 zone of euclase. Comparison of the two Patterson maps showed sufficient similarity to justify calculation of the electron-density projection of euclase along [001], using signs based on x and y coordinates of Al and Si from the Patterson projection, y coordinates for all remaining atoms from the 0kl refinement of euclase (Table 4, column 5), and x coordinates for the oxygen atoms from the final parameters for väyrynenite (Table 2, column 4). The x coordinate of Be was assigned as 0.200 from consideration of interatomic distances. The residual factor for this set of coordinates is 0.44; the resulting electron-density map resolved all atoms in the structure, yielding the x coordinates of Table 4, column 2. A final electron-density projection (Fig. 1b) gave the x coordinates of Table 4, column 3.

Least-squares refinement of the atomic parameters, based on the data of the hk0 zone, gave the x coordinates of Table 4, columns 4

The crystal structures of väyrynenite and euclase

Pa me	ira- ters	Biscoe and Warren (1933) ¹	Sta	ge of 1	refinem	nent^2	S e ato (ir	tandard rrors in mic para- meters n cycles) ³	$\begin{array}{c} \text{Individ} \\ \text{tropic to} \\ \text{ture coe} \\ B_j \text{ based} \\ \text{mete} \\ \text{colum} \end{array}$	ual iso- empera- officients on para- ers of un (5)
		(1)	(2)	(3)	(4)	(5) (final)		(6)	(7) hk0	(8) 0kl
Al	x	0.25	0.248	0.249	0.249	0.249	ε_x	0.0034		
	y	.44	.439	.443	.445	.4447	ε_y	.0040		
	z	.03	.946	.954	.956	.957	εz	.0052	0.10	0.19
Si	x	.15	.176	.177	.177	.177	ε _x	.0027		
	y	.10	.101	.100	.100	.1002	ε_y	.0030		
	\boldsymbol{z}	.53	.546	.542	.540	.539	ε_z	.0039	.06	.00
01	x	.39	.378	.381	.380	.380	Ex	.0067		
-	y	.05	.036	.036	.032	.0321	ε_y	.0080		
	\boldsymbol{z}	.78	.770	.761	.760	.761	ε_z	.0083	.18	.08
02	x	.39	.378	.383	.381	.380	Ex	.0071		
-	y	.35	.363	.374	.377	.3767	ε_{y}	.0080		
	z	.72	.643	.638	.650	.650	ϵ_z	.0090	.21	.16
03	x	.37	.347	.344	.346	.346	ε_x	.0062		
•	y	.19	.196	.199	.200	.2000	ε_y	.0080		
	z	.46	.530	.525	.524	.520	ε_z	.0100	.20	.34
O₄	x	.17	.101	.101	.100	.100	ε_x	.0066		
-	y	.03	.059	.051	.052	.0529	ε_{y}	.0080	1	
	z	.26	.236	.223	.222	.215	ε_z	.0100	.27	.21
0,	x	.18	.167	.158	.157	.158	ε_r	.0075		
(ŐH)y	.33	.274	.330	.332	.3324	E v	.0100		
	z	.22	.957	.122	.116	.118	ε_z	.0116	.42	.39
Be	x	.48*	.193	.180	.177	.176	ϵ_x	.0164)	
	y	.30	.250	.300	.301	.3000	ε_y	.0269		
	z	.50	.777	.478	.468	.470	ez	.0267	.46	.90
R	hk0		.300	.174	.148	.139			8	
	0kl	1	.552	.235	.177	.122			ł	
В	hk0		.20	.09	.14	.15				
	0kl	ļ	.28	.18	.14	.14				

Table 4. Atomic positional parameters, standard errors, and temperature coefficients for euclase

¹ Original parameters transferred to setting of present study.

² See text for the description of stage of refinement.

³ For method of calculation see CLARK and CHRIST (1959).

R The residual factor; $\varDelta F$ omitted in calculation if $F_{obs}=$ 0.

B The average isotropic temperature coefficient (in $Å^2$).

 ε_x The standard error in the positional parameter x of an atom. * Possibly due to a misprint in BISCOE and WARREN'S paper (1933).

and 5, after seven cycles of refinement with an overall temperature coefficient and seven additional cycles including individual isotropic temperature coefficients. Several small-angle reflections affected by

	nk0 ana 0k	for the h	factors j	ructure j	uted s	lcula	and ca	observed	of a	narison	5 Com	bla ^K	ТA	
	h k 1 Po	۴ _с հ	P.,	h k l	۳.	F	h k l	¥.	Ρ.	h k l	P. 001112	P 010 01	a	1
$ \begin{array}{c} \mathbf{F}_{0} \qquad \mathbf{F}_{0} \qquad \mathbf{h} \qquad \mathbf{k} \qquad $	$ \begin{array}{c} n_{\rm ch} \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	for the $k = 0$ = 0 = 0 = 0 = 0 = 0 = 0 = 0 = 0 =	Factors :	$ructure \int $		Loula	and cai h k 1 0 <td>observed r - - - <td></td><td>a + b + b + b + b + b + b + b + b + b +</td><td></td><td></td><td></td><td>Table 5.</td></td>	observed r - - - <td></td> <td>a + b + b + b + b + b + b + b + b + b +</td> <td></td> <td></td> <td></td> <td>Table 5.</td>		a + b + b + b + b + b + b + b + b + b +				Table 5.

26

The crystal structures of väyrynenite and euclase

Table 5. (Continued)

1 k 1	7.	۰.	6 k 1	F	Fc	h k l	Fa	Fc	h k 1	P _o	Pc	6 k 1	P _o	r _e	b k 1	P _o	y _e
€ 0	12 11 17 8 11 	$ \begin{array}{r} 12\\ 12\\ 3\\ -8\\ 9\\ -2\\ -5\\ -1\\ 3\\ 10\\ 2\\ -1\\ -10\\ \end{array} $	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	878		$\begin{array}{cccccccccccccccccccccccccccccccccccc$	9611111	- 8 - 10 - 3 - 4 8 4 - 8 - 4 8 - 4 8 - 4 8 1 - 4 8 1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	6 	2 5 2 2 5 2 2 2 7 1 5 2 2 7 1 5 2 2 7 1 5 2 2 7 1 5 2 2 7 1 5 2 2 7 1 5 2 2 7 1 5 2 2 7 1 5 2 2 7 1 5 2 1 2 1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	8 97 16 		$\begin{array}{cccccccccccccccccccccccccccccccccccc$	13 4 5 6 9 7 7 4	- 13
Ø buit	 from	lesst-square	a refinement														

extinction were not included in the least-squares refinement. The y coordinates obtained from the refinement of the two zones of data are virtually identical. The standard errors in atomic parameters are given in Table 4, column 6. Observed and calculated structure factors for the hk0 and 0kl zones of euclase are listed in Table 5. The final residual factors are $R_{hk0} = 0.14$ and $R_{0kl} = 0.12$.

Description of the structures

A. Väyrynenite

The crystal structure of väyrynenite consists of zigzag chains with composition $[Be_2(OH)_2(PO_4)_2]_n^{-4n}$ (Fig. 2a) cross-linked by Mn atoms between the chains and by hydrogen bonds. There are two crystallographically equivalent chains in the unit cell, oriented parallel to [100], and related to each other by a center of symmetry so that the *b* cell dimension represents two chain-widths. The Mn atoms lie between the chains and are octahedrally coordinated by two oxygen atoms and one hydroxyl oxygen from one chain (O₁, O₂, O₅,) and by one oxygen atom from each of three adjacent chains (O'₁, O'₄, O₄), as shown in Figs. 1a and 1c.

Within the $[Be_2(OH)_2(PO_4)_2]_n^{-4n}$ chain (Fig. 1*a*) each Be atom is tetrahedrally coordinated by two oxygen atoms (O₂, O'₃) and by two hydroxyl oxygens (O₅, O'₅); each P atom is tetrahedrally coordinated by four oxygen atoms (O₁, O₂, O₃, O₄). The characteristic features of the chain are as follows: (1) PO₄ tetrahedra do not share corners with other PO₄ tetrahedra, but share two oxygen atoms with two adjacent $BeO_2(OH)_2$ tetrahedra; (2) no oxygen atom is shared by more than two tetrahedra; (3) all four corners of the $BeO_2(OH)_2$ tetrahedron are shared, two with adjacent $BeO_2(OH)_2$ tetrahedra and two with PO₄ tetrahedra, thereby forming a puckered six-membered ring consisting of Be-O₂-P-O₃-Be-O'₅ (Fig. 1*a*); (4) the hydroxyl oxygen, O₅, is shared between two $BeO_2(OH)_2$ tetrahedra.

The crystal structures of väyrynenite and euclase

Table 5. (Continued)

i k	1	۲.	Fc	h k l	F ₀	F.	h k 1	F	۴	h k l	۶.	Pe	h k l	Р ₀	Fc	h k l	F _0	۶.
8 0	9	12	12	0 15 9	8	- 7	0 1 10	9	- 8	0 16 10	6	2	0 6 11	-	- 2	0 0 12	13	- 13
1 2	9	11	12	0 16 9 0 17 9	7	- 7	0 2 10 0 3 10	-	- 10	0 18 10	-	- 2	0 9 11	9	12	0 2 12	Ĩ.	- 5
* 3	2	17	- 8	0189	2	- 6	0 4 10 0 5 10	-	7	0 19 10 0 20 10	1	- 2	0 9 11 0 10 11	16	- 9	0 3 12 0 4 12	5	1 6
12	9	11	9	0 20 9	-	- 2	0 6 10	-	8	$0\ 21\ 10$ $0\ 22\ 10$	7	- 7	0 11 11 0 12 11	2	26	0 5 12 0 6 12	6	- 3
11	9		- 2	0 22 9		í	0 8 10	10		0 23 10	÷	- 5	0 13 11	Ξ	1	0 7 12	-	- 5
13	9	-	- 1	0 24 9	7	-	0 10 10	-	- 3	0 0 11	Ξ.	- 2	0 15 11	-	-	0 9 12	<u> </u>	2
610	9	9	10	0 25 9	-	- 2	0 12 10	-	6	0 2 11	7	- 6	0 17 11	÷.	_ 0	0 11 12	-	- 2
112	9	- 2	- 1	02790289	2	- 4	0 13 10 0 14 10	2	- 4	0 3 11 0 4 11	в	- 3	0 18 11 0 19 11	-	- 6	0 12 12 0 13 12	1	- 3
1	9	10	- 10	0 0 10	14	- 16	0 15 10	-	1	0 5 11	-	0	0 20 11	-	1			
P is	ittei	i from	least-squar	es refinemen	t													

extinction were not included in the least-squares refinement. The y coordinates obtained from the refinement of the two zones of data are virtually identical. The standard errors in atomic parameters are given in Table 4, column 6. Observed and calculated structure factors for the hk0 and 0kl zones of euclase are listed in Table 5. The final residual factors are $R_{hk0} = 0.14$ and $R_{0kl} = 0.12$.

Description of the structures

A. Väyrynenite

The crystal structure of väyrynenite consists of zigzag chains with composition $[Be_2(OH)_2(PO_4)_2]_n^{-4n}$ (Fig. 2a) cross-linked by Mn atoms between the chains and by hydrogen bonds. There are two crystallographically equivalent chains in the unit cell, oriented parallel to [100], and related to each other by a center of symmetry so that the *b* cell dimension represents two chain-widths. The Mn atoms lie between the chains and are octahedrally coordinated by two oxygen atoms and one hydroxyl oxygen from one chain (O₁, O₂, O₅,) and by one oxygen atom from each of three adjacent chains (O'₁, O'₄, O₄), as shown in Figs. 1a and 1c.

Within the $[Be_2(OH)_2(PO_4)_2]_n^{-4n}$ chain (Fig. 1*a*) each Be atom is tetrahedrally coordinated by two oxygen atoms (O₂, O'₃) and by two hydroxyl oxygens (O₅, O'₅); each P atom is tetrahedrally coordinated by four oxygen atoms (O₁, O₂, O₃, O₄). The characteristic features of the chain are as follows: (1) PO₄ tetrahedra do not share corners with other PO₄ tetrahedra, but share two oxygen atoms with two adjacent BeO₂(OH)₂ tetrahedra; (2) no oxygen atom is shared by more than two tetrahedra; (3) all four corners of the BeO₂(OH)₂ tetrahedron are shared, two with adjacent BeO₂(OH)₂ tetrahedra and two with PO₄ tetrahedra, thereby forming a puckered six-membered ring consisting of Be-O₂-P-O₃-Be-O'₅ (Fig. 1*a*); (4) the hydroxyl oxygen, O₅, is shared between two BeO₂(OH)₂ tetrahedra. The identification of O_5 as the hydroxyl oxygen was originally made from the fact that O_5 is the only oxygen atom in the structure which is not part of the tightly bonded PO_4 complex. The distance of 2.76 Å between O_5 and O'_3 (Fig. 1c) shows the presence of a hydrogen bond (PIMENTEL and McCLELLAN, 1960, p. 260). Calculation of bond strengths based on a simple model (Table 6), assuming: (1) all Mn-O bonds = $\frac{1}{3}$, (2) all P-O bonds = $1\frac{1}{4}$, (3) all Be-O bonds = $\frac{1}{2}$, shows



Fig. 2. A comparison of the $[Be_2(OH)_2(PO_4)_2]_n^{-4n}$ chain of väyrynenite (a) with the $[Be_2(OH)_2(SiO_4)_2]_n^{-6n}$ chain of euclase (b): large single circles, oxygen; large double circles, hydroxyl oxygen; small open circles, beryllium; small solid circles, silicon or phosphorus.

that local charge balance is maintained if the hydrogen bond is assigned a strength of $\frac{1}{3}$. This value is consistent with the average O-H distance of 1.0 Å given by PIMENTEL and McCLELLAN (1960) for hydrogen bonds with O-O distances in the neighborhood of 2.76 Å.

The perfect {010} cleavage of väyrynenite (MROSE and VON KNOR-RING, 1959) is between the chains and requires breaking only four Mn-O bonds for each unit cell; the good but difficult {100} cleavage is normal to the chains and requires breaking two Be-OH bonds, two Be-O or

Compound	Anion	Coordinated cations	Sum of bond strengths
Väyrynenite	0 ₁ , 0 ₄	2 Mn, 1 P	1.92
Euclase	O_1, O_4	2 Al, 1 Si	2.00
Väyrynenite	O,	1 Mn, 1P, 1 Be	2.08
Euclase	O_2	1 Al, 1 Si, 1 Be	2.00
Väyrynenite	0,	1 P, 1 Be, (1 H)	1.75, (2.08)
Euclase	O_3	1 Si, 2 Be	2.00
Väyrynenite	O ₅ *	1 Mn, 2 Be, (1 H)	1.33, (2.00)
Euclase	0 ₅ *	1 Al, 1 Be, (1 H)	1, (2.00)

Table 6. Summary of approximate bond strengths in väyrynenite and euclase The following bond strengths are assumed: Mn-O = 1/3, Al-O = 1/2, P-O = 5/4, Si-O = 1, Be-O = 1/2; väyrynenite, $H-O_2 = 1/3$, $H-O_5 * = 2/3$.

* Hydroxyl oxygen.

P-O bonds, and four Mn-O bonds; the fair {001} cleavage is between the chains but requires breaking eight Mn-O bonds and four hydrogen bonds. The observed order of the cleavages is in agreement with the bond strengths calculated on the basis of the charge balance discussed above.

Interatomic distances and bond angles found in väyrynenite are listed in Tables 7 and 8, together with standard errors calculated by the method described in CLARK, CHRIST, and APPLEMAN (1962). The average P-O distance in the PO₄ tetrahedron is 1.53 Å; for comparison, the average P-O distance found in the well refined LiMnPO₄ structure (GELLER and DURAND, 1960) is 1.54 Å for discrete PO₄ tetrahedra. Although the shortest tetrahedral P-O distances in väyrynenite, as would be expected, are those involving oxygens not shared with BeO₂(OH)₂ tetrahedra, the differences in bond lengths within the PO₄ tetrahedron may not be significant in view of the standard errors given in Table 7. The distortion of the PO₄ tetrahedron shown by the O-P-O angles listed in Table 8 is apparently real, and similar distortion has been reported in several recent studies of phosphate structures, summarized by GELLER and DURAND (1960).

The $BeO_2(OH)_2$ tetrahedron is not as regular as the PO_4 tetrahedron; the most noticeable distortion in the former is the length of the edge between the hydroxyl oxygens. The average tetrahedral Be-O distance of 1.65 Å found here is the same as the average Be-O distance

Table 7. Interatomic distances and standard errors in väyrynenite and euclase(See Fig. 1)

V	äyrynenite	E	luclase
•,, <u></u>	Distances i	n tetrahedron around i	Be
Be-O ₂	$1.63~{ m \AA}$	Be-O ₂	1.60 Å
$Be-O_{5}^{\prime}*$	1.69	Be-O ₃	1.63
$Be-O'_3$	1.65	$Be-O'_3$	1.64
Be-O ₅ *	1.63	BeO ₅ *	1.68
Average	1.65 ₀ Å	Average	1.63 ₈ Å
Be-O bonds	\pm 0.03 Å	Be-O bonds	\pm 0.04 ⁴ Å
	Distances in t	etrahedra around P an	d Si
$P - O_1$	$1.53~{ m \AA}$	Si–O ₁	1.61 Å
$P = O_{2}$	1.53	Si-O,	1.62
P03	1.55	Si-O ₃	1.65
$P-O_4$	1.51	Si-O4	1.62
Average	1.53. Å	Average	1.62. Å
PO bonds	$\pm 0.01_5$ Å	Si-O bonds	$\pm 0.01_5$ Å
	Distances in o	ctahedra around Mn a	nd Al
$Mn-O_1$	$2.08~{ m \AA}$	Al-O ₁	1.86 Å
$Mn - O'_1$	2.24	$Al - O'_1$	1.98
Mn←O₄	2.14	Al-O ₄	1.87
$Mn - O'_4$	2.18	Al-O'	1.93
Mn-O ₂	2.23	$Al - O_2^*$	1.91
Mn-O ₅ *	2.37	Al-O ₅ *	$1.85(\pm 0.01_9 {\rm ~\AA})$
Average	2.20, Å	Average	1.90 ₀ Å
Mn-O bonds	$+ 0.01_{5}$ Å	Al-O bonds	± 0.01 Å
		(excluding Al-	O_5)
Oxyg	en-oxygen distance	s for oxygens bonded t	o the same Be
$O_2 - O'_3$	$2.64~{ m \AA}$	$O_2 - O'_3$	$2.59~{ m \AA}$
$O_2 - O_5^*$	2.58	$O_2 - O_5^*$	2.57
$O_2 - O_5'^*$	2.72	$O_2 - O_3$	2.74
O' ₃ -O ₅ *	2.73	O'O_5*	2.69
O'_3O'_5*	2.65	O ₃ -O ₅ *	2.62
$O_5^* - O_5'^*$	2.81	$O'_3 - O_3$	2.78
Average	2.68_8 Å	Average	2.66_5 \AA

	Tab	le 7. ($Continued$)	
V	äyrynenite	E	Luclase
Oxygen	oxygen distances f	or oxygens bonded to	the same P or Si
$0_1 - 0_2$	$2.52~{ m \AA}$	$0_1 - 0_2$	$2.68~{ m \AA}$
$0_1 - 0_3$	2.50	$O_1 - O_3$	2.64
$0_1 - 0_4$	2.52	0 ₁ 0 ₄	2.65
0 ₂ -0 ₈	2.48	$O_2 - O_3$	2.64
$0_{2}-0_{4}$	2.48	$O_2 - O_4$	2.63
0304	2.50	$O_3 - O_4$	2.68
Average	2.50_0 Å	Average	2.65_3 Å
Oxygen-o	oxygen distances fo	r oxygens bonded to t	he same Mn or Al
0 ₁ 0' ₁	$2.72~{ m \AA}$	$O_1 - O'_1$	$2.47~{ m \AA}$
$0_1 - 0_2$	3.63	$O_1 - O_2$	2.84
0 ₁ O ₄	3.20	$0_1 - 0'_4$	2.74
0 ₁ -0 ₅ *	3.28	$O'_1 - O_5^*$	2.73
0'1O4	2.94	$O'_1 - O_4$	2.61
$0'_{1} - 0'_{4}$	2.99	$O_1' - O_4'$	2.69
0'1O5*	4.09	0 ₁ -0 ₅ *	2.92
$0_2 - 0_4$	2.93	$O_2 - O_4$	2.82
$0_2 - 0'_4$	2.89	$O_2 - O'_4$	2.59
$0_2 - 0_5^*$	2.58	$O_2 - O_5^*$	2.65
0 ₄ 0' ₄	2.92	$O_4 - O'_4$	2.54
0 ₄ 0 ₅ *	3.26	O4O5*	2.64
Average	3.11 ₉ Å	Average	2.68, Å

Distances between oxygen atoms not bonded to the same cation (only distances < 3.00 Å listed)

0 ₅ *O'3	2.76 Å	O ₃ O ₅ *	$2.93~{ m \AA}$
hydrogen bond			
(between chains)		(between chains)	
0–0 distances \pm	0.02 ₁ Å	$O-O ext{ distances (e} \pm 0.02_0 ext{ Å; } O-O_2 \pm 0.02_2 ext{ Å}$	xcluding $O-O_5^*$) $_5^*$ distances

* Hydroxyl oxygen.

I

found by JEFFREY, PARRY, and Mozzi (1956) in hexagonal beryllium oxide, which has a wurtzite-type structure. The octahedron around the Mn atom in väyrynenite is highly distorted; individual O-Mn-O angles deviate as much as 35° from the average of 90.8° .

 Table 8. Bond angles and standard errors in väyrynenite and euclase

 (See Fig. 1)

Väyr	ynenite	E	uclase
	Bond angles in t	etrahedron around	Be
O_2 -Be- O'_3	107.6°	O_2 —Be— O'_3	106.6°
$O_2 - Be - O_5^*$	104.6	O_2 —Be— O_5 *	103.7
O_2 -Be- O'_5 *	110.8	O_2 -Be- O_3	116.1
O'_3 —Be— O_5 *	112.9	O'_3 —Be— O_5 *	108.4
O'_{3} -Be- O'_{5} *	105.1	O ₃ -Be-O ₅ *	104.5
O_5^* -Be- $O_5'^*$	115.8	O_3 —Be— O'_3	116.5
Average	109.5°	Average	109.3°
O-Be-O angles	\pm 1.0 °	O-Be-O angle	$ m es~\pm 1.5^\circ$
	Bond angles in tetr	rahedra around P a	nd Si
$O_1 - P - O_2$	110.9°	O ₁ -Si-O ₂	112.4°
$O_1 - P - O_3$	108.5	O ₁ —Si—O ₃	108.3
$O_1 - P - O_4$	111.8	O ₁ —Si—O ₄	110.4
O ₂ P-O ₃	107.3	O ₂ -Si-O ₃	107.6
$O_2 - P - O_4$	109.0	O_2 -Si- O_4	108.0
O ₃ PO ₄	109.2	O_3 —Si— O_4	110.1
Average	109.5°	Average	109.5°
O_P_O angles	\pm 0.7 $^{\circ}$	0-Si-O angle	$\pm 0.7^{\circ}$

* Hydroxyl oxygen

B. Euclase

The crystal structure of euclase consists of zigzag chains with composition $[Be_2(OH)_2(SiO_4)_2]_n^{-6n}$ (Fig. 2b) cross-linked by Al atoms between the chains. The chains are oriented parallel to [100], and are disposed in the unit cell in the same way as those of väyrynenite. The Al atoms between the chains are octahedrally coordinated by two oxygen atoms from one chain (O₁, O₂), by one oxygen atom (O₄) and one hydroxyl oxygen (O₅) from one adjacent chain, and by one oxygen atom from each of two different neighboring chains (O'₁, O'₄), as shown in Figs. 1b and 1d.

Within the $[Be_2(OH)_2(SiO_4)_2]_n^{-6n}$ chains (Fig. 1b) each Be atom is tetrahedrally coordinated by three oxygen atoms (O₂, O₃, O₃) and by one hydroxyl oxygen (O₅); each Si is tetrahedrally coordinated by four oxygen atoms (O₁, O₂, O₃, O₄). The characteristic features of the chain are as follows: (1) SiO_4 tetrahedra do not share corners with other SiO_4 tetrahedra, but share two oxygen atoms with three adjacent $BeO_3(OH)$ tetrahedra; this follows from the fact that (2) one oxygen atom, O_3 , is shared by one SiO_4 and two $BeO_3(OH)$ tetrahedra; (3) three corners of the $BeO_3(OH)$ tetrahedron are shared, two with adjacent $BeO_3(OH)$ tetrahedra and one with the SiO_4 tetrahedron, thus forming a puckered six-membered ring consisting of $Be-O_2-Si-O_3 Be-O'_3$ (Fig. 1b); and (4) the fourth corner of the $BeO_3(OH)$ tetrahedron, which is the hydroxyl oxygen O_5 , is not shared with any other tetrahedron.

The hydroxyl oxygen was originally identified with O_5 from the fact that O_5 is the only oxygen atom in the structure which is not part of the tightly bonded SiO₄ complex. The shortest approach of this atom to any oxygen atom not included in the same coordination polyhedron is the 2.93 Å distance between O_5 and O_3 (Fig. 1d). In the absence of other evidence, this distance is too long to be indicative of hydrogen bonding (PIMENTEL and McCLELLAN, 1960, p. 260). Calculations of bond strengths based on a simple model (Table 6), assuming: (1) all Al-O bonds = $\frac{1}{2}$, (2) all Si-O bonds = 1, and (3) all Be-O bonds = $\frac{1}{2}$, show that local charge balance is maintained if the proton is associated entirely with O_5 .

The sharing of one oxygen atom between three separate tetrahedral groups, in the manner that O_3 in the euclase structure is shared between one SiO₄ and two BeO₃(OH) tetrahedra, is not a common structural arrangement. However, precisely the same linkage occurs in the structure proposed by BRAGG and ZACHARIASEN (1930) for phenacite, Be₂SiO₄. In that structure every oxygen atom is likewise bonded to one SiO₄ tetrahedron and two BeO₄ tetrahedra.

The perfect $\{010\}$ cleavage of euclase (MROSE and VON KNORRING, 1959), as in the case of väyrynenite, is between the chains in the structure and requires breaking only four Al-O bonds for each unit cell. The crystals of euclase used in the present study also have a good $\{100\}$ cleavage, moderately easy to produce, and a fair but very difficult $\{001\}$ cleavage. The $\{100\}$ cleavage across the chains requires breaking four Be-O bonds or two Be-O and two Si-O bonds, and four Al-O bonds; the $\{001\}$ cleavage between the chains requires breaking 12 Al-O bonds. The observed order of the cleavages is in accord with bond strengths calculated on the basis of the charge balance discussed above.

The structure found for euclase in the present study is essentially that proposed by BISCOE and WARREN (1933) and illustrated in Fig. 1b of their paper. That figure, labeled "projected upon (a, b) plane", should actually be labeled "projected upon (b, c) plane". It corresponds to a projection on the (001) plane in the setting of the present study. An erroneous x coordinate (0.48) is listed for Be (Table 4, column 1) and was used in Fig. 2 of BISCOE and WARREN (1933). With this exception, the atomic coordinates derived by those authors (Table 4, column 1) are in reasonable agreement with those obtained in the present work (Table 4, column 5).

Interatomic distances and bond angles obtained in the present study of euclase are listed in Tables 7 and 8, arranged for comparison with corresponding distances and angles found in väyrynenite. Standard errors were calculated by the method described in CLARK, CHRIST, and APPLEMAN (1962). The average Si-O distance in the SiO_4 tetrahedron is 1.62_5 Å, in good agreement with the average distances of 1.62, Å and 1.63, Å reported by SMITH, KARLE, HAUPTMAN, and KARLE (1960) for the two independent SiO₄ tetrahedra in spurrite, $Ca_5(SiO_4)_2CO_3$. The distortion induced in the SiO₄ tetrahedron of euclase by the long $Si-O_3$ distance is probably real; it is not unexpected, since O_3 is the only oxygen in the SiO₄ tetrahedron shared with two BeO₃(OH) tetrahedra. The range of the remaining Si-O distances is within the standard error of the determination. The $BeO_3(OH)$ tetrahedron is also distorted, principally owing to the long $Be-O_5$ distance. The average Be-O distance in the tetrahedron, 1.64 Å, is virtually the same as that found in väyrynenite. The octahedron around the Al atom in euclase is fairly regular, with a maximum deviation of 9.5° from the average O-Al-O angle of 90.1°. The average Al-O distance of 1.90, Å is comparable to the average distance of 1.915 Å found by BUSING and LEVY (1958) in diaspore, AlO(OH).

Comparison of the structures

A comparison of the crystal structures of väyrynenite and euclase (Fig. 1, Tables 2 and 4), shows that the arrangement of PO_4 tetrahedra around Mn in väyrynenite is almost identical with the arrangement of SiO_4 tetrahedra around Al in euclase. The differences between the two structures are chiefly a consequence of the change in electrostatic charge distribution induced by the replacement of Al^{+3} by Mn^{+2} in an octahedrally-coordinated site, and of Si^{+4} by P^{+5} in a tetrahedrally-coordinated site. In order to maintain local charge balance, the positions of Be and OH in väyrynenite must necessarily be quite different from those in euclase. This requirement is apparent from an analysis

 $\mathbf{34}$

of Table 6, where the approximate bond strengths in the two structures are given, calculated on the basis of a simple model.

The sums of the strengths of the bonds terminating on oxygen atoms O_1 , O_2 , and O_4 are almost the same for a structure containing Mn and P as for one containing Al and Si, owing to the fact that these oxygen atoms coordinate both the octahedral and the tetrahedral sites. However, oxygen atom O_3 coordinates the tetrahedral but not the octahedral site, whereas hydroxyl oxygen O_5 coordinates the octahedral but not the tetrahedral site. In order to maintain local charge balance O_3 must form one Be-O bond in väyrynenite and two Be-O bonds in euclase; O_5 must form two Be-O bonds in väyrynenite and one Be-O bond in euclase. This can be accomplished only if both the Be and the O_5 positions are different in the two structures.

As a result of these differences in Be and O_5 positions, the chains in euclase do not have exactly the same configuration as those in väyrynenite. In a single euclase chain (Fig. 2b) the apices of the PO_4 tetrahedra, O_4 , and of the BeO₃(OH) tetrahedra, O_5 , all point in approximately the same direction; but in väyrynenite (Fig. 2a) the O_5 - O_5 edges of the BeO₂(OH)₂ tetrahedra project from the chain in the opposite direction from the apices, O_4 , of the PO₄ tetrahedra. The väyrynenite configuration permits the formation of inter-chain hydrogen bonds between O_3 and the hydroxyl oxygen O_5 (Fig. 1a), thus restoring local charge balance around these two atoms (Table 8).

The cleavages in väyrynenite and euclase, as described in the previous sections, are entirely comparable. The crystal habits of the two minerals, however, differ in one important respect. Euclase crystals are elongated parallel to [100], the direction of the chain axes, whereas väyrynenite crystals are elongated parallel to [001] (MROȘE and VON KNORRING, 1959), the direction of strongest interchain bonding. Although euclase also has strong interchain bonding parallel to [001], it lacks the hydrogen bonding approximately parallel to [001], described above in väyrynenite. The difference in hydrogen bonding in the two structures apparently gives rise to the difference in elongation direction of the crystals.

References

J. BERGHUIS, IJBERTHA M. HAANAPPEL, M. POTTERS, B. O. LOOPSTRA, CARO-LINE H. MACGILLAVRY and A. L. VEENENDAAL (1955), New calculations of atomic scattering factors. Acta Crystallogr. 8, 478-483.

J. BISCOE and B. E. WARREN (1933), The structure of euclase HBeAlSiO₅. Z. Kristallogr. 86, 292-298.

- A. D. BOOTH (1948), Fourier technique in x-ray organic structure analysis. Cambridge, University Press, 106 p.
- W. L. BRAGG and W. H. ZACHARIASEN (1930), The crystalline structure of phenacite, Be₂SiO₄, and willemite, Zn₂SiO₄. Z. Kristallogr. 72, 518-528.
- W. R. BUSING and H. A. LEVY (1958), A single crystal neutron diffraction study of diaspore, AlO(OH). Acta Crystallogr. 11, 798-803.
- JOAN R. CLARK and C. L. CHRIST (1959), Studies of borate minerals (VIII): The crystal structure of CaB₃O₃(OH)₅ · 2H₂O. Z. Kristallogr. **112**, 213–233.
- JOAN R. CLARK, C. L. CHRIST and D. E. APPLEMAN (1962), The crystal structure of CaB₃O₅(OH). Acta Crystallogr., in press.
- J. D. H. DONNAY and W. NOWACKI (1954), Crystal data. Geol. Soc. America Mem. 60, 719 p.
- H. T. EVANS, JR. (1959), A program for crystallographic two-dimensional Fourier synthesis on a Burroughs 205 system. Technical Bulletin No. 59, Electrodata Division, Burroughs Corp., Pasadena, California, 46 p.
- S. GELLER and J. L. DURAND (1960), Refinement of the structure of LiMnPO₄. Acta Crystallogr. 13, 325-331.
- Internationale Tabellen zur Bestimmung von Kristallstrukturen (1935), v. 2. Berlin, Gebrüder Borntraeger, 239 p.
- G. A. JEFFREY, G. S. PARRY and R. L. MOZZI (1956), Study of the wurtzite-type binary compounds. I. Structures of aluminum nitride and beryllium oxide. Jour. Chem. Physics 25, No. 5, 1024-1031.
- MARY E. MROSE and O. VON KNORRING (1959), The mineralogy of väyrynenite, $(Mn,Fe)Be(PO_4)(OH)$. Z. Kristallogr. 112, 275–288.
- G. C. PIMENTEL and A. L. MCCLELLAN (1960), The hydrogen bond. San Francisco, W. H. Freeman and Company, 475 p.
- R. RIDGWAY (1912), Color standards and color nomenclature. Washington, D. C., 43 p.
- J. V. SMITH, ISABELLA L. KARLE, H. HAUPTMAN and J. KARLE (1960), The crystal structure of spurrite, $Ca_5(SiO_4)_2CO_3$. II. Description of structure. Acta Crystallogr. 13, 454–458.
- L. H. THOMAS and K. UMEDA (1957), Atomic scattering factors calculated from the TFD atomic model. Jour. Chem. Physics 26, No. 2, 293-303.