THE ELASTIC PROPERTIES OF PYROXENES

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A pulse method has been used to measure the elastic moduli of five monoclinic pyroxenes: diopside (baikalite), aegirite, aegirite-augite, augite, and diallage. The constants are found to be related to the structure. Mean values are calculated for Young's modulus and the shear modulus of monomineralic ocks; it is found that the mean values for all five differ little.

The elastic properties of minerals are of interest in two ways. Firstly, the elastic properties can be correlated with the structure; and secondly, the elasticity of a rock is governed by the elastic constants of its constituent minerals.

Several papers [1-4] deal with the relation of structure to elasticity for fairly complex crystals; in general, the elastic constants increase with the strength of the lattice bonds and are especially large along the directions of the strongest bonds (these are usually the ones most difficult to deform). It has been shown [3] that the most rigid elements in some silicates are ones built up from silicon-oxygen tetrahedra (chains, layers, and bonds) and also the related columns or layers of cation octahedra.

The second aspect has been dealt with only in a few papers [5, 6], which have dealt with a few monomineralic rocks largely because the elastic constants of most rock-forming minerals are unknown. The task here is made even more difficult by the presence of inclusions, pores, microcracks, and (very often) textures in all real rocks. These features greatly distort measurements of rock elasticity, especially if the methods of measurement involve substantial deformation. However, there is no doubt that the elasticity of a rock is largely governed by the elastic constants of the constituent minerals [7]; the other features merely distort the effects somewhat.

Our object was to determine the elastic constants of some monoclinic pyroxenes, which are major minerals in some ultrabasic rocks. Results for pyroxenes enable one to test earlier [3a] deductions as to the relation of elasticity to structure and, in conjunction with averages of the constants, to obtain limiting values for the elasticity of pyroxenites (monomineralic pyroxene rocks).

We had available reasonably perfect monocrystals of diopside (baikalite, Semizhilka quarry, ore-body No. 3, Sludyanka), augite (W. Georgia), diallage (from a gabbro-pegmatite in the Matkal massif, S. Ural), and aegirite-augite (from pegmatite, Material'naya shaft, Khibina massif). The diopside and augite had good faces. All showed cleavage on {110}, and diallage and aegirite-augite on {100} in addition. The specimens were set up from the cleavage and the main faces.

An ultrasonic pulse method was used; the elastic constants were calculated in the usual way [8, 9].

All of these minerals are monoclinic; β is 105°51' for diopside, 105°50' for augite, and 106°49' for aegirite [10]. The settings were converted to orthogonal coordinates as follows: the crystallographic b axis became the Y axis, the c axis the Z axis, and the direction normal to the YZ plane corresponding to a right-handed system became the X axis, which lay at an angle $\beta - 90°$ to the crystallographic a axis.

Table 1 gives the speeds found, together with the probable errors and results for aegirite [3a].

Table 2 gives the c_{ik} and the c_{ik}/ρ; the errors, as estimated in [3], can be 3% for c_{22}, c_{44}, c_{66}, and c_{46}, and 7-10% for c_{11}, c_{33}, c_{55}, c_{13}, c_{12}, and c_{23}, while the values of c_{15}, c_{25}, and c_{35} are no more than approximate.

The velocities of shear waves for aegirite-augite are much lower than for the other pyroxenes; these differences result in low values for c_{44}, c_{55}, and c_{66}, and also for the mean Young's modulus and shear modulus. It would be desirable to repeat these measurements on another crystal of similar composition, in order to establish whether they result from some feature of this particular specimen or from some general property of solid solutions in the aegirite-augite system.

These pyroxenes show several notable features in their elasticity. The elastic constants are largest along

TABLE 1. Velocities of Elastic Waves (km/sec)

Velocity	Directions of prop.	Directions of displ.	Diopside (baikalite)	Augite	Aegirite	Aegirite-augite	Diallage
V_1	001	001	8.60±0.13	8.15±0.12	8.21±0.12	7.99±0.10	8.00±0.12
V_2		100	3.95±0.04	3.82±0.05	3.78±0.05	3.60±0.05	4.30±0.05
V_3		010	4.51±0.05	4.58±0.05	4.23±0.05	3.41±0.05	4.39±0.05
V_4	110	110	7.98±0.15	7.34±0.15	7.60±0.12	7.06±0.09	6.90±0.12
V_5		$1\bar{1}0$	3.94±0.05	3.86±0.05	4.07±0.05	3.18±0.05	4.00±0.05
V_6		001	4.35±0.06	4.09±0.06	3.95±0.05	3.47±0.05	4.13±0.05
V_7	010	010	7.25±0.09	6.81±0.10	7.20±0.11	6.66±0.07	6.72±0.11
V_8		001	—	4.66±0.06	3.97±0.05	3.69±0.05	4.41±0.05
V_9		100	—	4.32±0.05	3.48±0.05	3.93±0.05	3.95±0.05
V_{10}	101	101	6.94±0.11	8.36±0.12	6.75±0.11	7.99±0.10	7.78±0.13
V_{11}		$10\bar{1}$	4.45±0.05	4.34±0.05	4.37±0.05	4.17±0.06	4.65±0.05
V_{12}		010	4.17±0.05	4.49±0.05	3.68±0.04	3.77±0.05	3.86±0.05
V_{13}	100	100	7.90±0.12	7.48±0.13	7.30±0.12	6.86±0.10	6.87±0.12
V_{14}		010	4.60±0.06	4.10±0.05	3.68±0.04	3.79±0.05	3.98±0.05
V_{15}		001	4.10±0.05	3.81±0.05	3.71±0.06	3.47±0.05	4.25±0.05
V_{16}	011	011	7.82±0.12	7.44±0.12	7.45±0.14	7.05±0.10	7.20±0.12
V_{17}		$0\bar{1}1$	4.83±0.06	4.72±0.05	4.65±0.05	4.24±0.05	4.67±0.05
V_{18}		100	4.19±0.05	3.87±0.05	3.86±0.05	3.41±0.05	4.16±0.05

TABLE 2. Elastic Moduli of Pyroxenes (10^{11} dyne/cm^2)

ik	Diopside ($\rho=3.31$)		Augite ($\rho=3.32$)		Diallage ($\rho=3.30$)		Aegirite-augite ($\rho=3.42$)		Aegirite ($\rho=3.50$)	
	c_{ik}/ρ	c_{ik}	c_{ik}/ρ	c_{ik}	c_{ik}/ρ	c_{ik}	c_{ik}/ρ	c_{ik}	c_{ik}/ρ	c_{ik}
11	6.16	20.4	5.47	18.16	4.65	15.39	4.55	15.56	5.31	18.58
22	5.27	17.5	4.54	15.07	4.52	14.96	4.44	15.18	5.18	18.13
33	7.18	23.8	6.56	21.78	6.37	21.08	6.32	21.61	6.67	23.44
44	2.03	6.75	2.10	6.97	1.93	6.39	1.17	4.00	1.79	6.29
55	1.77	5.88	1.54	5.11	1.88	6.22	1.36	4.65	1.45	5.10
66	2.12	7.05	1.68	5.58	1.58	5.23	1.44	4.92	1.35	4.74
12	2.54	8.44	2.21	7.34	1.72	5.69	2.37	8.11	1.95	6.85
13	2.66	8.83	2.18	7.24	1.13	3.74	1.93	6.60	2.05	7.07
23	1.45	4.82	1.02	3.39	0.92	3.05	2.00	6.84	1.79	6.26
15	−0.58	−1.93	0.60	1.99	0.44	1.46	0.74	2.53	0.26	0.98
25	−0.59	−1.96	0.50	1.66	0.43	1.42	0.76	2.60	0.27	0.94
35	−1.01	−3.36	0.74	2.46	0.36	1.19	0.56	1.92	0.61	2.14
46	−0.33	−1.13	0.13	0.43	−0.26	−0.86	0.12	0.41	0.22	0.77

[001], which is the direction of the chains of silicon-oxygen tetrahedra; c_{33} is greater than c_{11} and c_{22}. This occurs in all five. The c_{33}/ρ are closely similar, as had previously been supposed [3].

Belov's view is that the structures of silicates are built up from cation octahedra [11, 12]; the silicon-oxygen structures are merely associated with these and are adapted to them. On this basis one would expect that c_{33}/ρ and c_{44}/ρ for the pyroxenes should vary substantially with cation radius and valency, whereas in fact they vary only slightly. This indicates that the silicon-oxygen structures are important in relation to the elasticity; the bond strengths in these SiO_4 structures are the same in layers, chains, and so on, and they make up a large part of the total strength along the line of the structure.

The results in Table 2 have been used to calculate the mean shear modulus and Young's modulus for polycrystalline aggregates; the usual methods of averaging the c_{ik} (Vogt) and s_{ik} (Roys) were used. The usual methods were used to pass from the c_{ik} to the s_{ik} [13].

Neither method gives correct values, because the two are based respectively on equality in the deformations and stresses at the grain boundaries [14]. Numerous tests for metals and alloys have shown [15] that Vogt's method always gives high values and Roys's low ones, so it is natural to take a mean of the two for an untextured monomineralic rock. Table 3 gives these averages; E and G clearly vary little from one pyroxene to another, being 14.1-15.8 for E and 5.6-6.2 for G (both in 10^5 kg/cm^2). Table 3 would indicate that μ for a pure pyroxenite should be 0.25-0.28. The results for the E and G of aegirite-augite are somewhat low; these need to be verified. These E and G represent limiting values for monomineralic pyroxenites; pores, cracks, and weathering will result in lower E and G. Any texture (banding) could lead to slight variation in E and G with direction. A special study is needed for these features.

TABLE 3. Mean Values of E, G, and μ (10^5 kg/cm^2) for Pyroxenes

Mineral	Vogt average			Roys average			Mean of the two		
	E	G	μ	E	G	μ	E	G	μ
Diopside	16.97	6.71	0.266	15.75	6.28	0.254	16.37	6.49	0.260
Aegirite	15.34	6.01	0.276	14.64	5.72	0.280	14.99	5.87	0.277
Aegirite-augite	12.69	4.87	0.303	11.65	4.51	0.292	12.17	4.68	0.299
Augite	15.34	6.12	0.253	13.99	5.66	0.236	14.66	5.90	0.244
Diallage	15.19	6.29	0.207	14.43	5.96	0.211	14.81	6.12	0.209

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