A method of determining the angular direction represented by a point in the directions-image of an object under the microscope.

By JOHN WILLIAM EVANS, LL.B., D.Sc., F.G.S.

Lecturer in Petrology, Imperial College of Science and Technology, and Birkbeck College.

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THE determination of the direction in which the light illuminating a point in a directions-image (the so-called image in convergent light) was travelling when it reached the objective has hitherto been carried out by first measuring the position of the point in the image and thence calculating the angular direction it represents. It is usually assumed that the distance of a point in the image from the centre of the field is in proportion to the sine of the angle between the path of the light that illuminates the point and the axis of the microscope. The position of the point is accordingly generally fixed in polar co-ordinates by measuring the central distance by means of a micrometer screw or scale, and determining the azimuth, relatively to a definite direction on the stage or object, by means of a radial engraved line or pointer, which can be rotated round the centre of the field or remains fixed while the stage is rotated. Sometimes a net of radiating lines and concentric circles is employed and gives at once not only the central distance, but also, provided the stage be in a definite position, the azimuth as well. F. Becke[1] obtained a similar result with a rotating stage by drawing the interference-figures, by means of a camera lucida, on a rotating disc connected by a cord with the stage, so that both moved simultaneously through the same angles.

Others have employed rectilinear co-ordinates, making the necessary

[1] F. Becke, Min. Petr. Mitt., 1895, vol. xiv, pp. 563–565.

measurements either by means of two micrometer screws at right angles to each other, or a net with square meshes.[1]

Whatever be the accessory employed for measurement, it must be inserted in a position occupied by a directions-image, or a conjugate focus of such an image, so that it may be visible in focus to the observer simultaneously with this image. It is usually placed in the focus of the eyepiece, when a Bertrand (Amici) lens is employed to give the directions-image; or in the focus of the Becke (Klein) lens, when that is substituted.[2] It may, on the other hand, be placed in the principal focus of the objective, or even in its internal focus,[3] or in a conjugate focus below the condenser. E. Sommerfeldt[4] had a scale engraved on the plane surface of the lowest lens of the condenser. F. E. Wright[5] had a graduated glass hemisphere constructed, the spherical surface of which was engraved with circles having their centres on the normal to the plane surface, and with lines radiating from it. This he employed for a number of different purposes, including that of an apertometer. If, however, the radius were sufficiently large, say forty or fifty millimetres, and the plane surface at right angles to the microscope axis, the graduations would be visible simultaneously with the directions-image of a crystal plate resting on it with a film of highly refracting liquid between. There would, however, be some difficulty in illuminating all the directions in a hemisphere of such size.

Whatever the position in which the means of measurement is placed, it requires careful calibration with the help of some form of apertometer such as that invented by Abbé, or by the observation of mineral plates, certain directions in which make known angles with the normal and can be easily recognized in the directions-image. In the latter case the refraction at the surface of the plate must of course be allowed for.

As the sine law is only approximate, the calibration must be carried out for more than one central distance and even for different azimuths, and this is obviously none the less the case when the scale or net purports to give directly the angular distance of the path of the light from

[1] F. E. Wright, Amer. Journ. Sci., 1907, vol. xxiv, pp. 336–337; ibid., 1910, vol. xxix, p. 428; Journ. Wash. Acad. Sci., 1911, vol. i, pp. 60–61; Min. Petr. Mitt., 1908, vol. xxvii, pp. 293–300.

[2] F. Becke, Min. Petr. Mitt., 1894, vol. xiv, pp. 376–378.

[3] H. Lenk, Zeits. Kryst. Min., 1896, vol. xxv, pp. 379–380.

[4] E. Sommerfeldt, Zeits. wiss. Mikr., 1905, vol. xxii, pp. 356–362.

[5] F. E. Wright, 'The Methods of Petrographic-Microscopic Research.' Carnegie Institution, Washington, 1911, pp. 174–175.

the axis, instead of the central distance of the corresponding point in the image.

I propose, on the other hand, to obtain the angular position of a point in the directions-image by the following procedure, which obviates the necessity for calibration and avoids all difficulties due to imperfections in the construction of the lens, the adjustment of the microscope, or the rotation of the stage.

A circular disc is prepared comparable in size with the microscope stage. On this a network of concentric and radial lines is delineated, the radial lines at intervals of 5°, except near the centre, where they are at intervals of 10°, and the concentric lines at distances from the centre equal to $r \tan \theta$, where r is a convenient length, say 50 mm., and θ represents successive angles differing by 5°. The net is therefore a gnomonic projection of the points where a sphere is intersected by lines at equal intervals of azimuth and of angular distance from the normal to the plate through its centre. The circles are numbered with the corresponding value of θ, and the angles of azimuth are numbered counter-clockwise, like the degrees on the rotating stage or the circle on which the movements of rotating nicols are recorded.

The plate is notched at the circumference, or perforated, at three points so that it can be rapidly fitted on corresponding supports on the stage in one definite position relatively to it, and as quickly removed. In this position it is raised a little above the stage so that it does not interfere with a glass slip and rock-section resting on the stage, or the clip, or clips holding it.[1]

When it is desired to determine the angular position represented by a point in a directions-image of a crystal section (usually a thin section of a mineral in a rock-slice), the end of a movable pointer, inserted in any position where it is in focus simultaneously with the directions-image, is made to coincide with it. When the position of the point in the directions-image has been thus recorded, the microscope is raised by the coarse adjustment, and, after the plate has been placed on its supports, adjusted so that the lower surface of the objective is at a distance exactly equal to r from the plate. This may be effected by placing a rod

[1] A plate with a similar arrangement of concentric lines appears to have been employed as an apertometer by W. Volkmann (Aus Theorie und Praxis des Mikroskopes, Berichte über Appar. u. Anlag. von Leppin & Masche, Berlin, 1911, vol. viii, p. 5, noticed by E. A. Wülfing, Fortschritte Min. Krist. Petr., 1913, vol. iii, pp. 71, 78), but I have not been able to see the original description.

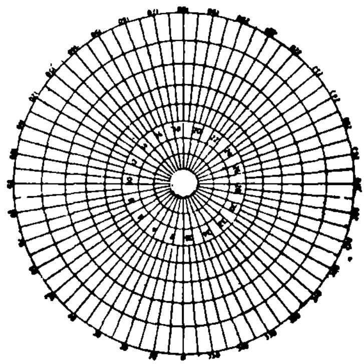
of that length on the plate, at right angles to it, and focusing the microscope down till the lowest surface of the objective touches the end, or a mark indicating this point may be engraved on the rackwork. The distance to which the microscope can be racked back places an obvious limit to the magnitude of r, but it is usually possible to make it equal to 50 mm. If an objective with a working distance of not more than 6 mm. be at a distance of at least 50 mm. from the plate, the net is clearly visible in the microscope when the Bertrand or Becke lens is inserted, or the microscope is otherwise converted into a hodoscope (p. 45 above).

The relation of any point in the field to the image of the net will then indicate the angular position of the path of the light that illuminates it, and likewise that of the light which previously illuminated the same point in the directions-image of the crystal section. The position of the end of the pointer relatively to the image of the net will therefore give at once the angular position required.

The most important advantage of this method is that every peculiarity in the construction or adjustment of the microscope will affect the light coming from the plate as it does that illuminating the directions-image, so that no calibration or correction will be required. In microscopes with a rotating stage the plate and object both rotate with the stage so that all measurements of azimuth with the net will have reference to the same zero direction in the object and plate, whatever may be the position of the stage. The plate should be so adjusted that the zero direction in it coincides with the right and left cross-wire, and is on the right when the index reading of the stage is at zero.

It is of course desirable that the plate should be as nearly as possible at right angles to the axis of the microscope and the axis of rotation, and that the centre of the plate should lie in these axes, which should of course coincide with one another. It is hardly necessary to explain that the angular position determined in this way is that at which the light meets the outer surface of the objective. To obtain the angular position in a crystal section or other object, correction must be made for refraction at its surface. In the case of a crystal section, if θ be the angular distance of the path in the crystal plate from the normal, and ϕ that of the path in air, and μ is the refractive index of the crystal section for the vibrations of the light under examination, $\mu \sin\theta = \sin\phi$.

The procedure may be varied, if desired, by placing the end of another pointer on the plate in such a position that its image coincides with that of the end of the pointer employed to record the position of a particular point in the directions-image. The corresponding angular position can then be read off directly from the plate by observing the position relatively to the net of the end of the pointer resting on the plate.



In the plate shown in the accompanying text-figure, which was made by Messrs. J. Swift & Son, a circular hole was, at my suggestion, left in the centre, so that the crystal section could be observed through it. I do not think, however, that there is much advantage in this, since, with a little practice, the plate can be rapidly placed in position and removed. I also stipulated that the plate should not be so large as to cover up the graduations on the stage, but there seems no reason why it should not do so, and have as great a diameter as the construction of the microscope permits, so that the angular distance from the axis that can be determined should be as large as possible instead of being restricted to 45°.

Similar results may be obtained by substituting for the plate a larger net, with r equal to say 200 mm., on a wall or vertical screen. When the pointer has been placed in position on the directions-image in the manner already explained, the glass slip with the crystal section and the substage are removed and the microscope racked down through the aperture in the stage, and then brought to a horizontal position opposite the centre of the net and at right angles to it with the objective at a distance r from it.

The angular position represented by the end of the pointer may then be determined, either by observing its relation to the image of the net or by an assistant moving, under the directions of the observer, the end of a pointer resting on the actual net until it is seen to be in apparent coincidence with the end of the pointer in the microscope. In the latter case, the angular position required may be read off from the net itself.

If this method be employed, it would be of advantage to use a microscope with a mechanical stage so that the object could, after removal to allow of the racking down of the microscope, be brought back to exactly the same position and orientation, if it were desired to make further observations. It is obviously unsuited to a microscope in which the stage and not the nicols rotates, as the net cannot well be made to rotate with the stage.

The use of a scale on a wall for calibration of the directions-image of a microscope has long been practised at Heidelberg (E. A. Wülfing, Fortschritte Min. Krist. Petr., 1913, vol. iii, p. 78).