The relation between different laws of twinning that result in the same twin-crystal.

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IN the following notes I have endeavoured to give in a convenient and simple form a complete account of the relations connecting different twinning-laws or operations, which, when applied to the same crystal-structure, produce indistinguishable results.

A rigorous analysis of the possible relations between the compound structures of twin-crystals, based on the principles of the equality of the structural distances in the plane of composition, will be found in the author's papers, 'The geometry of twin-crystals' (Proc. Roy. Soc. Edinburgh, 1912–1913, vol. xxxii, pp. 416–432, 433–457) and 'Die Geometrie der Zwillingskrystalle' (Zeits. Kryst. Min., 1913, vol. lii, pp. 327–371). See also the paper 'Twin-planes and cross-planes' in this Magazine (1910, vol. xv, pp. 390–397).

I. *Twin-operations.*—A twin-crystal may be defined as a compound crystal with one or more twin-planes. A twin-plane is a plane which possesses the character that every line in it stands in exactly the same relation to two differently oriented crystal-structures of the same substance. The normal to a twin-plane is a twin-axis. It also has the same relation to both structures.

The connexion between the two portions of a twin-crystal may be conveniently represented by considering that one portion of a simple crystal has been subjected to one of the following 'twinning-operations':

(1) A rotation round a line, the twin-axis, through half a circle; or, as I prefer to express it, 'a reversal relatively to a line'. This may be described as 'line-twinning' or 'rotation-twinning', and denoted by the letter L. The plane at right angles to the line is here the twin-plane.

(2) A reflection on a plane, the twin-plane ; or a 'reversal relatively to a plane'. This may be termed 'plane-twinning' or 'reflection-twinning', and denoted by P. The normal to the plane is the twin-axis.

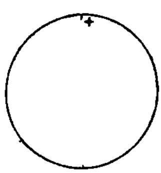
(3) Inversion[1] through a point; or a 'reversal relatively to a point'. Here the structure is completely reversed, so that every plane, line, or point is shifted to the opposite side of a central point which remains unmoved. This may be referred to as 'point-twinning' or 'inversion-twinning', and denoted by I. Here every plane has the characters of a twin-plane, and every line that of a twin-axis.

(4) Rotation round a line through a quarter of a circle.

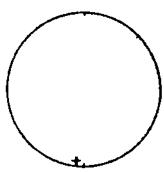
(5) Rotation round a line through a sixth of a circle.

The first three, however, of these operations suffice to explain every twin-crystal. These essential twinning operations are best illustrated in the twins of quartz and pyrargyrite referred to on pp. 235–237, but examples will be found throughout this communication.

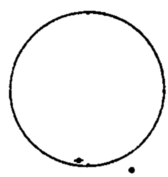
II. *Twinning in the absence of symmetry.*—If there be no symmetry in the untwinned crystal, or none related to a twin-axis in any of the ways hereafter described, a twin-crystal can be explained by only one twinning operation, and this must be one of the first three mentioned above.



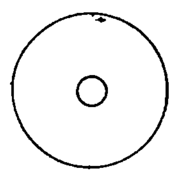
1 Uu
FIG. 1.



1 Uu L
FIG. 2



1 Uu P
FIG. 3.



1 Uu I
FIG. 4

Figs. 1 to 4 illustrate the three modes of twinning when they occur in a crystal without symmetry. Each diagram may be considered as a stereographic (or gnomonic) projection showing a single face constituting a 'form'. If the face is on the *upper* side of the crystal, the projection point is marked by the centre of a plus sign, while if it is on the *under* side, its projection is indicated by the centre of a minus sign in the place where the projection of a *parallel* face on the *upper* side would have been situated—*not* that of the upper face which is the reflection of the lower face in the plane of projection according to the usual practice

[1] This must be distinguished from inversion as defined in the geometrical method of Stubbs and Ingram.

in the case of stereographic projection.[1] In other words, the procedure in gnomonic projection is followed. The left-hand diagram (fig. 1) shows the orientation of that portion of the twin-crystal which is considered to be unaffected by the twinning operations; the next (fig. 2) gives the orientation of the other portion after rotation through a half circle round the normal to the plane of projection. Figs. 1 and 2 represent, therefore, the orientation of the two portions of a crystal with rotation-twinning. Fig. 3 gives the orientation of the second portion after reflection on the plane of projection. Consequently, figs. 1 and 3 show the relative orientation of the two portions of a twin-crystal with reflection-twinning. Fig. 4 shows in like manner the orientation of the second portion after inversion-twinning, so that in this case figs. 1 and 4 illustrate the relative orientation of the two parts of the twin.

The result is different in the three cases, but in each the plane of projection is a twin-plane and its normal a twin-axis, though in the last any other plane and its normal might be taken as twin-plane and axis.

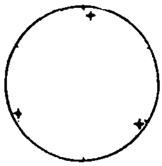
The numeral '1' of the symbols accompanying the diagrams indicates that the normal to the plane of projection is *not* an axis of symmetry. The capital 'U' (unilateral) shows that it does not lie in a plane of symmetry, and the small 'u' (uniterminal) that opposite directions in the normal are not equivalent to one another, and have therefore different physical characters. The letters 'L', 'P', and 'I' denote, as already stated, twinning by rotation, reflection, and inversion respectively. The occurrence of inversion-twinning is indicated in the diagram by a small circle round the centre of the larger one.

If, on the other hand, there be some form of symmetry present in the untwinned crystal, a twin-crystal may in many cases be described by any one of two or more twinning-operations. These are said to be 'equivalent' to one another.

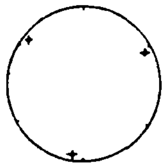
III. *Axis of trigonal symmetry as twin-axis.*—If there be only an axis of trigonal symmetry, the three essential modes of twinning, with this as twin-axis, will still give different results. In the case of rotation-twinning, however, the same result will be obtained by rotation through one-sixth of a circle as by rotation through half a circle.

In figs. 5, 6, 7, and 8 the fact that the normal to the plane of projection is an axis of trigonal symmetry is indicated by the numeral '3' in the symbol. Here each form consists of the three faces shown in the diagrams.

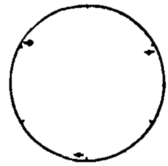
[1] J. W. Evans, A modification of the stereographic projection. Mineralogical Magazine, 1910, vol. xv, pp. 401–402.



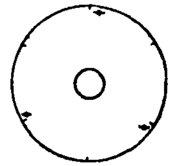
3 Uu
FIG. 5.



3 Uu L
FIG. 6.

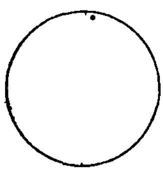


3 UuP
FIG. 7.

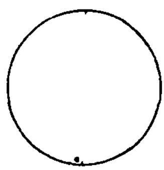


3 Uu I
FIG. 8.

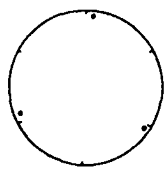
IV. *Centre of Symmetry.*—If a centre of symmetry be present, every axis of rotation-twinning is also an axis of reflection-twinning, and vice versa. Conversely, if the same line be an axis of both rotation- and reflection-twinning, and these are equivalent to one another, the untwinned crystal must possess a centre of symmetry.



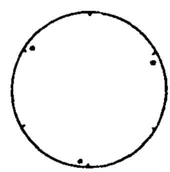
1 Uc
FIG. 9.



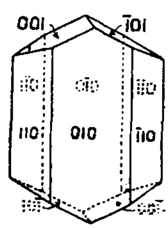
1 Uc L P
FIG. 10.



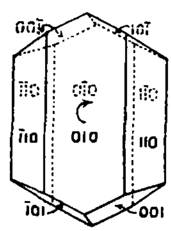
3 Uc
FIG. 11.



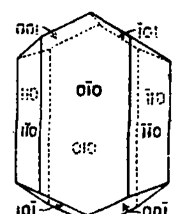
3 Uc L P
FIG. 12.



1 Uc
FIG. 13.



1 Uc L
FIG. 14.



1 Uc P
FIG. 15.

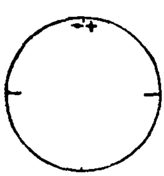
Figs. 9 and 10 are projections, similar to those already described (p. 225), of a single crystal-form. They illustrate the twinning of a crystal possessing a centre of symmetry. Here every face of a form is accompanied by an equivalent opposite parallel face, and both of these faces are represented in the projection not by plus and minus signs but by a single dot. In the symbols, the letter 'c' indicates the presence of a centre of symmetry, and the conjunction of the letters 'L P' that the same configuration results from rotation round the twin-axis as from reflection on the twin-plane.

In figs. 13, 14, and 15, the same relation is illustrated by orthogonal projections on the brachypinakoid of the two component parts of a crystal of plagioclase showing albite-twinning. Fig. 13 shows the orientation of the portion to which no twinning operation has been applied; fig. 14, the result of rotation round the normal to a brachypinakoid face, and fig. 15, that of reflection on the same face. The indices demonstrate the difference between these two operations, which, however, produce exactly the same apparent result. The edges of the under crystal faces are shown by interrupted lines, and their indices are distinguished in the same manner.

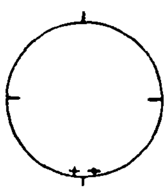
Figs. 11 and 12 illustrate the twins of phenakite on the basal plane. Here there are a trigonal axis and a centre of symmetry.

If there be a centre of symmetry, inversion-twinning is impossible.

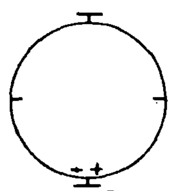
V. *Line of Symmetry.*—A line of symmetry[1] may also result in the presence of equivalent twinning-operations.



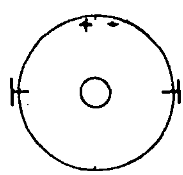
1 Uh
FIG. 16



1 Uh L
FIG. 17



1 Uh P
FIG. 18



1 Uh I
FIG. 19

(1) If there be an axis of rotation-twinning, or of reflection-twinning, or of both rotation- and reflection-twinning at right angles to a line of symmetry, there will be another axis of twinning of the same nature at right angles both to the line of symmetry and to the first twin-axis. The simplest examples would occur in twins of crystals which, like cane-sugar, have no symmetry except an axis of digonal symmetry.

This is illustrated in figs. 16, 17, and 18, where the letter 'h' denotes the presence of a line of symmetry in the plane of projection, in the position indicated in the diagrams by two short radial lines *inside* the circle. In 1 Uh L, fig. 17, the crystal has been rotated by the twinning operation round the normal to the plane of projection, which thus

[1] It is convenient to use the term 'line of symmetry' to include axes with an *even* number of symmetry—digonal, tetragonal, or hexagonal—but not those with 'contra-directional' digonal or hexagonal symmetry, as defined on p. 238. A line of symmetry is analogous in many respects to the normal to a plane of symmetry; which is a line of symmetry, when a centre of symmetry is present; and, in its absence, an axis of contra-directional digonal or hexagonal symmetry.

becomes an axis of rotation-twinning. The same result is obtained by a rotation round an axis parallel to the plane of projection at right angles to the line of symmetry. The position of this second axis of rotation-twinning is indicated by two short radial lines *outside* the circle. In 1 Uh P, fig. 18, the crystal has been reflected in the plane of projection, the normal to which has become a twin-axis of reflection-twinning. The same result follows from a reflection about a plane at right angles to the plane of projection, and passing through the line of symmetry. Its normal, which lies in the plane of projection and is at right angles to the line of symmetry, is therefore a second axis of reflection-twinning, the position of which is shown by two short radial lines, outside the circle, terminated by two longer lines at right angles. Fig. 19, 1 Uh I, will be considered later.

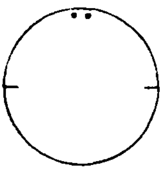
The case where there is also a centre of symmetry and each twin-axis is accordingly an axis both of rotation and reflection-twinning, is illustrated by the Karlsbad-twins of orthoclase. Here there are two axes of twinning, one the vertical axis, the other the normal to the orthopinakoid. Both are at right angles to the ortho-axis, which is here an axis of digonal symmetry and therefore a line of symmetry, and lie in the clinopinakoid, which is a plane of symmetry; and they are at right angles to each other.

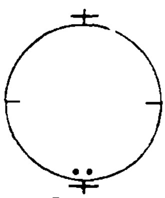
Figs. 20 and 21, 1 Bc and 1 Bc L P, are projections of the character, already described (p. 225), of a single crystal-form in the component parts of a twin of the Karlsbad-type; that is to say, with two twin-axes at right angles to a line of symmetry and lying in the plane of symmetry. One of these twin-axes is at right angles to the plane of projection, and the other parallel to it. In the symbols the letter 'B' indicates that the normal to the plane of projection is parallel to a plane of symmetry, and the letters 'L P' that it is an axis of both rotation- and reflection-twinning. The position of the other axis of both rotation- and reflection-twinning, which is parallel to the plane of projection, is shown by two crosses outside the circle. Both twin-axes are seen to be at right angles to a line of symmetry parallel to the plane of projection.

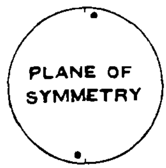
Orthogonal projections of a crystal of orthoclase on the plane at right angles to the vertical axis are shown in figs. 24 to 28. Fig. 24 corresponds to fig. 20, and figs. 25–28 to fig. 21. Figs. 25–28 show how the same configuration may be brought about by different operations, which can be inferred from the indices of the faces shown. Fig. 25 shows the positions of the faces after (a) rotation on the vertical axis; fig. 26 after (b) reflection on the horizontal plane to which it is normal;

fig. 27 after (*c*) rotation on the normal to the orthopinakoid; and fig. 28 after (*d*) reflection on the orthopinakoid. All these twinning-operations produce an indistinguishable result.

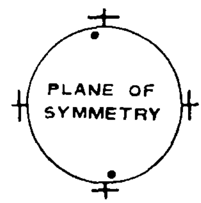
Figs. 22 and 23 are projections on the plane at right angles to the digonal axis—that is to say, on the plane of symmetry, the clinopinakoid —of the same form in the same twin-structure as in figs. 20 and 21. In the same manner, figs. 29 to 33 are orthogonal projections on the


1 Bc
FIG. 20.

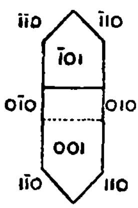

1 Bc L P
FIG. 21.

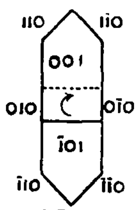


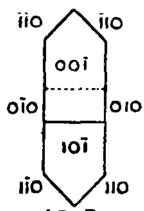
2 Uc
FIG. 22.

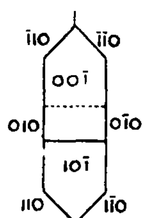


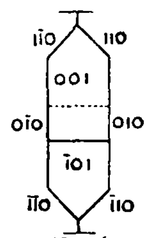
2 Uc L′ P′
FIG. 23.

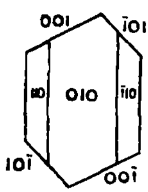

1 Bc
FIG. 24.

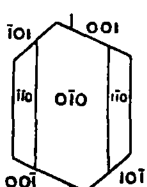

1 Bc L
(*a*) FIG. 25.

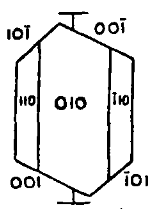

1 Bc P
(*b*) FIG. 26.

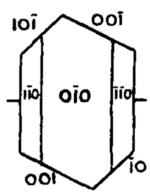

1 Bc L′
(*c*) FIG. 27.

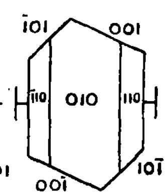

1 Bc P′
(*d*) FIG. 28.


2 Uc
FIG. 29.


2 Uc L′
(*a*)
FIG. 30


2 Uc P′
(*b*)
FIG. 31.


2 Uc L″
(*c*)
FIG. 32.


2 Uc P″
(*d*)
FIG. 33.

clinopinakoid corresponding to figs. 24 to 28. The letters L′ P′ and L″ P″ indicate the presence of axes of both rotation- and reflection-twinning lying in the plane of projection.

If (as in plagioclase) there were no symmetry except the centre of symmetry, there could be only one twin-axis in the same simple twin, corresponding either to (*a*) and (*b*) or to (*c*) and (*d*). There are in fact two different kinds of twins in triclinic felspars corresponding to the

Karlsbad-twin of orthoclase. In plagioclase the twin-axis is usually the vertical axis, and in anorthoclase, apparently, the normal to the macropinakoid.

If the centre of symmetry were absent and only the axis of digonal symmetry remained, the two twin-axes would both be either axes of rotation-twinning (*a*) and (*c*), or both axes of reflection-twinning (*b*) and (*d*), but not axes of both kinds of twinning. We shall see later that if only the plane of symmetry remained, one of the twin-axes would be an axis of rotation-twinning and the other of reflection-twinning, so that (*a*) and (*d*) would be equivalent twinning-operations, and so would (*b*) and (*c*), but the two former would not give the same result as the two latter.

(2) If a line of symmetry be an axis of reflection-twinning, the twin-crystal can also be explained by inversion-twinning, and, conversely, if a twin-crystal can be explained by inversion-twinning every line of symmetry is an axis of reflection-twinning.

This relation is illustrated by figs. 16 and 19 (1 Uh and 1 Uh l), p. 228, where the line of symmetry and the axis of reflection-twinning, which coincides with it, are parallel to the plane of projection.[1]

The same relation is also shown in figs. 34 to 45, and 53 to 58, where the lines of symmetry and axes of reflection-twinning are normal to the plane of projection. In figs. 40 to 45 they occur both normal and parallel to that plane.

Here the numerals 2, 4, and 6, indicate that the normal to the plane of projection is a digonal, tetragonal, or hexagonal axis of symmetry.

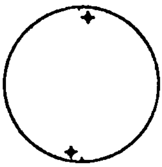
There are twins of eulytine, diamond, and tetrahedrite, which can be described as twins by reflection on the cube faces. The crystallographic axes are therefore axes of reflection-twinning. As they are at the same time lines of symmetry, being digonal axes, the twins may also be explained by inversion-twinning, so that every point of one component portion corresponds to an exactly similar point of the other on the opposite side of the point of inversion. See p. 239, and figs. 79 to 82.

Another familiar case, the inversion- and reflection-twins of quartz, is described later. See page 237, and figs. 59, 62, 63, 66.

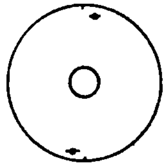
A line of symmetry cannot be an axis of rotation-twinning.

VI. *Plane of symmetry.*—A plane of symmetry will also give rise to equivalent twin-operations.

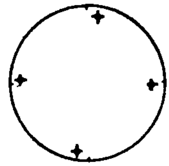
[1] The manner in which a line of symmetry and an axis of reflection-twinning are respectively indicated in these projections is explained on pp. 228, 229.



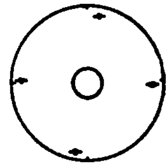
2 Uu
FIG. 34.



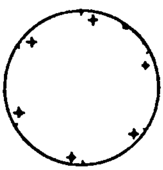
2 Uu P I
FIG. 35.



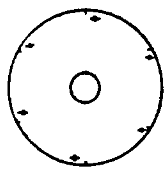
4 Uu
FIG. 36.



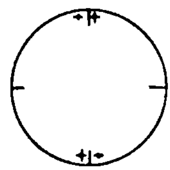
4 Uu PI
FIG. 37.



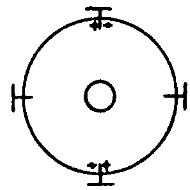
6 Uu
FIG. 38.



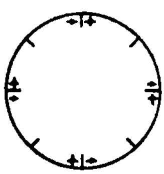
6 Uu P I
FIG. 39



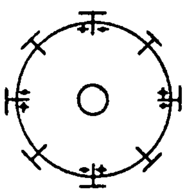
2 Uh
FIG. 40.



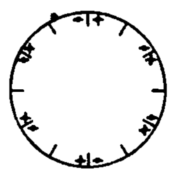
2 Uh PI
FIG. 41.



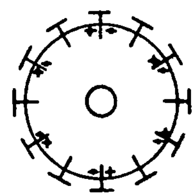
4 Uh
FIG. 42.



4 Uh P I
FIG. 43.



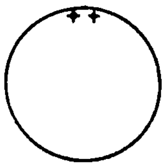
6 Uh
FIG. 44



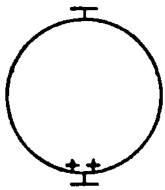
6 Uh P I
FIG. 45.

(1) If there be an axis of rotation-twinning parallel to a plane of symmetry, there will be parallel to the same plane of symmetry an axis of reflection-twinning at right angles to the axis of rotation-twinning; and, vice versa, an axis of reflection-twinning parallel to a plane of symmetry implies the existence of an axis of rotation-twinning at right angles to it and parallel to the same plane of symmetry. Examples may occur in any crystal possessing a plane of symmetry but no centre of symmetry; for instance, clinohedrite (see figs. 46 to 48). In these figures there is a plane of symmetry at right angles to the plane of projection, and therefore parallel to its normal. Figs. 46 and 47 represent the components of a twin in which the normal to the plane of projection is an axis of rotation-twinning parallel to a plane of symmetry. There is accordingly also an axis of reflection-twinning parallel to the same plane of symmetry and at right angles to the axis of rotation-twinning, and therefore parallel to the plane of projection. Figs. 46 and 48 represent the components of a twin in which the normal to the plane of projection is an axis of reflection-twinning, and there is an axis

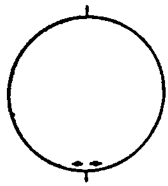
of rotation-twinning at right angles to it, parallel to the plane of projection. Both twin-axes are parallel to the plane of symmetry.



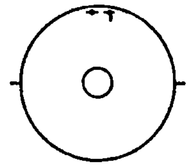
1 Bu
FIG. 46.



1 Bu L
FIG. 47.



1 Bu P
FIG. 48.



1 Bu I
FIG. 49.

If there be an axis of both rotation- and reflection-twinning parallel to a plane of symmetry, there will be another axis of the same kind of twinning parallel to the same plane of symmetry and at right angles to the first axis. This case is, however, the same as the case of a similar axis of twinning at right angles to a line of symmetry, for a centre of symmetry must be present (see p. 227), and when that is the case every plane of symmetry has a line of symmetry at right angles to it. It has already been illustrated by the projections of Karlsbad twin-crystals in figs. 20 to 33. If the centre of symmetry and line of symmetry did not exist, but only the plane of symmetry, each axis would, as stated on p. 231, be an axis of only one mode of twinning, and while one would be an axis of rotation-twinning, the other would be an axis of reflection-twinning, as in figs. 46 to 48.

(2) If the normal to a plane of symmetry be an axis of rotation-twinning, the same twin-crystal can be formed by inversion-twinning; and if a twin-crystal can be explained by inversion-twinning, the normal to every plane of symmetry is an axis of rotation-twinning (figs. 46, 49, 53 to 58, 67, 70, 71, 74, 79 to 90). Thus the twins of eulytine, diamond, and tetrahedrite, already referred to (pp. 231, 239, and figs. 79 to 82), possess axes of rotation-twinning normal to the six planes of symmetry parallel to the rhombic-dodecahedron faces.

The normal to a plane of symmetry cannot be an axis of reflection-twinning. The similarity of the relations between equivalent twinning-operations due to the presence of a line or a plane of symmetry, may be seen at a glance in the following table, which also shows how closely the normal to a plane of symmetry is comparable to a line of symmetry.

Line of Symmetry.

If there be an axis of *rotation-*, or of *reflection-twinning*, or of *both*, at right angles to a line of symmetry, there will be another axis of *rotation-*, or *reflection-twinning* respectively, or of *both*, at right angles to the line of symmetry and to the first axis of twinning.

If a line of symmetry be an axis of reflection-twinning, the twin-crystal is an inversion-twin.

In an inversion-twin, every line of symmetry is an axis of reflection-twinning.

The Normal to a Plane of Symmetry.

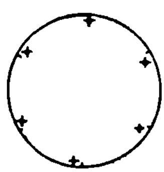
If there be an axis of *rotation-*, or of *reflection-twinning*, or of *both*, at right angles to the normal to a plane of symmetry, there will be an axis of *reflection-*, or *rotation-twinning* respectively, or of *both*, at right angles to the normal to the plane of symmetry and to the first axis of twinning.

These twin-axes will of course be parallel to the plane of symmetry.

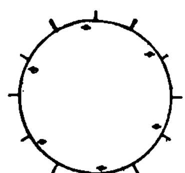
If the normal to a plane of symmetry be an axis of rotation-twinning, the twin-crystal is an inversion-twin.

In an inversion-twin, every normal to a plane of symmetry is an axis of rotation-twinning.

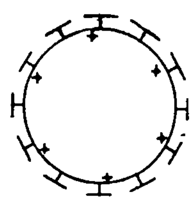
VII. *Axis of simple, or 'co-directional',* n-*fold symmetry.*—If there be an axis of rotation-, or of reflection-twinning, or of both, at right angles to the axis of n-fold symmetry, it will be one of n twin-axes of the same nature at right angles to the same axis of symmetry and making equal angles with one another. This is illustrated in figs. 50 to 52.



6 Uu
FIG. 50.



6 Uu L′
FIG. 51.



6 Uu P′
FIG. 52.

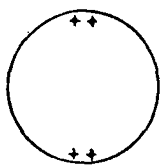
These figures show an axis of hexagonal symmetry. Figs. 50 and 51 represent two components of a twin-crystal where there are six axes of rotation-twinning at right angles to the hexagonal axis and parallel to the plane of projection; and figs. 50 and 52 represent the two components of a twin-crystal where there are six axes of reflection-twinning in a similar position. The letters L′ and P′ indicate the presence of these axes of rotation- and reflection-twinning respectively, the positions of which are shown in the projection in the manner already described on p. 229.

The relations described in Section V (1), pp. 228–231, are of course a special case of those dealt with under this Section.

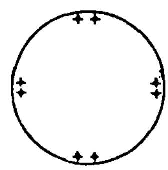
VIII. *Axis of simple, or 'co-directional',* n-*fold symmetry, with* n *lines of symmetry at right angles to it.*—If an axis of simple n-fold symmetry, with n lines of symmetry at right angles to it, be an axis of rotation-, or of reflection-twinning, or of both, there will be n axes of twinning of the same nature at right angles to each line of symmetry and to the axis of n-fold symmetry (combination of V (1) and VII).

These relations are illustrated by figs. 40 to 45, 59 to 61, 63 to 65, and 75 to 78, p. 236, which will be considered in more detail later.

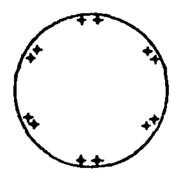
IX. *Axis of simple, or 'co-directional', n-fold symmetry, with n-planes of symmetry passing through it.*—If an axis of simple n-fold symmetry be an axis of rotation-, or of reflection-twinning, or both, there will be an axis of reflection-, or rotation-twinning respectively, or both, at right angles to the axis of symmetry, parallel to every plane of symmetry. (Combination of VI (1) and VII.)



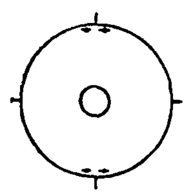
2 Bu
FIG. 53.



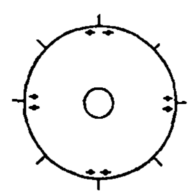
4 Bu
FIG. 55.



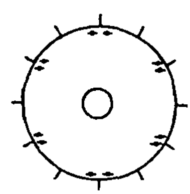
6 Bu
FIG. 57.



2 Bu P I
FIG. 54.



4 Bu P I
FIG. 56.



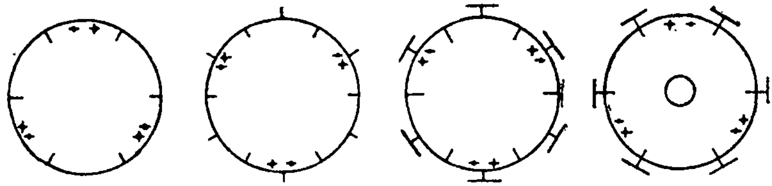
6 Bu P I
FIG. 58.

This is illustrated by figs. 53 to 58, and by figs. 67 to 69, 71 to 73, and 75 to 78, which likewise will be considered in more detail later.

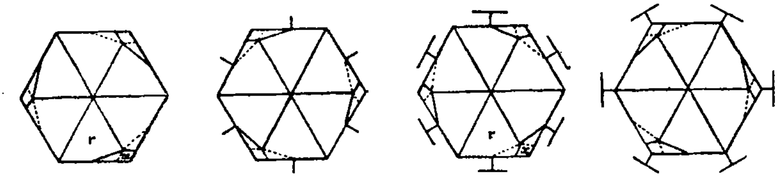
If the axis of symmetry be an axis both of rotation- and of reflection-twinning, there must be a centre of symmetry, and therefore a line of symmetry at right angles to every plane of symmetry, so that the case coincides with one already considered in VIII.

In all the twin-crystals included in Sections VII, VIII, and IX, the principal axis of symmetry has the same orientation in the two component parts of the twin. This is also the case in all inversion-twins.

These relations are well illustrated by the twin-crystals of quartz, pyrargyrite (with proustite), and calcite, that fall in this category. They

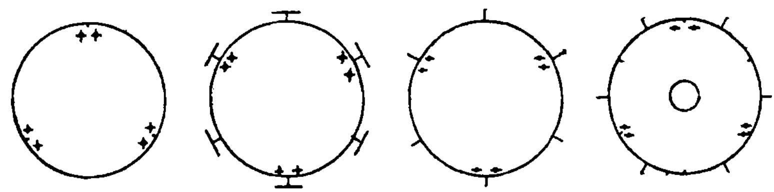


3 Uh FIG. 59. | 3 Uh L FIG. 60. | 3 UhP FIG. 61 | 3 Uh I FIG. 62.

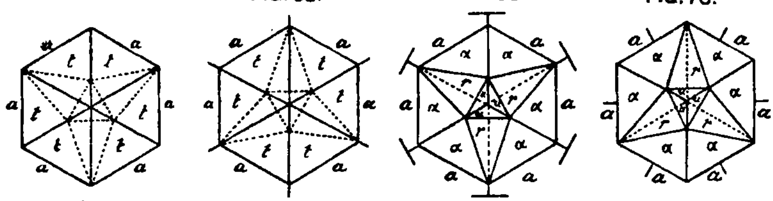


3 Uh FIG. 63. | 3 Uh L FIG. 64. | 3 UhP FIG. 65. | 3 Uh I FIG. 66.

Figs. 59-66.—Quartz.

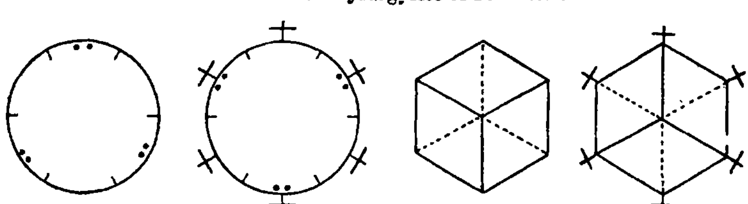


3 Bu FIG. 67 | 3 Bu L FIG. 68. | 3 BuP FIG. 69. | 3 Bu I FIG. 70.



3 Bu FIG. 71. | 3 Bu L FIG. 72. | 3 BuP FIG. 73. | 3 Bu I FIG. 74.

Figs. 67-74.—Pyrargyrite or Proustite.



3 Bc FIG. 75. | 3 Bc L P FIG. 76 | 3 Bc FIG. 77. | 3 Bc L P FIG. 78.

Figs. 75-78.—Calcite.

are shown in figs. 59 to 74, and the close analogy between them will be seen from the table which follows.

Figs. 59 to 62, 67 to 70, 75 and 76, are stereographic projections of the character already described (p. 225), on the basal plane. Figs. 63 to 66, 71 to 74, 77 and 78, are orthogonal projections on the basal plane.

Most of these twins are described by Prof. Lewis in his 'Treatise on Crystallography' (1899, pp. 514–515, 519–523, 527), those of pyrargyrite and proustite by Sir Henry Miers (this Magazine, 1888, vol. viii, pp. 73–82). Their relations to one another are shown in the following table.

	Quartz.	*Pyrargyrite.*	*Calcite.*
Symmetry.	Axis of threefold symmetry, the principal axis. Three lines of symmetry (digonal axes) at right angles to the principal axis. These are the lateral crystallographic axes.	Axis of threefold symmetry, the principal axis. Three planes of symmetry passing through the principal axis. Their normals are the lateral crystallographic axes.	Axis of threefold symmetry, the principal axis. Three lines of symmetry (digonal axes) at right angles to the principal axis. These are the lateral crystallographic axes and they are normal to planes of symmetry passing through the principal axis.
Types of twin-crystals in which the principal axes of the two components are parallel.	(1) The principal axis is an axis of rotation-twinning; there are three other axes of rotation-twinning at right angles to the principal axis and to the lines of symmetry.[1] (3 Uh, 3 Uh L.)	The principal axis is an axis of rotation-twinning; there are three axes of reflection-twinning at right angles to the principal axis, and parallel to the planes of symmetry.[1] (3 Bu, 3 Bu L.)	The principal axis is an axis of rotation- and reflection-twinning; there are also three axes of rotation- and reflection-twinning at right angles to the principal axis and to the lines of symmetry, and parallel to the planes of symmetry.[1] (3Bc, 3BcLP.)
	(2) The principal axis is an axis of reflection-twinning; there are also three other axes of reflection-twinning at right angles to the principal axis and to the lines of symmetry (3 Uh, 3 Uh P.)	The principal axis is an axis of reflection-twinning; there are also three axes of rotation-twinning at right angles to the principal axis, and parallel to the planes of symmetry. (3 Bu, 3 Bu P.)	The corresponding twin-crystal is that already described.
	(3) Inversion-twin The three lines of symmetry are axes of reflection-twinning. (3 Uh, 3 Uh I.)	Inversion-twin. The normals to the three planes of symmetry are axes of rotation-twinning. (3 Bu, 3 Bu I.)	As there is a centre of symmetry, no inversion-twinning is possible.

[1] These twin-crystals may also be formed by rotation round the principal axis through one-sixth of a circle. (See p. 240.)

X. *Axis of contra-directional* n*-fold symmetry.*—(Rotation-inversion axis of Hilton);[1] that is to say, an axis of symmetry in a crystal without central symmetry, such that a rotation round it through an angle $\frac{2\pi}{n}$ brings every line in the structure into coincidence with the former position of an equivalent line, but, in the case of a line having different properties in opposite directions, with its directions reversed. Here n must be even[2]—2, 4, or 6. See figs. 79, 81, 83, 85, 87, 89, 91. In the symbols accompanying the figures, such an axis is indicated by k. Examples of contra-directional axes are, the ortho-axis of clinohedrite, 2 Uk, the lateral axes of hemimorphite, 2 Bk, the crystallographic axes of eulytine and tetrahedrite, 4 Bk, and the vertical axis of chalcopyrite, 4 Bk, and of benitoite, 6 Bk; also the vertical axes 4 Uk and 6 Uk in the classes that differ from the last two minerals by the absence of vertical planes of symmetry, but of which no examples are known.

If there be an axis of rotation- or reflection-twinning at right angles to an axis of contra-directional symmetry, there will be in all $n/2$ axes of rotation-twinning and $n/2$ axes of reflection-twinning at right angles to the axis of symmetry, alternating and making equal angles with one another. Examples are given under XI and XII, where the axis of contra-directional symmetry itself is also an axis of twinning, which is not the case when the lateral twin-axis is neither at right angles nor parallel to a line of symmetry, or to a plane of symmetry passing through the contra-directional axis of symmetry.

The relations described in this Section include those stated in VI (1), for in the absence of a centre of symmetry, a normal to a plane of symmetry is always a contra-directional axis of n-fold symmetry, in which $n/2$ is odd, in other words n is 2 or 6. This is illustrated by figs. 83, 85, 87, 89.

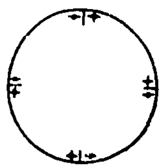
XI. *Axis of contra-directional* n*-fold symmetry, where* n/2 *is even, and where there are* n/2 *lines of symmetry passing through the axis, alternating with* n/2 *planes of symmetry and bisecting the angles between them.*—In

[1] H. Hilton, this Magazine, 1907, vol. xiv, pp. 261–263.

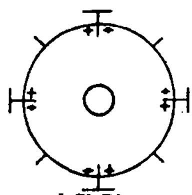
[2] In a simple, or co-directional, axis of symmetry, there is, on rotation through a portion of a circle, coincidence, not only of every line of the crystal with the former position of an equivalent line, but also of the directions in them. A contra-directional axis, in which $n = 4$ or 6 is also a co-directional axis with $n = 2$ or 3, as the case may be, but a contra-directional axis in which n is 2 is not a co-directional axis. If the character of an axis of symmetry is not expressly mentioned, it must be assumed that a simple or co-directional axis is intended.

this case n must be 4, and the contra-directional axis is represented by the symbol 4 Bk (fig. 79).

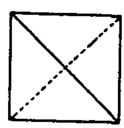
If in such a crystal the axis of contra-directional symmetry be an axis of reflection-twinning, the $n/2$ lines of symmetry at right angles to it will be axes of reflection-twinning, and the $n/2$ normals to the planes of symmetry will be axes of rotation-twinning. The same twin-crystal can also be formed by inversion, or by rotation round the axis through a quarter of a circle. The occurrence of such twins in eulytine, tetrahedrite, and diamond has already been described (pp. 231, 233). They are also met with in chalcopyrite. They are illustrated by the stereographic



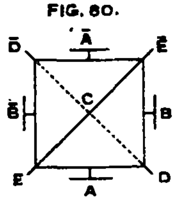
4 Bk
FIG. 79.



4 Bk Pl
FIG. 80.



4 Bk
FIG. 81.



4 Bk Pl
FIG. 82.

projections in figs. 79 and 80, and the orthogonal projections in figs. 81 and 82.

These may be taken to represent the reflection-twinning of chalcopyrite on the basal plane, or that of a cubic tetrahedron on the face of the cube. In each there are two other axes of reflection-twinning (AĀ and BB̄). At the same time in the former there are also two axes of rotation-twinning (DD̄ and EĒ) at right angles to the prism of the first order, and in the latter, six axes of rotation-twinning (four of which are not shown) at right angles to the rhombic-dodecahedron faces.

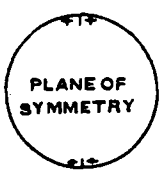
An axis of contra-directional symmetry when $n/2$ is even, cannot be an axis of rotation-twinning, as it is always a line of symmetry (see pp. 228 note, 231, 238 note).

XII. *Axis of* n-*fold contra-directional symmetry where* n/2 *is odd and* *there are* n/2 *lines of symmetry at right angles to it, lying in* n 2 *planes of symmetry passing through it.*—Here n must be 2 or 6. The former case is represented by the normal to a plane of symmetry in hemimorphite, 2 Bk, the latter by the principal axis of benitoite, 6 Bk. See figs. 83 and 85.

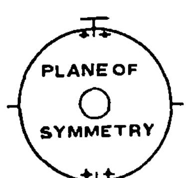
If the axis of contra-directional symmetry be an axis of rotation-twinning, the $n/2$ lines of symmetry at right angles to it will be axes of reflection-twinning and the $n/2$ normals to the planes of symmetry will be axes of rotation-twinning.

The same twin-crystal can also result from inversion, and where n is 6, by rotation round the axis of contra-directional symmetry through a sixth of a circle. These twins are illustrated in figs. 83 to 86. The twins of hemimorphite on a lateral pinakoid offer an example where n is 2.

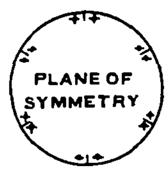
An axis of contra-directional symmetry with $n/2$ odd cannot be an



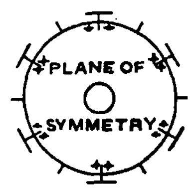
2 Bk
FIG. 83.



2 Bk L I
FIG. 84.



6 Bk
FIG. 85



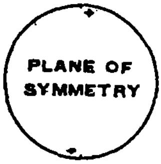
6 Bk L I
FIG. 86.

axis of reflection-twinning, because it is the normal to a plane of symmetry (see p. 233).

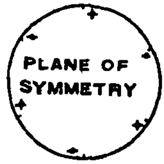
In the absence of planes of symmetry passing through a contra-directional axis, or of lines of symmetry at right angles to it, the fact that the axis of contra-directional symmetry is a twinning-axis, does not involve the occurrence of any other twinning except inversion-twinning and, in the cases specified below, twinning by rotation through an angle less than a half circle. This is illustrated by figs. 87 to 92.

XIII. We have seen that a rotation through a quarter of a circle round an axis of contra-directional fourfold symmetry is a twinning-operation (see p. 239 and figs. 79 to 82 and 91, 92).

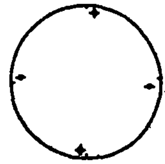
XIV. In the same way a rotation through a sixth of a circle round a co-directional axis of threefold or trigonal symmetry, or a contra-directional axis of sixfold or hexagonal symmetry, is also a twinning operation (see pp. 226, 237 note, 240, and figs. 5, 6, 11, 12, 59, 60, 63, 64, 67, 68, 71, 72, 75 to 78, 85, 86, 89, 90).



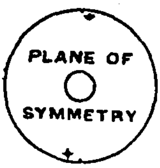
2 Uk
FIG. 87.



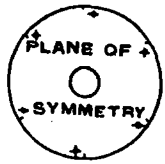
6 Uk
FIG. 89.



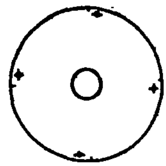
4 Uk
FIG. 91.



2 Uk L I
FIG. 88.

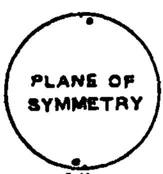


6 Uk L I
FIG. 90.

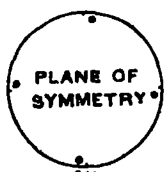


4 Uk P I
FIG. 92.

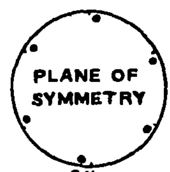
XV. If a crystal possesses symmetry about a centre, a line of symmetry (that is, an axis with an even symmetry number, see p. 228 note) cannot be a twin-axis. For being a line of symmetry it cannot be an axis of line- or rotation-twinning, and being normal to a plane of symmetry (see p. 233) it cannot be an axis of plane- or reflection-twinning. At the



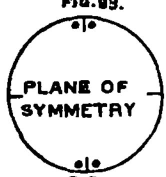
2 Uc
FIG. 93.



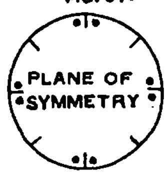
4 Uc
FIG. 94.



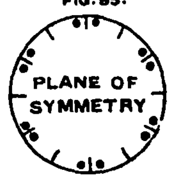
6 Uc
FIG. 95.



2 Bc
FIG. 96.



4 Bc
FIG. 97.



6 Bc
FIG. 98.

same time the presence of a centre of symmetry renders point- or inversion-twinning impossible. This is illustrated by figs. 93 to 98. Examples are the crystallographic axes of staurolite, 2 Bc, rutile, 4 Bc, scheelite, 4 Uc, spinel, 4 Bc, pyrite, 2 Bc, and beryl, 6 Bc (Bravais-Miller axes), the ortho-axis of augite, 2 Uc, the vertical axis of apatite, 6 Uc, and lateral axes of calcite, 2 Uc (Bravais-Miller).

Résumé of the explanations of Diagrams and Symbols.

The circle may be regarded as the primitive circle of a stereographic projection in which faces of a crystallographic form of a general type are represented. A plus sign with a dot at its centre indicates an upper face (fig. 1), and a minus sign with a central dot a lower face (fig. 8). As in the gnomonic projection, the projection of a lower face is at the point where the projection of a parallel upper face would be, if it were present. A simple dot without a plus or minus sign indicates the presence of both an upper face and the lower face parallel to it (see fig. 9).

Two short opposite radial lines drawn inwards from the circumference of the circle show the position of a line of symmetry (fig. 16) in the plane of projection. If the plane of projection is a plane of symmetry, the words 'plane of symmetry' are written in the circle (fig. 83). Other elements of symmetry are evident at once from the character and distribution of the projections of the faces.

The left-hand or upper diagram in each case shows the orientation of the untwinned crystal, the other or others, that after the application of a twinning-operation.

Unless otherwise stated, the plane of projection is a twin-plane and its normal the corresponding twin-axis.

Two simple opposite radial lines drawn outwards from the circumference of the circle indicate the position of a twin-axis of rotation-twinning in the plane of projection (fig. 17). Two similar, but shorter lines, each terminating in the centre of a longer line at right angles, show the position in the plane of projection of a twin-axis of reflection-twinning (fig. 18). In the same manner two crosses on the exterior of the circle show the position of a twin-axis of both rotation- and reflection-twinning (fig. 21). A small circle in the centre of the diagram indicates the presence of inversion-twinning (fig. 4).

The same signs, representing the presence of twin-axes in the plane of projection, are also employed in the orthogonal projections.

The number at the commencement of each symbol indicates the number of the times the crystal will coincide with its former position in the course of a complete revolution round the normal to the projection. If the number be 1, the twin-axis is not an axis of symmetry. If the number be 2, it is a twofold or digonal axis, and so on. The only exception is when the symbol concludes with the letter k. In that case half the coincidences are imperfect, equivalent lines being coincident, but their directions opposed. The normal to the projection may then be described as an axis of contra-directional symmetry of n-fold symmetry when n is the number prefixed. The principal axis of chalcopyrite, for instance, is an axis of contra-directional tetragonal symmetry. The capital letter B following the numeral indicates the presence of one or more planes of symmetry at right angles to the plane of projection, and U the absence of such a plane of symmetry. The small letter c at the end of a symbol denotes the presence of a centre of symmetry; h indicates that, while there is no centre of symmetry, there is a line of symmetry in the plane of projection, and by rotation round this, one termination may be brought into coincidence with the original position of the other. It has already been explained that k indicates that the normal to the plane of projection is an axis of contra-directional symmetry. Here, also, the two terminations are similar, but they are not superposable, unless the axis is parallel to a plane of symmetry.

The small letter u indicates that the terminations of the normal to the plane of projection are unlike.

The different classes of symmetry may be denoted by the same symbols as those of the axis or axes which possess the highest symmetry in each class (this Magazine, 1910, vol. xv, pp. 398–400). The number is then written with Roman numerals, except in the case of the cubic system where the four trigonal axes are employed and the capital letter C is substituted for the numeral.

The capital letter P placed at the end of the symbol indicates that one component has been subjected to a twinning-operation by reflection on the plane of projection; L, that it has been rotated on the normal to the same plane, and I, that it has suffered inversion through the centre (p. 225). The presence of two of these letters shows that the two operations indicated give the same result, and are therefore equivalent to one another. P′ or P″ and L′ or L″ indicate reflection- and rotation-twinning respectively with a twin-axis lying in the plane of projection.

The figures have been executed under my direction by Miss Margaret Reeks, Technical Artist to the Imperial College of Science and Technology, to whom I wish to express my indebtedness.