

Merosymmetry versus merohedrism.

By AUSTIN F. ROGERS, Ph.D.

Department of Geology, Stanford University, California.

[Read November 3, 1938.]

WHILE crystal classes based upon symmetry furnish a clear-cut classification of crystals free from ambiguity and uncertainty, there are cogent reasons for recognizing the larger groups called systems.

It is important to recognize crystal systems, not only on account of physical properties, but also on account of the difficulty of assigning some crystals to their appropriate class. For example, a crystal consisting of a tetragonal prism and a pinakoid may belong to any one of seven classes. For the time being it is placed in the tetragonal system. When it is carefully investigated and its symmetry determined it may be assigned to the proper class.

Contrary to statements that have been made,¹ we can have, and do have, a two-fold classification in geometrical crystallography—the thirty-two classes based upon symmetry, and the six systems based upon properties that certain classes have in common.

In the opinion of the writer the most satisfactory set of names for the crystal classes is the one based upon names of general forms. These names of Groth are comparatively simple, consistent, and lend themselves to international usage. Moreover, they are entirely free from ambiguity and this can scarcely be claimed for any other set of names. The use of these class names in Groth's 'Chemische Krystallographie' (1906–19) indicates that in all probability they will become standard, since this important five-volume reference work is so extensively used by structural crystallographers the world over.

Class names based upon merohedrism or merohedry (general terms to include both hemihedrism and tetartohedrism or the corresponding terms hemihedry and tetartohedry) have been very widely used for the past century, but many crystallographers have abandoned these terms since they lead to inconsistencies.²

¹ Victor Goldschmidt (trans. by M. A. Peacock), *Amer. Min.*, 1931, vol. 16, pp. 18–33. [M.A. 4–439.]

² W. J. Lewis *A treatise on crystallography*. Cambridge, 1899, p. 259.

Application of the principle of merohedrism failed to establish all of the possible crystal classes. At one time, for example, sucrose was assigned to a possible tetartohedral division of the orthorhombic system.¹ Hemimorphism was formerly placed on an entirely different footing from other kinds of hemihedrism.² In spite of the preliminary researches of Hessel and Bravais, it was not until the work of the Finnish crystallographer Gadolin in 1871 that we had a really satisfactory classification of crystals. By the use of the single principle of symmetry Gadolin deduced the thirty-two classes of symmetry.

And not until the symmetry classes were derived was it possible to establish the crystal systems satisfactorily. The diclinic system of the middle nineteenth century disappeared, not because of the impossibility of diclinic axes of reference (I have recently suggested that the term diclinic syngony be used for such cases³), but because there were no crystal classes that could be assigned to it. Gadolin used seven systems, but Fedorov, by means of his syngony-ellipsoid, showed that there are only six crystal systems, or syngonies as he preferred to call them. Fedorov's work has been confirmed by the writer, who defines the six crystal systems in terms of zones.⁴

Although often discarded and regarded as having historic interest only, terms involving merohedrism sometimes have been retained for pedagogic reasons. The late A. H. Phillips,⁵ for example, says: 'The old method of deriving the hemihedrons, etc., from the holohedrons has been retained and may meet with criticism, but it is a simple method of determining what forms are possible to combine on crystals of lower symmetry. I have always found it a very material aid to the student, leaving no false impressions.' Now, one's recognition of merohedrism seems to depend largely upon the emphasis placed upon crystal systems and the question arises as to whether crystal systems or crystal classes are more fundamental. If space permitted, arguments might be given to show that crystal classes are more fundamental than crystal systems. The mere fact that there is disagreement as to the number of crystal systems is almost proof of this.

But even if this is admitted, there is need of a series of general terms for classes of varying degree of symmetry within the crystal systems.

¹ G. H. Williams, *Elements of crystallography*. New York, 1890, p. 169.

² P. Groth, *Physikalische Krystallographie*. First edition, Leipzig, 1876.

³ A. F. Rogers, *Amer. Min.*, 1935, vol. 20, p. 210.

⁴ *Idem*, *Introduction to the study of minerals*. Third edition, New York, 1937, pp. 38-39.

⁵ A. H. Phillips, *Mineralogy*. New York, 1912, p. v.

This need has usually been supplied by the use of terms based upon merohedrism or merohedry. The class with the highest grade of symmetry in a system was designated holohedral, those obtained by suppressing one-half of the faces and extending the other half were called hemihedral, and those obtained by extending one-fourth of the faces were called tetartohedral.

Since the selection of faces is always a symmetrical selection, and this is rarely stated, much less emphasized, it seems advisable to use the terms merosymmetry, hemisymmetry, and tetartosymmetry, rather than merohedrism, hemihedrism, and tetartohedrism. The key-note of geometrical crystallography, if I understand it aright, is symmetry. The first chapter of Fedorov's Курсъ кристаллографіи (Course of crystallography, 3rd edition, St. Petersburg, 1901) is entitled О симметріи (On symmetry).

The term *merosymmetry* (from the Greek μέρος, part or fraction) was introduced by Nevil Story-Maskelyne in a series of lectures before the Chemical Society of London, which appeared in the *Chemical News*¹ for 1875. These lectures formed the basis for Story-Maskelyne's classic work 'Crystallography, a treatise on the morphology of crystals' which appeared in 1895. Since the publication of this important book the term holosymmetry often has been used, due largely to the influence of Miers² and Spencer,³ but to the best of my knowledge there has been no particular recognition of the value of Story-Maskelyne's idea. This is probably due largely to the fact that Story-Maskelyne himself was not altogether consistent. Instead of using hemisymmetric, tetartosymmetric, and ogdosymmetric (he used six crystal systems, so there is one case of ogdosymmetry) he introduced such terms as haplohedral and diplohedral, holosystematic and hemisystematic, which unnecessarily complicated his nomenclature. However, Story-Maskelyne did clearly set forth satisfactory divisions of the crystal systems and was one of the first to recognize the importance of the thirty-two symmetry classes.

The purpose of this communication is to advocate the general use of the terms holosymmetry and merosymmetry with its subdivisions hemisymmetry, tetartosymmetry, and ogdosymmetry. By substituting the suffixes *-symmetry* and *-symmetric* for *-hedrism* and *-hedral* we have a satisfactory series of terms. We then have the advantages claimed for merohedrism by Phillips and at the same time emphasize symmetry. It is a pity that this suggestion was not made a quarter of a century ago,

¹ N. Story-Maskelyne, *Chem. News*, London, 1875, vol. 31, pp. 3-4 . . . 232.

² H. A. Miers, *Mineralogy*. London, 1902.

³ L. J. Spencer, *Encyclopaedia Britannica*, 11th edition, 1910, vol. 7, p. 569.

but geometrical crystallography will doubtless be studied for centuries to come. If the proposal is a good one, the change should be made even though it is rather late.

On the table of the thirty-two crystal classes (p. 256) I have attempted to put down what I consider to be a simple consistent set of terms based upon merosymmetry. In the left-hand column are the six crystal systems with the two sub-systems of the hexagonal system. In the next column are the class numbers which I have used for some time.¹ These are slight modifications of Groth's numbers² arranged so as to place classes 21 and 22 in the hexagonal sub-system instead of in a separate trigonal system as Groth did. The class numbers are convenient for reference and it should be emphasized that the sequence of the classes is only in minor part arbitrary. The arrangement is such that every class has a lower class number than any of the classes of which it is a sub-group.³

Next are the class names of Groth which are based upon Fedorov's names of general forms. The names of the isometric classes are simplified; hexoctahedral, for example, instead of hexakisoctahedral. In the fourth column are what Shearer, the structural crystallographer, has happily called the symmetry number, which will be recognized as the number of faces in the general form or what in group theory is called the order of the group (point-group).

Next are the class names based upon merosymmetry. Since there are six crystal systems there are six holosymmetric classes. Asymmetric is used for the pedial class instead of hemisymmetric, because strictly speaking there is no symmetry at all, hence no hemisymmetry. In the monoclinic system the adjectival terms axial and planar are adapted from Boldyrev.⁴ The convenient term holoaxial was introduced by Miers.⁵ Polar is used, but not equatorial, since it implies the use of the stereographic projection. The terminology should, I believe, be independent of any particular method of projection. The term alternating has not been used, since it seemed to be unnecessary. For two of the classes, nos. 19 and 23, triple names unfortunately seemed to be necessary.

While the terms I have used here are probably free from ambiguity,

¹ A. F. Rogers, *Amer. Min.*, 1928, vol. 13, p. 573. [M.A. 4-54.]

² P. Groth, *Physikalische Krystallographic*. 4th edition, Leipzig, 1905.

³ A. F. Rogers, *Proc. Amer. Acad. Arts Sci.*, 1926, vol. 61, pp. 198-200. [M.A. 3-239.]

⁴ A. K. Boldyrev and V. V. Dolivo-Dobrovolsky, *Ann. Inst. Mines, Leningrad*, 1934, vol. 8, p. 155. [M.A. 6-79.]

⁵ H. A. Miers, *Mineralogy*. London, 1902, p. 45.

Table of the thirty-two Crystal Classes

System.	Class. no.	Class names (based upon the general form).	Sym. no.	Alkermative names (based upon nesrosymmetry).	Example.	
Triclinic	1	Pedial	1	Asymmetric	$H_2SiCl_3 \cdot 4H_2O$ Albite	
	2	Pinacoidal	2	Holosymmetric		
Monoclinic	3	Sphenoidal	2	Axial hemisymmetric	$C_{12}H_{22}O_{11}$ (Sucrose) Climohedrite Gypsum	
	4	Domatic	2	Planar hemisymmetric		
	5	Prismatic	4	Holosymmetric		
Orthorhombic	6	Rhombic disphenoidal	4	Holoaxial hemisymmetric	Epsomite Hemimorphite Barite	
	7	Rhombic pyramidal	4	Polar hemisymmetric		
Tetragonal	8	Rhombic dipyramidal	8	Holosymmetric	Cahnite $Ba(SbO)_2(O_2H_2O)_4 \cdot H_2O$ Chalcopyrite $NH_4(OH)_2$ Scheelite Ag $\cdot H_2O$ Zircon	
	9	Tetragonal disphenoidal	4	Sphenoidal tetartosymmetric		
	10	Tetragonal pyramidal	4	Polar tetartosymmetric		
	11	Tetragonal scalenohedral	8	Sphenoidal hemisymmetric		
	12	Tetragonal trapezohedral	8	Holoaxial hemisymmetric		
	13	Tetragonal dipyramidal	8	Dipyramidal hemisymmetric		
	14	Ditetragonal pyramidal	8	Polar hemisymmetric		
	15	Ditetragonal dipyramidal	16	Holosymmetric		
Hexagonal sub-system	16	Trigonal pyramidal	3	Ogdisymmetric	$NaIO_3 \cdot 3H_2O$ Diopside α -Quartz Tourmaline Calcite Benitoite Nepheline β -Quartz Aristite Iodyrite Beryl Ullmannite Pyrite Tetrahedrite Garnet	
	17	Rhombohedral	6	Rhombohedral tetartosymmetric		
	18	Trigonal trapezohedral	6	Holoaxial tetartosymmetric		
	19	Tetragonal pyramidal	6	Polar ditrigonal tetartosymmetric		
	20	Hexagonal scalenohedral	12	Rhombohedral hemisymmetric		
	Hexagonal system	21	Trigonal dipyramidal	6		Trigonal tetartosymmetric
		22	Ditrigonal dipyramidal	12		Trigonal hemisymmetric
		23	Hexagonal pyramidal	6		Polar hexagonal tetartosymmetric
		24	Hexagonal trapezohedral	12		Holoaxial hemisymmetric
		25	Hexagonal dipyramidal	12		Dipyramidal hemisymmetric
26		Dihexagonal pyramidal	12	Polar hemisymmetric		
27		Dihexagonal dipyramidal	24	Holosymmetric		
Isometric	28	Tetartohial	12	Tetartosymmetric	Ullmannite Pyrite Tetrahedrite Garnet	
	29	Gyroidal	24	Holoaxial hemisymmetric		
	30	Diploidal	24	Central hemisymmetric		
	31	Hextetrahedral	24	Polar hemisymmetric		
	32	Hexoctahedral	48	Holosymmetric		

I realize that other terms could have been chosen. There are in some cases a number of choices. For example, class 3 might have been called holoaxial hemisymmetric or polar hemisymmetric; class 13 might have been designated as central hemisymmetric or equatorial hemisymmetric. This difficulty in naming a class will serve to emphasize the great value of Groth's names of classes which are names of general forms. In consequence of this only one name for each class is possible after the form names are established. You will note that I regard names based upon merosymmetry as alternative names which are often useful, especially in discussing the crystal classes of a particular system. The classes are for the time being regarded as divisions of crystal systems. But these names are not to replace class names based upon general forms.

Then finally, in the last column, examples are given of substances crystallizing in each class, except for classes 21 and 29. Acid silver phosphate is sometimes given for class 21, but there has never been any proof that it belongs in this class. Cuprite and sylvite have been assigned to class 29, but they almost certainly are holosymmetric. Other candidates for class 29 are $\gamma\text{-Al}_2\text{O}_3$, and $\gamma\text{-Fe}_2\text{O}_3$, but Dr. Tom Barth¹ is almost certain that they are holosymmetric, since they are probably isomorphous with each other, and $\gamma\text{-Al}_2\text{O}_3$ forms solid solutions with holosymmetric MgAl_2O_4 . Cahnite, $\text{Ca}_4\text{B}_2\text{As}_2\text{O}_{12}\cdot 4\text{H}_2\text{O}$, from Franklin Furnace, New Jersey, belongs to the tetragonal disphenoidal class² and hence in the table it may replace $\text{Ca}_2\text{Al}_2\text{SiO}_7$, a laboratory product assigned to this class by Weyberg.

We now have four useful sets of terms for the 32 symmetry classes:

- (1) Class names based upon general forms.
- (2) Alternative names based upon merosymmetry.
- (3) Example names of substances crystallizing in each class.
- (4) Class numbers.

These are listed in order of their probable importance. We need synonyms here as in other fields. Examples of mineral names were used by Miers, who spoke of the calcite class, for example. As suggested earlier in this article, I find the class numbers very convenient. After some use they become quite familiar.

The French school of crystallographers following Bravais make the symmetry of the space-lattice the basis of the crystal system and so employ seven systems instead of six. This appeals to many modern structural crystallographers. In this usage holohedral crystals are defined as those

¹ Personal communication, October 1938.

² Lazard Cahn, personal communication, May 1938.

having the same symmetry as the lattice and merohedral crystals as those with less symmetry than that of the lattice. In the opinion of some crystallographers this is the main justification for the use of merohedrism.

Now it is true that there are *seven* styles of crystal architecture and this should be emphasized even more than the 14 Bravais space-lattices. But to insist upon seven crystal systems based upon the symmetry of lattices is to give the term crystal system a meaning different from the generally accepted use of that term. In whatever terms it may be defined, a crystal system is thought of as a series of closely related crystal classes with geometrical and physical properties in common. The difficulty to which I refer does not seem to be appreciated by structural crystallographers. Let me explain it.

In five crystal systems (triclinic, monoclinic, orthorhombic, tetragonal, and isometric) there is no difficulty, for there is but one primary lattice for each system. But for five of the twelve classes with a single hexagonal zone there is a serious objection to this method of defining a crystal system. In the case of each of five classes of what I call the rhombohedral sub-system (nos. 16–20 of the tabulation on p. 256) there are two possible space-lattices, the hexagonal and the rhombohedral. Now this places us in the awkward position of knowing the crystal class of a given crystal before we know its crystal system. I contend that the determination of the symmetry of a crystal should automatically determine the crystal system. We cannot properly speak of systems of symmetry, but the symmetry classes are the constituent members of the crystal systems. One of the principal values of crystal systems is their use as groups larger than crystal classes, as suggested in the second paragraph of this article. It is important, I believe, to retain the original and generally accepted use of the term crystal system.

My friend and former student Dr. J. D. H. Donnay has kindly called my attention to a difficulty of another kind that is not so easy to solve; this also concerns the rhombohedral sub-system. Calcite, for example, is hemisymmetric according to my use of the term, since it belongs to the hexagonal system, but from the standpoint of the French school of crystallographers and many structural crystallographers it is considered to be holohedral, since the space-lattice is rhombohedral. This introduces a complication which I think can be cleared up by suitable nomenclature, but I have no suggestion to offer at present.

It is a pleasure to record my indebtedness to Dr. J. D. H. Donnay for a critical reading and discussion of the manuscript of this article.