The relation between area and volume in micrometric analysis.

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IN a recent note in this Magazine, Dr. R. B. Elliott[1] points out that inclined contacts of grains of considerably different index may bias the result of a micrometric analysis. That there may be some bias of this sort is hardly to be doubted, though I am inclined to feel that he greatly overestimates its importance. His demonstration, however, involves a fallacy so long and widely held as to be worth refuting.

Dr. Elliott begins by arguing that the micrometric analysis is an attempt to estimate directly the composition of the volume of rock actually contained in the thin section. The thickness of a thin section, however, has nothing to do with the question of whether the 'composition' of its surface may be regarded as an unbiased estimate of the composition of a volume of rock. The section could be wedge-shaped, hemispherical, or as thick as the lithosphere, provided only that we could satisfactorily identify the minerals whose sections form its surface.

The theoretical justification for 'Rosiwal' analyses as now performed has nothing to do with either thin sections or grain-size. The systematic traversing of the surface of the section is a form of graphical integration which may be made as precise as seems desirable. Of course this capacity for infinite improvement is mostly theoretical. One of the advantages of the point-counter[2] is that its error is well enough known to permit intelligent adjustment of count length to meet varying demands. The situation is by no means as clear for the various line-integrators, particularly the mechanically operated ones. In principle, however, it ought to be similar; the smaller the intervals between traverses the smaller the precision error of the results. But the question of precision is not a serious consideration at this stage of the argument. What matters here is that, barring identification difficulties (and erratic mechanical failure of the automatic integrators), the Rosiwal analysis is accurate *in the sense that it is unbiased.* If we can identify the mineral

[1] R. B. Elliott, Min. Mag., 1952, vol. 29, p. 833.

[2] F. Chayes, Amer. Min., 1949, vol. 34, pp. 1 and 600. [M.A. 11-277.]

sections forming the surface of the thin section, the micrometric analysis gives an unbiased estimate of their areal proportions, the 'composition' of the area.

But of what use are accurate areal proportions if we may not regard them as statements of composition by volume? Clearly they are of no use, and it is precisely in the passage from area to volume that most of the difficulty in interpreting Rosiwal analyses arises. The justification is simple, but, like the preceding argument, has nothing to do with rocks or the sizes and shapes of grains. It is just the usual formula, proved in every calculus text, for the volume of a solid, namely:

$$V = \int_a^b A\,dh \tag{1}$$

where A, the area of the cross-section perpendicular to an axis along which h is measured, is some function of h. In order to operate mathematically with (1) it is necessary to know this function, but the formula is valid so long as the function exists, whether we know it or not. If a and b represent the top and bottom of the grain normal to the plane of the thin section, A has one and only one value for every value of h between a and b. It follows then that if distances of a (or b) from the plane of the thin section are equally likely, the proportion of the area of any section occupied by the grain is an unbiased estimate of its relative volume, for the sum of an infinite number of such infinitely thin areas would *be* its volume.

In practice, of course, the plane of the thin section cuts each grain only once, and a single cross-section, though accurate in the sense of being unbiased, will almost certainly be highly imprecise. For practical purposes we are forced to make assumptions about the distribution of grains through the mass. The problems are many and varied, and some of them are extremely complex. But neither the thickness of the thin section nor the volume of the rock it contains has anything to do with the passage from the determination of relative areas to the estimation of relative volumes. In particular, the temptation to identify its thickness with the differential and its volume with the quantity Adh of equation (1) is to be scrupulously avoided.

Dr. Elliott's calculations are all based on the assumption that the micrometric analysis is an attempt to estimate directly the volumetric composition of the thin section, and the measuring procedure he follows is quite evidently influenced by this assumption. He says, for instance, (p. 833), that when contacts are inclined and adjacent grains differ considerably in index, '. . . we measure the maximum diameter of the high relief mineral, that is, from the tip of the bevelled edge on one side to the tip of the bevelled edge on the other. This will produce an inaccuracy, *for we ought to measure from the centre of the bevel on one side to the centre of the bevel on the other'* [italics added]. But according to the preceding argument, this last is just precisely what we should *not* do. The proper procedure is to focus as sharply as possible on the surface of the slide and change keys when the intersection of the grain contact with the slide surface passes the cross-hair intersection. In general, only that part of any grain which does actually reach the surface along the line of traverse is to be measured or counted.

This procedure is an exact analogue of that originally proposed by Delesse[1] more than a century ago. He says (p. 384) 'On doit avoir soin de dessiner seulement les minéraux qui trouvent dans le plan même de section', and again, on the same page, '. . . on ne dessine que celles qui sont réellement coupées par la surface considérée.' Precedent for supposing that micrometric measurement is in some way concerned with a volume rather than a surface may be found in Rosiwal (p. 163),[2] but the notion is quite as unwarranted today as it was in 1898.

When the intersections of grain boundaries with the surface of the section can be clearly located, the bias described by Dr. Elliott simply does not exist if the measurement is properly made. The combination of high relief, fine grain, and inclined contacts—sometimes even any one of the three—may make it difficult or impossible to locate the intersections of contact planes with the surface of the section. Though the resulting uncertainty has nothing whatever to do with the relation between area and volume, it may influence our ability to measure the area in either of two ways. It may simply lower the precision of the measurement without affecting its potential accuracy; when this is so the obvious remedy is more extensive measurement if the loss of precision cannot be tolerated. If we have reason to suppose, however, that the error does introduce a systematic bias into the areal measurement, mere extension of the measurement will not help. If the bias can be neither tolerated nor eliminated, the material is not amenable to micrometric analysis. The failure, however, is not in the fundamental relation between areal and volumetric composition, as one might suppose from Dr. Elliott's treatment of the problem. It is simply our inability to obtain an unbiased estimate of the composition of the area.

[1] A. Delesse, Annales des Mines, 1848, sér. 4, vol. 13, p. 379.

[2] A. Rosiwal, Verhandl. Geol. Reichsanst. Wien, 1898, vol. 5–6, p. 143.