The Holmes effect and the lower limit of modal analysis.

By F. Chayes.

Geophysical Laboratory, Carnegie Institution of Washington, Washington, D.C., U.S.A.

[Taken as read 7 June 1956.]

Summary.—Thin-section analysis is essentially an areal measurement, the measurement area usually being the upper surface of the section. If transmitted light is used for the measurement, the apparent areas of opaque grains will in general be somewhat larger than their true areas on the measurement surface. For strictly spherical opaque particles in a transparent matrix the expected excess of apparent over true area is shown to be $(\pi r^2 k)/(2r+k)$ where r is the spherical radius and k is the thickness of the thin section. A table shows the relation between true and apparent area as a function of r/k.

A. HOLMES[1] seems to have been the first to note the bias involved in measuring areas of opaque grains in a transparent medium by means of transmitted light. The apparent area of the opaque grain will always be that of its maximum cross-section in the slide, whereas one should be measuring only its 'true' area, e.g. its area at the *surface* of the thin section. Thus in general one will overestimate the relative area of opaque granules on the measurement surface.

For particle shapes of low symmetry the size of the Holmes bias is rather troublesome to calculate. This is true also for highly symmetrical shapes oriented in such a way that the symmetry cannot be utilized. For spherical particles, however, the calculation is straightforward, though somewhat tedious. It is worth carrying through as an indication of the general order of magnitude of the effect for particle shapes of lower symmetry.

A sphere of radius greater than the thickness of the thin section $(r > k)$ is shown in fig. 1; the observer is situated somewhere along the positive region and the light source along the negative region of the Y-axis. The distance of the measurement surface (the upper surface of the thin section) from the base of the sphere is given by y, and an opaque area will appear in the field of vision whenever $0 < y < (2r+k)$. For $y \leqslant r$ the apparent opaque area will be identical with the true one. In the

[1] Petrographic methods and calculations. Thomas Murby and Co., London, 1920, p. 317.

region $r \leqslant y \leqslant (r+k)$ the apparent area will be constant and equal to the area of the equatorial plane, while the true area, the one the observer should be measuring, decreases steadily as $y \to (r+k)$. In the region $(r+k) \leqslant y \leqslant 2r$ both the true and the apparent areas decrease steadily as $y \to 2r$, but they do so at different rates and the apparent area is

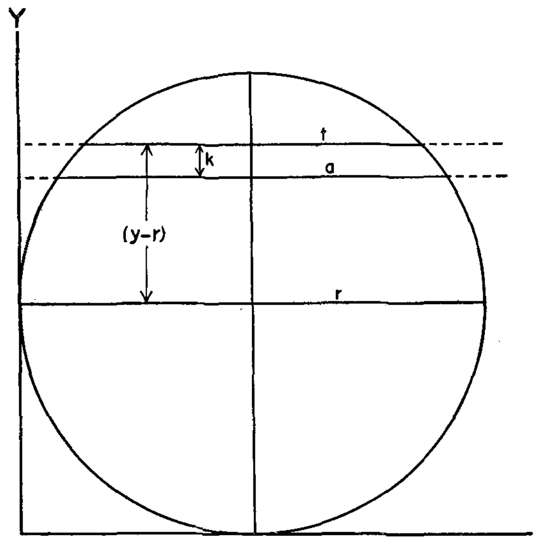

FIG. 1. Thin section of thickness k cutting opaque spherical particle of radius r. The true opaque area at the measurement surface is a circle of radius t; the apparent opaque area is a circle of radius a.

always larger than the true. Finally, in the region $2r \leqslant y \leqslant (2r+k)$, the true area is zero—the sphere does not intersect the measurement surface of the thin section—but the apparent area persists until $y = (2r+k)$, at which point it vanishes. Thus at any y value in the region $0 < y < (2r+k)$ the apparent area is never less than the true, and in the region $r < y < (2r+k)$ it is always larger. Areal measurements made in transmitted light will inevitably be subject to a positive bias and our problem is to estimate the size of this bias. In order to do so we first find the expected or average values for the true and apparent opaque areas. The bias is simply the difference between the two.

Our interest is only with those sections of the sphere in which the apparent area is greater than zero; this will be true of all sections in the region $0 < y < (2r+k)$ and only of such sections. We therefore specify that elevations of measurement areas above the base of the sphere are to be chosen simply at random, but with the limitation $0 \leqslant h \leqslant (2r+k)$, so that the probability that some particular elevation, y, is equal to or less than any arbitrary value, h, is simply $Pr\,\{0 < y \leqslant h\} = h/(2r+k)$. The cumulative frequency is accordingly

$$F(h) = \frac{1}{2r+k} \int\limits_0^h dy \tag{1}$$

which is clearly equal to unity when $h = (2r+k)$.

Now the area of any particular circular section at the measurement surface will be:

$$\left.\begin{aligned} &A_t = \pi\{r^2-(r-y)^2\} = \pi\{r^2-(y-r)^2\} = \pi(2ry-y^2) \\ &\text{in the region} \qquad 0 \leqslant y \leqslant 2r, \\ &\text{and} \quad A_t = 0 \text{ in the region} \qquad 2r \leqslant y \leqslant (2r+k) \end{aligned}\right\}. \tag{2}$$

From equations (1) and (2) we have that the expected value or long-range average for the true area is

$$E(A_t) = \frac{\pi}{2r+k} \int\limits_0^{2r} (2ry-y^2)\,dy = \frac{4\pi r^3}{3(2r+k)}. \tag{3}$$

The apparent area will be

$$\pi(2ry-y^2) \text{ if } 0 \leqslant y \leqslant r, \qquad \pi r^2 \text{ if } r \leqslant y \leqslant (r+k),$$

and

$$\pi\{r^2-(y-r-k)^2\} = \pi\{2r(y-k)-(y-k)^2\} \text{ if } (r+k) \leqslant y \leqslant (2r+k).$$

Hence the expected value of the apparent area will be

$$E(A_a) = \frac{\pi}{2r+k} \int\limits_0^r (2ry-y^2)\,dy + \frac{\pi r^2}{2r+k} \int\limits_r^{r+k} dy +$$

$$+\frac{\pi}{2r+k} \int\limits_{r+k}^{2r+k} \{2r(y-k)-(y-k)^2\}\,dy = \frac{4\pi r^3}{3(2r+k)} + \frac{\pi r^2 k}{2r+k}. \tag{4}$$

The bias, or expected excess of apparent area over true, obtained by subtracting eq. (3) from eq. (4), is $B = E(A_a)-E(A_t) = \pi r^2 k/(2r+k)$. The apparent area will be $E(A_a)/E(A_t)$ times the true one, so that in the rather special case in which a correction would be justified the proper procedure would be to multiply the observed value by

$$C = E(A_t)/E(A_a) = 4r/(4r+3k).$$

Although the figure illustrates only the case in which $r > k$, it can readily be shown that the proof holds also for $r < k$. We are thus in a position to calculate the appropriate correction factor, C, for any ratio of spherical radius to thin-section thickness. Table I shows C for selected

TABLE I. Correction factors for cross-section areas of opaque spheres in a transparent matrix, measured in transmitted light. r/k = ratio of spherical radius (r) to section thickness (k); 'correction' factor $C = 4r/(4r+3k)$.

r/k.	r (mm.) for k = 0·03 mm.	C.	r/k.	r (mm.) for k = 0·03 mm.	C.
200	6·0	0·996	2	0·06	0·727
100	3·0	0·993	1	0·03	0·571
50	1·5	0·985	0·50	0·015	0·400
33	1·0	0·978	0·33	0·010	0·308
20	0·6	0·964	0·25	0·0075	0·250
10	0·3	0·930	0·20	0·0060	0·210
5	0·15	0·897	0·10	0·0030	0·118
4	0·12	0·842	0·067	0·0020	0·082
3	0·09	0·800	0·033	0·0010	0·043

values of r/k in the range $200 \geqslant r/k \geqslant 0{\cdot}033$. The second column of the table shows r in millimetres for $k = 0{\cdot}03$ mm., the standard section thickness used in most petrographic laboratories. The third column of the table, of course, applies strictly only to opaque spherical particles in a transparent matrix.

The Holmes effect is not likely to be of much importance if opaque constituents are present only in small amount, and in grains of reasonable size, as in most common rocks. Whenever the amount of opaque material is large or the 'grain size' of the opaque particles is small, however, the situation is very different. Estimates of magnetite dust in plagioclase, for instance, could easily be absurdly high. If the radius were 0·015 mm., a value not unreasonably small, one would overestimate the dust by a factor of 2·5. For a radius of 0·01 mm. the 'observed' amount would be high by a factor of 3·25. And for a radius of 0·003 mm., one-tenth the thickness of the slide, the true opaque area would be a little less than one-eighth of the apparent area.

Similar difficulties may be anticipated in the analysis of fine-grained sediments, such as black shales, which contain considerable amounts of finely divided organic or other opaque material. Under such circumstances microscopic analyses may be internally consistent—and hence useful for many geological purposes—provided specimens being compared with each other do not differ materially in grain size. It is not to be expected, however, that the results will tell much about the modal compositions of the parent materials. In general they will overestimate the amount of opaque material; even a good guess as to the size of the overestimate, however, will require more information than is ordinarily available and perhaps more than can be obtained.

R. B. Elliott[1] has argued that something similar to the Holmes effect may be generated by the juxtaposition of transparent minerals differing greatly in refractive index. Although relief is strictly a property of the contacts between grains, we do generally identify it with the mineral whose index differs more from that of the mounting medium. Accordingly, if we are unable to locate the intersections of grain contacts with some specific measurement plane—usually the upper surface of the thin section—our measurements may well be subject to the Holmes effect, even though all the constituents involved are transparent.

Even when index difference is not enough to produce strong relief, inability to locate the measurement surface may introduce large constant errors. Most of the members of a swarm of perthitic albite blebs, for instance, may be below the surface of the thin section, yet all may show the appropriate Becke line against the host, and differences in birefringence or extinction will not be obscured by the fact that many of the blebs are partly or entirely beneath the measurement surface.

Provided there is no tendency for some particular transparent mineral to mantle the opaque granules, the Holmes effect could be overcome by measuring the amount of opaque material in reflected rather than transmitted light. This procedure would always eliminate the bias as far as the opaques are concerned, but if some one of the transparent minerals systematically mantled the opaque particles, the amount of this material relative to other transparent minerals would be underestimated in transmitted light.

The general case, in which neither mineral is opaque, is considerably more difficult and, I believe, more important. There is need of a good experimental study of the problem, and without such a study we are not likely to know just how serious it is. The difficulty arises because the analyst either does not or cannot confine his observations to the surface of the thin section. If the rock is so fine grained that individual particles are of the order of magnitude of imperfections in the surface (or depth of focus of the microscope), the lower limit of modal analysis has been reached. Even if we knew the appropriate corrections in the latter case, they would probably be so large that most of us would hesitate to use them.

[1] Min. Mag., 1952, vol. 29, p. 833.

It may be useful to point out in closing that what is involved is not grain size in any absolute sense. Rather, it is the ratio of grain size to thin-section thickness that governs the size of the bias. It may be calculated from table I, for instance, that particles of spherical radius 0·06 mm. will be subject to a 37·5 % overestimate in thin sections of standard thickness. If, however, the thickness of the thin section could be reduced by a factor of 3, the bias would be reduced to something less than 15 %. And if it were possible to prepare slides one-fifth of the standard thickness, the bias would be only 7·5 %.