New techniques for the universal stage.

# I. An extinction curve method for the determination of the optical indicatrix. 

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#### Abstract

Summary. This paper describes a development for the universal stage of the extinction curve method for determining the orientation of the optical indicatrix in biaxial crystals. The extinction curve may be defined as the locus of the poles of the extinction directions when the crystal is rotated between crossed polarizers about an arbitrary but fixed axis perpendicular to the microscope axis.

When two or more extinction curves, or parts of them, are plotted on a stereographic projection, their intersections give the directions of the axes of the indicatrix, of which at least two are directly accessible. The main application of this new orthoscopic method is to provide a means of refining the orientation of the indicatrix obtained by this or any other method. The accuracy that can be obtained for minerals of low birefringence such as the felspars is considerably greater than with the conventional orthoscopic method.

A complete account is given of the details of the plotting procedure.


THIS is the first of several papers devoted to the use of the universal stage. The topics discussed are concerned mostly with the development of new or improved orthoscopic methods, which may be applied widely to rock-forming minerals in thin sections. In these papers, written for use with 3-or 4-axis universal stages, the nomenclature used for referring to the different rotation axes is that of Berek (1924) for the 4 -axis stage.

One of the inherent difficulties in the conventional orthoscopic method of locating accurately the orientation of the indicatrix in biaxial crystals lies in determining how accurately the optic symmetry planes have been set normal to the $A_{4}$-axis of the universal stage. Conoscopic
methods afford a much higher degree of accuracy but can only be applied under favourable conditions.

The difficulties involved in the accurate location of an optic symmetry plane when corrections are required for refractive index differences between crystals and hemispheres have been discussed by Berek (1949) and by Hallimond (1950), who have described procedures whereby the plane may be located more accurately. Even where no refractive index correction is involved, however, the extinction position at any general inclined setting of the universal stage can always be determined with much greater precision than that with which an optic symmetry plane can be located by the orthoscopic method. This is because the extinction positions are determined by rotating the microscope stage for fixed settings of the inclined circles of the universal stage; whereas the orthoscopic setting of an optic symmetry plane is determined by extinction or near extinction being maintained as the crystal is rotated through an arc about the $A_{4}$-axis. Errors of the order of two degrees or more in the tilt of $A_{2}$, or of a degree or more on $A_{1}$ if $A_{2}$ has a high inclination, may still result in a state of near-extinction being observed on rotation about $A_{4}$. Therefore, if the axes of the optical indicatrix could be derived from the plotting of a series of accurately determined extinction positions, a considerable advance would be made in the accuracy that could be obtained by the use of orthoscopic methods with the universal stage.

Joel (1950, 1951), and Joel and Garaycochea (1957), using the simple microscope-stage goniometer of Bernal and Carlisle (1947), showed how the extinction curve of a crystal could be used to determine the orientation of the optical indicatrix of both uniaxial and biaxial crystals. It is shown here that extinction curve methods can be applied very successfully to the universal stage and that, because more than one horizontal rotation axis can be selected, ambiguities that are present in the method as originally described can be removed by using the universal stage. The method also becomes simpler.

Because the plotting of the extinction directions on a stereographic projection is not a very simple procedure, the method to be presented here cannot be described as a rapid one. Its principal application consists in refining the directions of the axes of the indicatrix, which may have been located approximately already by this or any other method. In certain circumstances, however, for example when working with finely twinned plagioclases, it provides a method of determining accurately the optic orientation where all other methods fail.

## The extinction curve.

When a beam of light travels through a birefringent crystal in any direction except along an optic axis, there are two vibration directions associated with each wave normal. These are perpendicular to each other and to the wave normal, and when the crystal is observed between crossed polarizers they appear as the two corresponding extinction directions in the crystal.


Frg. 1. One inclined great circle corresponds to each setting of the horizontal axis $P_{0}$. On each of these circles the poles of the extinction directions, obtained by rotating the stage about the microscope axis, are plotted. The extinction curve is the locus of these poles.

Consider now a crystal mounted on a stage free to rotate not only about the microscope axis but also about a horizontal axis perpendicular to it. For each setting on the horizontal axis it will be possible, by rotating the stage about the microscope axis, to determine the corresponding extinction directions. If this is done for a series of settings of the horizontal axis, and the extinction directions are inserted on a stereographic projection (fig. 1), then the poles thus obtained will lie along the 'extinction curve'.

The extinction curve can thus be defined as the locus of the poles of the extinction directions when the crystal is rotated between crossed polarizers about an arbitrary but fixed axis that is set perpendicular to the microscope axis. As can be seen, it can be represented very conveniently on a stereographic projection.

The generation of the extinction curve can be visualized in the follow-
ing way: $P_{0}$ is a point on the surface of an ellipsoid (the optical indicatrix) whose centre is $O$, and a diametral plane passes through $O P_{0}$. This plane intersects the ellipsoid along an ellipse whose axes $D_{1}$ and $D_{2}$ pass through the surface of the ellipsoid at four points, $E_{1}, E_{1}^{\prime}, E_{2}$, and $E_{2}^{\prime}$. The locus of these points $E$ when the diametral plane rotates round the line $O P_{0}$ is the extinction curve. Defined in this way, the extinction curve lies on the ellipsoid; but it can now be projected radially on to a sphere, and then to a stereographic projection.

The most important property of the extinction curve is the following: the poles of all the principal vibration directions of the indicatrix ( $X, Y$, and $Z$ for a biaxial crystal ; the optic axis and all directions perpendicular to it in a uniaxial crystal) lie on the extinction curve. This is evident because when the crystal is rotated about its fixed horizontal axis, each of the principal vibration directions must in turn come into the plane of the microscope stage, and when this occurs they will become one of the two principal axes of that ellipse. It is because the poles $X, Y$, and $Z$ lie on the extinction curve that it may be used to determine the orientation of the indicatrix.

Before a general description of the extinction curve is given, some simple cases will be considered.

Suppose, first, a uniaxial crystal with its optic axis perpendicular to the microscope axis is rotated about this optic axis. To obtain extinction the stage has to be rotated until the optic axis is parallel or perpendicular to the vibrations transmitted by the polarizer. If the crystal is now rotated about its horizontal axis $P_{0}$ (the optic axis), extinction will be maintained throughout. In this very special case the extinction curve consists of a great circle (the circular section of the ellipsoid, locus of ordinary vibration directions) and a point (the point $P_{0}$, pole of the optic axis, extraordinary vibration direction).

Consider now the same uniaxial crystal, but set in such a way that the rotation axis $P_{0}$ is no longer parallel or perpendicular to the optic axis. For any setting of the crystal about $P_{0}$, one of the two vibration directions will always have a refractive index $\omega$ (ordinary vibration), and the corresponding extinction directions will all lie in the circular section of the indicatrix. Thus, the extinction curve will still show a great circle (but inclined to $P_{0}$ ). The poles of the extraordinary extinction directions will lie on a curve that passes through the poles of the optic axis and of the rotation axis $P_{0}$, and the size of this curve will increase with the angular distance between these two poles (fig. 2). The complete extinction curve consists of three branches.

In the simplest case for a biaxial crystal, $X$ or $Z$ coincides with $P_{0}$ (the case where $Y$ is parallel to $P_{0}$ will be considered later). If the crystal is set to extinction by rotation about the microscope axis, $X$ or $Z$ (and hence $P_{0}$ ) will be parallel or perpendicular to the vibrations transmitted by the polarizer. If a subsequent rotation is made about the horizontal


Fig. 2.
Fig. 2. Extinction curve of a uniaxial crystal, when $Z$ is inclined to $P_{0}$.
Fig. 3. Extinction curve of a biaxial crystal ( $\mathrm{CuSO}_{4} .5 \mathrm{H}_{2} \mathrm{O}$, triclinic) for rotation about a given axis $P_{0} . X, Y$, and $Z$ are the principal directions of the indicatrix. In this particular setting the equatorial curve passes through $X$ and $Y$, while the polar curve passes through $Z$ and $P_{0}$.
axis the crystal will remain in extinction. Hence, the extinction curve will consist of a great circle and its poles (the poles of $X$ or $Z$ ).

From the properties of an ellipsoid it follows that if the axis of rotation $P_{0}$ does not coincide with one of the three principal axes, the large curve will no longer be a great circle but will be a centrosymmetrical curve deviating somewhat from it by an amount that depends on the direction of $P_{0}$ relative to the indicatrix. And the locus of the other extinction direction ( $90^{\circ}$ away from the first) will no longer be restricted to a point but will be a small curve.

For any given general direction of the rotation axis, the complete extinction curve (if both upper and lower poles are plotted) will consist of three branches: a curve encircling the sphere of projection and which is itself centrosymmetrical-this is the 'equatorial curve'; and two smaller curves, whose largest angular diameter never exceeds $90^{\circ}$; these curves, which are mutually symmetrical relative to the centre of the projection sphere, are the 'polar curves' (fig. 3). $\dagger$
$\dagger$ Because of the symmetry of the extinction curves, all stereograms in this paper, except figs. 2 and 3 , show the upper half of the projection only.

If the dihedral angles formed by the planes of the two circular sections are considered, the polar curves are confined to one pair of opposite angles (the ones that contain the rotation axis $P_{0}$ ) and the equatorial curve is confined to the other pair. Hence, the polar curves do not intersect the circular sections, while the equatorial curve does so, but only at their common intersections $Y$ and $\bar{Y}$. Consequently, the equatorial curve always contains the poles of two axes of the indicatrix, one of which is $Y$. The pole of the third axis of the indicatrix, and also the point $P_{0}$, are on the polar curve.

On rotation of the indicatrix about a horizontal axis, the refractive indices associated with the two vibration directions fall into two groups, one limited to the range $\alpha$ to $\beta$, and the other to the range $\beta$ to $\gamma$. One of these ranges is always fully covered, and corresponds to the equatorial curve; the other does not necessarily reach the $\beta$ end of the range, and corresponds to the polar curve. For instance, if $X$ is on the polar curve, then $Y$ and $Z$ will be on the equatorial curve, and in this case all the points of the latter will correspond to vibrations with refractive indices in the full range $\beta$ to $\gamma$.

When $Y$, or indeed any direction lying in either of the two circular sections, is set parallel to the rotation axis, a limiting case arises: the equatorial and polar branches of the extinction curve touch at four singular points on the same circular section. The extinction curve includes this circular section in such a way that one half is covered by the equatorial curve and the other half by the polar curves. In this case it becomes difficult to distinguish by simple inspection between the equatorial and the polar parts of the curve.

For the particular case of the $Y$-axis coinciding with the horizontal rotation axis, the whole extinction curve now consists of the great circle $X Z$ and the two circular sections of the indicatrix. This is due to the fact that throughout the rotation positions are reached in which the crystal is being observed along each of its optic axes. These considera. tions apply also to uniaxial crystals when the rotation axis $P_{0}$ lies in the circular section of the indicatrix. The extinction curve then consists of the circular section and a great circle perpendicular to $P_{0}$.

Examples of extinction curves, details of their use with a one-axis stage, and a brief mathematical treatment of the problem have been given by Joel and Garaycochea (1957). They showed how the orientation of the indicatrix of biaxial crystals could be determined by locating on the extinction curve three points $90^{\circ}$ distant from each other. Two such triangles are obtained in the general case, the 'true' triangle and
a 'ghost' triangle. Criteria were given by which the 'true' triangle $X Y Z$ could always be distinguished from the 'ghost' one. The accuracy with which $X, Y$, and $Z$ can thus be located, that is, the precision with which a tri-rectangular spherical triangle can be located with its vertices on the extinction curve, depends greatly on the shape of the extinction curve which in turn depends on the direction of the rotation axis relative to the indicatrix.

Johannsen (1918) refers briefly to an extinction curve in his extensive review of methods for the graphical determination of zonal extinction angles and their use in such problems as plagioclase determination, but he makes no attempt to use them. It is clear from the discussion of Duparc and Pearce (1907) that the use of these extinction curves for determining the orientation of the optical indicatrix was not known to the early workers in crystal optics.

## Application of extinction curves to the universal stage.

The extinction curve method was applied originally to a one-axis microscope-stage goniometer which can be compared to a universal stage in which the $A_{4}$-axis only is used. This instrument is very suitable for the microscopic examination of a mineral grain mounted on a fibre such as would be used for single-crystal X-ray work, and it permits a complete rotation of the crystal about the horizontal axis.

Clearly this method can be applied to a universal stage. If the $A_{4}$-axis alone were used, the universal stage would be at a disadvantage owing to the limited rotation about $A_{4}$, but use can be made of other axes. It is here that the new developments in the extinction curve method arise, and with the universal stage the method can now be applied to thin sections as well as to grains.

The methods described here refer only to biaxial crystals, for the uniaxial case is simple, and are applied to a 4 -axis universal stage. The operations carried out during the procedure involve the determination of a number of extinction directions and the plotting of two or more extinction curves. The method is based on the fact that more than one horizontal rotation axis may be selected. The extinction curve is drawn for each of these axes. Since the poles $X, Y$, and $Z$ must lie on each extinction curve, they will be determined by the intersections of the different extinction curves.

For any given rotation about the $A_{4}$-axis, the inaccessible region due to the restriction on the rotation will be the area shown in fig. $4 a$. However, any pole in this region inclined at more than about $30^{\circ}$ or $35^{\circ}$ to the
normal of the section can be made accessible by a suitable rotation about the $A_{3}$-axis, as shown in fig. $4 b$, to bring it near the N.-S. plane. In this way the inaccessible region is reduced to the area lying within a small circle concentric with the pole of the section and of angular radius $30^{\circ}$ to $35^{\circ}$. This is referred to as the 'blind circle'.


Fig. $4 a$.


Fig. $4 b$.

Fig. 4a. Same crystal and same setting as in fig. 3. Owing to the restriction on the rotation about $A_{4}$, extinction directions within the shaded area of the projection become inaccessible; $r=30^{\circ}$ to $35^{\circ}$.
Fig. 4b. Same crystal as in figs. 3 and $4 a$. By making use of other axes of the universal stage, the inaccessible region is reduced to the 'blind circle'; $r=30^{\circ}$ to $35^{\circ}$. At least two of the three principal directions of the indicatrix always lie outside the 'blind circle' and hence are accessible.

As the 'blind circle' cannot contain more than one of the points $X Y Z$, two of the axes and sometimes all three of them can be located directly by means of the extinction curves.

In the following sections the terms $\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}$ are used to define the angular rotations about the respective axes from their zero positions. For $A_{1}, A_{3}$, and $A_{5}$ (microscope stage axis) the divided scales are marked in such a way that clockwise rotations increase their readings (the zero reading on $A_{3}$ is $90^{\circ}$ ); hence, for these axes the $\alpha$ angles define a clockwise rotation from the zero setting. Readings on $\boldsymbol{A}_{\mathbf{2}}$ are made on the left- or right-hand arcs according to the tilt on that axis. Readings on this axis are therefore recorded $\alpha_{2} L$ or $\alpha_{2} R$. Similarly for $A_{4}$; readings are recorded as $\alpha_{4} N$ (stage inclined towards the observer) and $\alpha_{4} S$ (stage inclined away from the observer).

In the procedure that follows, the inner axes of the stage, $A_{1}$ and $A_{2}$, may be used in certain circumstances, such as for finely twinned plagioclases, to set the crystal into a more desirable orientation for determining
the extinction curve. Once these axes have been set, their positions should not again be altered until the operations have been completed. All subsequent operations are carried out using the outer axes $A_{3}, A_{4}$, and $A_{5}$. If there is no need to incline the section ( $\alpha_{2}=0$ ), then $A_{1}$ may be used for those parts of the procedure described here for $A_{3}$ and the technique may then be used with a 3 -axis stage, or indeed a 2 -axis stage. The 3 -axis stage can be used for all the operations described here, but if it is necessary to incline the $A_{2}$-axis, the plotting becomes very complex and it is recommended that such operations should always be carried out on the 4 - or 5 -axis instruments. For the 5 -axis stage the additional axis (inner east-west) is not required for the extinction curve procedure.

## Procedure.

The use of the outer axes $\left(\alpha_{2}=0\right)$. All axes of the stage are set to their zero positions, and the crystal to be examined is brought to the centre of the field. $A_{1}$ is set to a convenient position and its reading is noted. If there should be no preference for any particular position of $\mathrm{A}_{1}$, it is best to begin with $\alpha_{1}=0$.

Rotate the crystal to extinction by a clockwise rotation on $A_{5}$; record the $\alpha_{5}$ angle ( $\alpha_{4}=0$ ) ; these extinction directions will later be inserted on the stereogram as four points on the primitive.

Rotate $A_{4}$ to $10^{\circ} \mathrm{N}$., adjust $\mathrm{A}_{5}$ to extinction again, and note the $\alpha_{4}$ and $\alpha_{5}$ values.

Make further successive rotations of $10^{\circ}$ on $A_{4}$ to cover the complete available arc of rotation and for each setting note the $\alpha_{5}$ extinction angle. When these readings are plotted on the upper hemisphere of the projection, each $A_{4}$ setting will produce two poles $90^{\circ}$ apart on the respective great circles which represent planes inclined to the section at angles corresponding to the $\alpha_{4} N$ or $S$ values.

When sufficient extinction directions have been obtained for the particular setting of $A_{3}$ they are plotted as described below, and the accessible portion of the extinction curve is drawn through these poles.

A new rotation axis is now selected in order to obtain a second extinction curve. This can be done in a variety of ways, two of which do not involve complicated plotting: if the $A_{4}$-axis is set to its zero reading the second rotation may be performed about $A_{2}$; this enables a rotation to be made about an axis perpendicular to the first one. Or if $A_{2}$ is left fixed at its zero reading, the rotations may be performed around the $A_{4}$-axis after rotating the $A_{3}$-axis into a new position. In this way any
rotation axis in the plane of the section can be selected, and by further rotations about $A_{3}$ as many additional extinction curves may be obtained as are desired (fig. 5). No definite rules can be given as to the new settings of $A_{3}$ that will produce good intersections of the extinction curves, but


Fig. 5. Four extinction curves for a specimen of labradorite from porphyritic basalt, St. John's Point, County Down. $P_{1}, P_{2}, P_{3}$, and $P_{4}$ are the poles of the rotation axis $A_{4}$ for settings of $A_{1}$ equal to $0^{\circ}, 45^{\circ}, 90^{\circ}, 135^{\circ}$; and the numbers by the curves indicate the corresponding rotation axis. The parts of the curves that point towards $Y$ are the equatorial curves.
(as can be seen in fig. 5) a rotation of $A_{3}$ of $45^{\circ}, 90^{\circ}, 135^{\circ}$, or intermediate values, will üsually produce satisfactory results.

After two extinction curves have been recorded and the three axial points have been located, a process of refining their positions may be carried out; indeed, the conventional orthoscopic procedure could be used at first to locate these points approximately. For each of the
rotation axes selected, the regions close to the poles of the accessible indicatrix axes are explored and the extinction curves plotted in smaller steps of rotation about $A_{4}$.

After the three indicatrix axes have been located, they are identified in the usual manner by means of the compensating plates. It is useful to remember at this stage that the $Y$-axis always lies on the equatorial curve; even with the incomplete extinction curves obtained on the universal stage it is usually possible to distinguish between their equatorial and polar parts.

It should be noted that if the rotation axes are chosen so that they all lie in the same plane, which is generally the case, then the different extinction curves will have two additional points of intersection in the upper hemisphere, both of them in that same plane. If $\alpha_{2}=0$ throughout, these additional intersections will occur at four points on the primitive. Their presence should not cause any ambiguity. These points might be considered to be 'shadows of the ghosts'. $\dagger$ They need not occasion any alarm.

Use of the inner axes $\left(\alpha_{2} \neq 0\right)$. If it should be desired to set the crystal into a more convenient position for its rotation, use can be made of the inner axes $A_{1}$ and $A_{2}$. This would arise, for instance, if a cleavage plane or twin composition plane should be kept vertical during the rotation. Such preliminary settings are particularly desirable when dealing with finely twinned plagioclase where any considerable obliquity of the composition plane may cause the walls of contiguous lamellae to overlap and prevent the extinction positions from being determined. In this way the extinction curve and 'zonal' methods of plagioclase determination may be carried out simultaneously. In the case of normal twins the requirement for extinction angles in adjacent lamellae to be equal and opposite when the composition plane is vertical affords a check on the measurements. Other cases where it may be desirable to incline the $A_{2}$-axis arise when a grain to be examined has an optic axis nearly vertical ; the low partial birefringence at low inclinations of $A_{4}$ then makes an exact determination of the extinction positions difficult.

The procedure when the $A_{2}$ circle is inclined is as follows:
By means of $A_{1}$ set the crystal in the desired orientation, e.g. set the trace of the composition plane parallel to the N.-S. crosswire. Using $A_{2}$, set the composition plane vertical.

Set to extinction on $A_{5}$; as before, $A_{4}$ is used as rotation axis.
When further extinction curves are required these may be obtained
$\dagger$ See Joel and Garaycochea, 1957, p. 401.
by altering the setting of $A_{3}$. The composition plane will no longer be N.-S., nor will it remain vertical, but by selecting suitable rotations about $A_{3}$ it can be made to deviate less from the vertical than if $A_{2}$ had not been inclined.

## Plotting.

With a 4-axis universal stage and a rotating-stage microscope there are five axes of rotation available. In their zero positions three axes are vertical ( $A_{1}, A_{3}, A_{5}$ ) and two are horizontal ( $A_{2}, A_{4}$ ). The graduated circles of the first three are marked in such a way that clockwise rotations increase their readings. For the actual plotting, the stereographic net should be placed with its inclined great circles running E.-W.

The zero of $A_{1}$ is the south point of the primitive, and the zero setting of the $A_{3}$-axis (which reads $90^{\circ}$ ) is the west point. Therefore, for plotting the extinction directions, the tracing paper must first be rotated to a position corresponding to the reading on $A_{1}$; that is, a clockwise rotation equal to $\alpha_{1}$. In the plotting, the zero of the paper is rotated to the appropriate reading of $A_{1}$ or (if $\alpha_{2}=0$ ) of $A_{3}$. All the other readings are made then along the fixed circles inscribed on the net.

First set the tracing paper over the net and mark on it a point $S_{1}$ coinciding with the south point of the primitive and a point $W_{3}$ coinciding with the west point of it. The former is to be used as origin for the readings of $A_{1}$ and the latter for those of $A_{3}$ (provided $\alpha_{2}-0$ ). In both cases the paper should be turned clockwise.

As the plotting procedure is more involved when the $A_{2}$ circle is inclined than when it is on its zero position, the two cases will be dealt with separately.

If $\alpha_{2}=0$. Since the axis $A_{4}$ is E.-W., its readings are inserted on the tracing paper along the N.-S. great circle of the net: readings for tilts north (towards the observer) are inserted inwards from the south point of the primitive, that is, towards the north; and conversely for rotations in the opposite sense.

Rotations about $A_{4}$ bring the inclined great circles running E.-W. successively into positions perpendicular to the microscope axis. Therefore, after the appropriate great circle has been selected according to the $A_{4}$ reading, the extinction directions-as read on $A_{5}$-are inserted along it.

The zero position of $A_{5}$ is the one in which the $A_{4}$-axis runs E.-W. And it is the difference between the actual readings of $A_{5}$ at extinction and its zero-position reading that are relevant and constitute the $\alpha_{5}$ 3 ェ2
angles. For the purpose of inserting the extinction directions into the stereogram, positive values of $\alpha_{5}$ only need be considered; it is always possible to add or to subtract any multiple of $90^{\circ}$ to the $A_{5}$ reading, and thus to restrict the values of $\alpha_{5}$ to the range $0^{\circ}$ to $180^{\circ}$. This simplifies the plotting.

The values of $\alpha_{5}$ thus obtained are inserted along the appropriate


Fig. $6 a$.


Fig. 6 b.

Fig. 6a. Two examples of plotting: $\alpha_{1}=\alpha_{2}=\alpha_{3}=0^{\circ} ; \alpha_{4}=20^{\circ} \mathrm{N}, \alpha_{5}=-50^{\circ}$; $\alpha_{1} \cdot \alpha_{2}=\alpha_{3}=0^{\circ} ; \alpha_{4}=40^{\circ} \mathrm{S}, \alpha_{5} \quad 60^{\circ}$.
Fig. 6b. Example of plotting with $\alpha_{1} \neq 0^{\circ}$; the tracing paper has been rotated clockwise, through an angle equal to $\alpha_{1}$ before inserting $\alpha_{4}$ and $\alpha_{5}$. $\alpha_{1}=70^{\circ}$, $\alpha_{2}=\alpha_{3}=0^{\circ}, \alpha_{4}=40^{\circ} \mathrm{S}, \alpha_{5}=-60^{\circ}$. In fact the same plotting will be obtained in this case $\left(\alpha_{2}=0^{\circ}\right)$ if $\alpha_{1}=0^{\circ}$ and $\alpha_{3}=70^{\circ}$, or indeed for any $\alpha_{1}$ and $\alpha_{3}$ such that $\alpha_{1} \div \alpha_{3}=70^{\circ}$.
inclined great circle. For north tilts of $A_{4}$ the readings are made anticlockwise along the great circles from the west point of the primitive; for south tilts, anticlockwise from the east point (fig. 6).

If $\alpha_{2} \neq 0$. When it has been necessary to use the two inner axes the plotting becomes more difficult. If by means of $A_{1}$ the trace of a plane has been set N .-S., this position of $A_{1}$ should be taken as its zero position (the point $S_{1}$ of the stereogram coinciding with the $S$ point of the net). It should not be altered during the rest of the procedure. Then the plane was set vertical by means of $A_{2}$. Fig. $7 a$ shows the pole of $A_{4}$ for $\alpha_{2}=30^{\circ} L$ (the normal to the section is tilted $30^{\circ}$ to the right). Rotation about $A_{4}$ will in turn bring parallel to the microscope stage planes represented by great circles through this new position of $A_{4}$ (which is not the point $E$ of the net). Fig. $7 a$ shows one of these great circles along
which $\alpha_{5}$ has to be inserted. Fig. $7 b$ shows the plotting for a case in which all five axes have been used.

The plotting of this case can nevertheless be simplified considerably if it is carried out first as if $\alpha_{2}$ and $\alpha_{1}$ were zero. $\alpha_{3}$ readings are inserted around the primitive, $\alpha_{4}$ along the line N.-S., and $\alpha_{5}$ along the inclined


Fig. $7 a$.


Fig. $7 b$.

Fig. 7a. Example of plotting with $\alpha_{2} \neq 0^{\circ}: \alpha_{1}=0^{\circ}, \alpha_{2}=30^{\circ} L, \alpha_{3}=0^{\circ}, \alpha_{4}=20^{\circ} \mathrm{S}$, $\alpha_{5}=50^{\circ}$.
Fic. 7b. Example of plotting in the most general case: $\alpha_{1}=40^{\circ}, \alpha_{2}=30^{\circ} L$, $\alpha_{3}=60^{\circ}, \alpha_{4}=20^{\circ} \mathrm{S}, \alpha_{5}=50^{\circ}$.


Fig. 8. $X, Y$, and $Z$ are the poles of the axes of the indicatrix obtained by neglecting the $A_{1}$ and $A_{2}$ readings in the plotting. A rotation of amount $\alpha_{2}$ about $A_{2}$ brings them to $X^{\prime}, Y^{\prime}$, and $Z^{\prime}$. A further rotation of amount $\alpha_{1}$ about $A_{1}$ brings them to $X^{\prime \prime}, Y^{\prime \prime}$, and $Z^{\prime \prime}$, which are the poles of the axes of the indicatrix referred to the plane of the mineral section. In this example $\alpha_{1}=60^{\circ}$ and $\alpha_{2}=30^{\circ} L$.
great circles just as before. When the positions thus obtained for the poles $X, Y$, and $Z$ have to be referred to the zero setting of the section, this can be done by rotating them around the N.-S. line of the stereogram (they will move along small circles perpendicular to this line) by an angle equal to $\alpha_{2}$ (to the left if $\alpha_{2}$ was a reading on the left), and then by rotating them about the centre of the stereogram by an angle equal to $\alpha_{1}$ in an anticlockwise sense; fig. 8 shows this.

## Corrections.

The corrections due to inclined incidence when the refractive indices of crystal and hemispheres are different can be applied very simply if the crystal has a low birefringence. Its estimated average index can then be used for the correction.

If $\alpha_{2}=0$, the only correction necessary is the one to be applied to $\alpha_{4}$. The plotting is made easier if instead of applying the correction to the stereogram it is made to the $A_{4}$ setting of the stage, in the opposite sense, in order to make use of one of the great circles drawn on the net.

If $\alpha_{2} \neq 0$, the angle to be corrected is $\alpha^{\prime}$, given as usual by $\cos \alpha^{\prime}=$ $\cos \alpha_{2} \cdot \cos \alpha_{4}$. And the correction has to be applied radially towards (or away from) the pole of the normal to the section. Both the determination of $\alpha^{\prime}$ and the actual correction should be made on the stereogram in the customary way. More details on these corrections, especially for crystals of high birefringence, are given by Joel and Muir (1958).

## Accuracy of measurements with the universal stage.

With the standard orthoscopic method it is difficult to check on the accuracy with which the optic planes have been set. A check can be applied to the setting of the optic axial plane by rotating the $A_{5}$-axis into the alternative $45^{\circ}$ positions and testing the locations of the optic axes, but no check can be applied to the other two optic symmetry planes except that the three axes of the indicatrix should be $90^{\circ}$ distant from each other. Errors of two or three degrees in the location of $X Y Z$ are not uncommon. The use of twin crystals affords in many cases a reliable check on the accuracy with which the principal vibration directions $X Y Z$ have been located in both subindividuals, but if only two twin subindividuals have been plotted and if the plotted poles of any pair of equivalent principal vibration directions lie within about $20^{\circ}$ or less of each other, then a very small error in location of one of them may greatly affect the determination of the position of the twin axis. Also,
compensating errors may well hide inaccuracies in the location of $X Y Z$ in one or both subindividuals.

A high degree of accuracy may be obtained when it is possible to use conoscopic methods; these may be used satisfactorily in thin sections with minerals of high birefrigence such as olivine, provided that the crystals are of sufficient size and unzoned, and that fine twin lamellae are not present. If the mineral has a high refractive index, the upper surface of the section should be polished to a void irregular refraction of the emergent rays. Conoscopic methods used with grains can also be applied to minerals of relatively low birefringence such as the alkali felspars. In favourable cases the points of emergence of the optic axes may be located with an accuracy of about $0.5^{\circ}$. Where fine twin lamellae are present or the mineral is strongly zoned, conoscopic methods cannot be used.

One of the commonest uses of the universal stage is for the determination of the composition and assessment of the structural state of the plagioclase felspars. In many cases, both the composition and the structural state may be estimated with a reasonable degree of accuracy if the mineral is not too strongly zoned or if the twin lamellae are sufficiently broad to make possible accurate orthoscopic settings of the optic symmetry planes. The optical distinctions between 'high' and 'low' natural plagioclases are greatest for albite and diminish as the anorthite content increases. Because of this, the distinctions can be made most readily for minerals more sodic than $\mathrm{An}_{40}$. Felspars whose compositions lie between $\mathrm{An}_{15}$ and $\mathrm{An}_{40}$ frequently exhibit both strong zoning and fine twinning. Such felspars cannot be determined satisfactorily using the normal orthoscopic method. To meet these difficulties, Rittmann (1929) developed the 'zonal' method in which the universal stage was used to orient the crystal into the most convenient position for the measurement of its extinction angles in the conventional petrographic manner. This method has the advantage of being simple and rapid and gives very satisfactory results where the plagioclase is known to be in a low state. For plagioclases thought to be in a transitional or natural high state, however, a determination of refractive index (Schwarzmann, 1955) is required in addition to the measurements of the Rittmann method. These difficulties do not arise if the extinction curve method can be used to determine accurately the optical orientation of the various subindividuals of a twinned crystal.

With the extinction curve method any inaccuracies of measurement are shown up immediately by the course of the curves as the plotting
proceeds. If more than two curves are plotted they should all intersect in the poles XYZ; any inaccuracy of measurement can be estimated by the departure of these curves from concurrence. The accuracy that can be obtained depends on the accuracy with which extinction can be set, and this in turn depends on the birefringence and optic axial angle of the mineral. For minerals of very low birefringence the setting of extinction can be improved considerably by the use of a low-power objective and a Nakamura twin-crystal plate with a cap analyser.

In the case of the plagioclase felspars the locations of $X Y Z$ could be determined with an accuracy estimated to be of the order of about one degree. This is considerably better than could be obtained with the conventional orthoscopic procedure.

The example shown in fig. 5 is a favourable one as the extinction settings could be made readily. Even in less favourable cases, however, accurate results can be obtained because it is possible to choose several rotation axes in the crystal and so to plot as many extinction curves as are required.

## Conclusions.

Although the method described in this paper is not a rapid one, the plotting, which is the only difficult part of the procedure, can be done quite rapidly once the operator has become familiar with it; the method then requires only about twice the time demanded by the normal procedure of plotting optic symmetry planes. The amount of plotting required may be reduced considerably if the positions of the axes of the indicatrix can be located approximately first, for then only those portions of two or more extinction curves that are adjacent to the axial points need be determined. Applied in this way the method is very useful for refining the orientation of the indicatrix, especially so for minerals of low birefringence.

When applied to finely twinned plagioclases it provides the only method whereby the optical orientation may be obtained, but even this method may fail with very finely twinned plagioclases such as those from the rhomb porphyries.

The positions of the optic axes cannot be located directly by the extinction curve method, nor can it be used for determining directions lying within the 'blind circle', but this is the region where the normal orthoscopic procedure of determining optic symmetry planes can be applied with the most accurate results. Finally, it should be pointed out that the conventional orthoscopic method of setting an optic symmetry
plane of the indicatrix normal to the $A_{4}$-axis of the universal stage is a special case of the more general extinction curve method presented here.

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