Extinction curves and 2V: a stereographic solution.

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Summary. Utilizing the Biot–Fresnel construction in reverse, the magnitude and sign of the optic axial angle of a biaxial crystal are determinable from the stereogram of the extinction curves. Refinements in the determination of the extinction curves include the use of conoscopic observations, and of crystal cylinders instead of irregular fragments or entire crystals. Direct 2V measurements on some of the material investigated by this method show that this indirect and graphical approach permits the location of each optic axis to within ¼° of its true position.

FOR any propagation direction in any biaxial crystal the extinction direction, measured with respect to the morphology, depends upon three factors: the orientation of the indicatrix with respect to the morphology, the relation of the propagation direction to the indicatrix, and the magnitude and sign of the optic axial angle. The relation between the extinction directions, on the one hand, and a large number of propagation directions confined to one plane, on the other, can be clearly depicted on a stereogram. The resulting extinction curves can, in general, as has been shown by Joel and Garaycochea (1957, pp. 399–404), be used to determine the orientation of the indicatrix with respect to the form of the crystal. It is further demonstrated here that the third of the above factors, the magnitude and sign of the optic axial angle, may also be determined on the basis of the same observations.

The experimental technique employed here is essentially similar to that of Joel (1950, p. 206) and Wilcox[1] (1960, 1961) but with the difference that the axis of the microscope, a Dick type with rotating nicols, is horizontal, and that of the one-circle stage goniometer vertical. This arrangement permits the immersion of the crystal fragment in an oil-filled glass cell whose two parallel sides are perpendicular to the propagation direction: deviation of the wave-normal on entry to and exit from the immersion medium is thereby eliminated.

[1] The work described in the present paper was carried out during September and October 1960 and apparently more or less simultaneously with that of Wilcox whose investigations run along generally similar lines.

The method of plotting the extinction curves used throughout is similar to that employed by Johannsen (1914, pp. 406–409) in his illustration of extinction curves, and has P_0, the axis of rotation of the crystal fragment, in the centre of the stereogram. This results in one complete polar curve (Joel and Garaycochea, 1957, p. 400) always appearing on the upper hemisphere and is thus considered somewhat more convenient than the method employed by Joel (1950, pp. 207–208) and by Joel and Garaycochea (1957, pp. 399, 402) in which each polar curve is divided between the upper and lower hemispheres. Further, with P_0 central, the 'ghost triangle' (Joel and Garaycochea, 1957, pp. 401–402) always has two apices, V and W, on a diameter, and the third apex, U, polar to this diameter, on the primitive circle. The 'ghost triangle' can be located at a glance as follows (fig. 1): U, on the equatorial curve and 90° from P_0, lies at the intersection of the equatorial curve and the primitive circle; V and W, on the equatorial and polar curves respectively, at intersections of these curves with the plane whose pole is U.

This 'ghost triangle' whose existence has been proved mathematically (*ibid.*, pp. 404–405) may be regarded simply in terms of the indicatrix in the following manner. The radius vector to the indicatrix parallel to the axis of rotation, P_0, of the crystal, will lie on an infinite number of central elliptic sections of the indicatrix but in general will in only one case be a principal axis, major or minor, of the ellipse. In this case only will the vector P_0 define a vibration direction. It is then associated with two other directions: U, the second vibration direction, and N, the wave-normal (fig. 1). When the crystal is rotated 90° about P_0, the former vibration direction U, perpendicular to P_0, now becomes the wave-normal, and the associated vibration directions, V and W, must lie in the plane perpendicular to U; that is, in the plane P_0N. Should P_0 lie in a special position, in a principal plane or along a principal axis of the indicatrix, then, of necessity, U must be a principal vibration; consequently, so must V and W. However, since when P_0 is central the apices can be located at a glance, the existence of the 'ghost triangle' may be virtually ignored when searching for the indicatrix axes without danger of confusion.

The principal axes of the indicatrix can be located simply in the following manner. Rotate the projection on the net about P_0 noting meanwhile two variables, the point C where the polar curve, P_0W, intersects the net diameter, FG, and, secondly, the great circle, AB, of which C is the pole, until a point is reached where AB cuts the equatorial curve, UVU, in two points, D and E, 90° apart. As has been shown (*ibid.*, pp. 401–405), there can, in general, apart from the apices of the 'ghost triangle', be only one such set of three points. These three

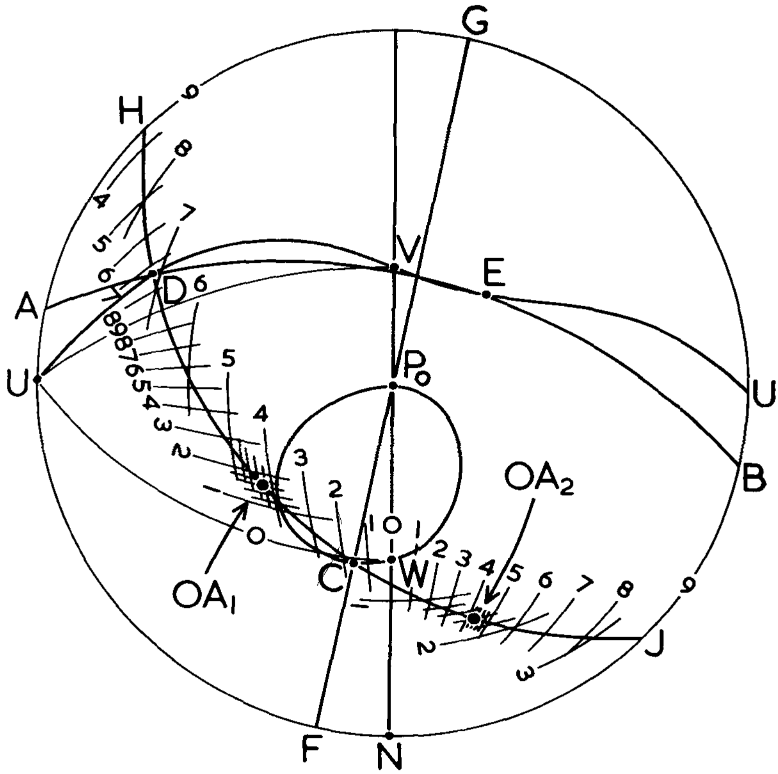

FIG. 1. General stereogram, with the axis of rotation, P_0, of the crystal as centre, showing: UVU, the equatorial, and P_0W, the polar extinction curves; U, V, W, the apices of the 'ghost triangle'; C, D, E, the principal axes of the indicatrix; HJ, the optic axial plane; and OA_1, OA_2, the final location of the optic axes. C, on the diameter, FG, is polar to the great circle, AB, which intersects the equatorial curve in the two points, D and E, 90° apart. The numbered partial great circles are symmetrically placed at 10° intervals on either side of the wave-normal: vibration great circles, $N:P_0$ and $U:W$, and pass through the wave-normals, N and U respectively.

points, C, D, and E, of which two are on the equatorial and one on the polar curve, are the principal axes of the indicatrix and can each be named α or β or γ as the case may be by one of the methods outlined by Joel and Garaycochea (*ibid.*, p. 403).

Having located α, β, and γ as above, the optic axial plane, HJ, whose pole is β (E in fig. 1), may be drawn in the stereogram and the positions of the optic axes deduced by utilizing the Biot–Fresnel construction in reverse as follows. In the wave-front, P_0U, the two vibration directions, P_0 and U, bisect the angles between great circles drawn through the wave-normal, N, and the two optic axes. The optic axes therefore lie on a pair of great circles through N and symmetrically disposed to the great circles NP_0 and NU. Similarly, they must also lie on another pair of great circles through U and symmetrically disposed to the great circles UV and UW. For convenience, the construction has been described in terms of the wave-normals and vibration directions associated with the 'ghost triangle' but, in practice, either of these sets may be rendered valueless by virtue of sub-parallelism of the wave-normal concerned and the optic axial plane. In such a case, and, in fact, in every case, any pair of suitably placed wave-normals in the primitive circle, and their associated vibrations may be utilized (fig. 2).

The following two-stage procedure is found convenient in carrying out the above graphical determination of the optic axes (figs. 1 and 2). For any convenient wave-normal: vibration set, pairs of great circles, through the wave-normal and symmetrically disposed to the vibration directions, are constructed at 10° intervals and each great circle is labelled according to its angular distance from one vibration; those great circles passing through the two vibrations themselves are thus labelled 0 (= 0°) and 9 (= 90°). A similar set of great circles associated with the second wave-normal: vibration set is then constructed. The wave-normal: vibration sets should be so chosen that the points of intersection of the optic axial plane and the two great circles labelled 0 (= 0°) are not too close together. In general, the farther apart these two points are the more accurately can the optic axes be located. The great circles need not, of course, be drawn in full: only their relations in the immediate vicinity of the optic axial plane are of importance in this connexion. On inspection, two portions of the optic axial plane can then be found that are simultaneously symmetrically placed with respect to both sets of numbered great circles. Finally, having found the approximate position of the optic axes by this means, both sets of great circles may be drawn at 2° or smaller intervals where they cross the critical parts of the optic axial plane. The optic axes, OA_1 and OA_2, may then be accurately located by inspection. The foregoing graphical procedure, it may be noted, differs radically from that of Wilcox (1959, p. 1290; 1961), which, while capable of producing equally accurate results, is based upon a trial-and-error approach.

A check on the accuracy of both the above graphical construction and the observations may now be made by ensuring that the optic axes so found are symmetrical with respect to any other wave-normal: vibration set.

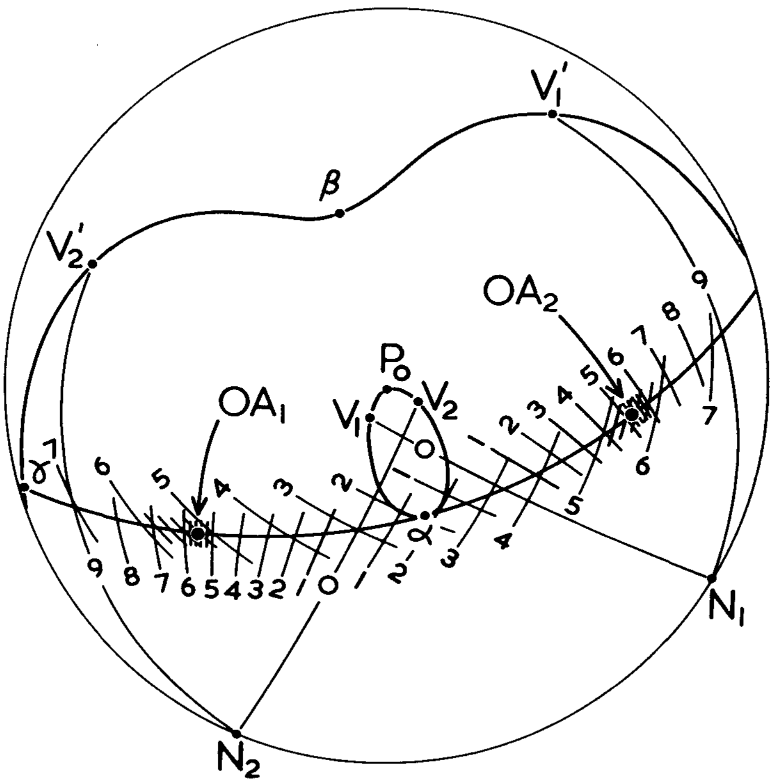

FIG. 2. Determination of the location of the optic axes, OA_1 and OA_2, from the extinction curves, $\gamma\beta$ and $P_0\alpha$, of a cylinder of topaz, utilizing the wave-normals, N_1 and N_2, and their associated vibrations, V_1, V_1' and V_2, V_2' respectively. α, β, γ are the principal axes of the indicatrix. $2V = +65°$.

Refinements of the method.

Although the accuracy of this method is potentially high and is superficially restricted only by plotting limitations—given an accurate net, plotting can be done comfortably to within 30′—there are nevertheless several sources of error, each of which may make itself felt to a greater or less degree in any individual investigation.

An important source of error, particularly where birefringence is high, although Wilcox (1960) finds that this 'does not particularly handicap the determination' by his technique, is the effect of varying refractive index with change in propagation direction and the consequent general lack of correspondence in the refractive indices of the crystal vibration and immersing medium for any individual observation. For non-normal incidence on the crystal surface, the general rule when using cleavage or other fragments or entire crystals, this gives rise in general to a deviation of the wave-normal within the crystal from the direct light-source:observer path; the goniometer reading does not then indicate the correct direction of wave propagation in the crystal. This may be corrected in two ways: by ensuring either that the refractive index of the immersing medium corresponds at every reading to that of the crystal vibration parallel to the polariser, or that, for all propagation directions through the crystal specimen, the incidence is normal. The former seems to involve a laborious sequence of oil changes and checks throughout the observations; the latter could be achieved, admittedly at the expense of some time spent prior to making the observations, by using a crystal cylinder whose axis is P_0. The cylinder rotating about its axis would then, if perfect, ensure normality of incidence at the centre of the field regardless of the refractive index of the immersing medium. However, in view of the roughness produced in grinding, and to keep the errors as small as possible, the refractive index of the oil should fall within the range of refractive indices of the vibrations encountered.

Suitable cylinders were produced by drilling with a thin-walled copper tube of 2·18 mm inner diameter. Abrasive powder, of coarseness appropriate to the hardness of the crystal being drilled, was fed in periodically, along with water as lubricant.

The determination of the extinction position for each wave-normal presents its own particular problems. White light should, of course, never be used in view of the resultant poor definition of the extinction position, particularly where the propagation direction is near an optic axis and, in general, in crystals of high dispersion. Even in monochromatic light, when propagation is in the vicinity of an optic axis, definition of the extinction position can be poor. This inaccuracy can, however, be reduced if crystal cylinders are used: it is found then that, in monochromatic light, if the Bertrand lens is added to the normal low-power magnification system used in the orthoscopic observations, a conoscopic figure is obtained; extinction now occurs when the centre of the isogyre passes through the cross-wires. With propagation in the vicinity of an optic axis, the conoscopic method gives more accurate results than the orthoscopic, but with propagation closer to the β axis of the indicatrix the isogyres become so broad that the converse is true. Each individual method should thus be utilized where it is the more sensitive.

A further benefit to be derived from conoscopic observations made on the cylinders is that goniometer readings may be taken where the wave-normal is parallel to a principal plane of the indicatrix, or at its closest to a bisectrix or optic axis. Application of this information to the final stereogram serves to check the accuracy of the location thereon of the various features concerned.

The effects of zoning and twinning may often be overcome by cutting the cylinder so that, provided P_0 is not thereby placed in an unfavourable orientation, the zone- or twin-boundaries are more or less perpendicular to the axis of rotation, P_0. Each zone or twin-unit may then be dealt with individually by orthoscopic and, if the units are sufficiently large, by conoscopic methods.

It may often be found in practice that, having apparently located the optic axes accurately, within the limits of the graphical method, the bisectrices do not in fact bisect the optic axial angle. This will normally be attributable to inaccuracy in location of the axes α, β, γ of the indicatrix and, consequently, to misplacing of the optic axial plane, errors which can frequently arise if P_0 is in an unfavourable orientation. If P_0 is too close to either of the axes α or γ of the indicatrix, the equatorial curve is, in the vicinity of the critical points, too close to a great circle for accurate direct location of α, β, γ. This may be remedied by performing the complete graphical operation for several positions of α, β, γ, and of the optic axial plane, within their range, usually limited, of possible positions until a solution is obtained where the bisectrices do bisect the optic axial angle. This, of course, does not involve redrawing the great circle pairs which, having been drawn once, will normally serve for the whole determination. By this procedure, not only is 2V determined but greater accuracy is also achieved in locating the indicatrix axes in otherwise unfavourable cases.

P_0 should preferably be fairly close to one circular section of the indicatrix and not far removed from β. This setting produces an eccentric polar curve and an equatorial curve departing greatly from a great circle. Alternatively, if it can be done accurately, P_0 should be deliberately set to coincide with a symmetry plane of the indicatrix. The apices of the 'ghost triangle' then define the axes, α, β, γ.

Extinction curves plotted for cylinders of the following minerals, using the above refinements where necessary, gave the values of 2V indicated: anhydrite (+43°), barytes (+41°), epidote (−76°). Since the result, 2V = +41°, for three barytes cylinders in different orientations was consistently some $3\frac{1}{2}$° to 5° higher than normal textbook values for the pure mineral a check was made by direct conoscopic measurement of 2V on a plate from the same crystal. The plate was fixed on the one-circle stage goniometer with the indicatrix axis β in the axis of rotation and was immersed in the glass cell in a fluid equal in refractive index to the β vibration of the mineral. This direct measurement gave 2V = +40° 35′, showing that the optic axes were located to within $\frac{1}{4}$° of their true position on the basis of the extinction curves.

A final check on the validity of this method was made using two minerals, albite and topaz, from each of which a cylinder and a sphere were made. The cylinders, on the basis of their extinction curves, gave 2V values of +78° and +65° respectively. Direct conoscopic measurements of 2V on the spheres, on a one-circle goniometer, gave values of +78° 04′ and +65° 17′ respectively.

Thus, from the extinction curves, each optic axis can be located with a degree of accuracy which, considering the indirect and graphical nature of the determination, compares favourably with more direct methods.

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References.

Joel (N.), 1950. Min. Mag., vol. 29, p. 206.
—— and Garaycochea (I.), 1957. Acta Cryst., vol. 10, p. 399.
Johannsen (A.), 1914. Manual of petrographic methods. McGraw-Hill Book Co. Inc., New York.
Wilcox (R. E.), 1959. Amer. Min., vol. 44, p. 1272.
—— 1960. Bull. geol. Soc. Amer., vol. 71, p. 2003.
—— 1961. Personal communication.