Conical extinction curves: a new universal stage technique[1]

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[Read 28 May 1964]

Summary. New generalized extinction curves, derived from wave-normals located on a circular cone, are presented. They may be used with the universal stage for the accurate location of up to all three of the indicatrix axes, α, β, γ, of any biaxial crystal, or the optic axis of any uniaxial crystal.

In recent years, many interesting methods, both graphical and mathematical, have been advanced for locating the principal axes, α, β, γ, of the optical indicatrix from extinction curves (Joel, 1950, 1951; Joel and Garaycochea, 1957; Joel and Muir, 1958; Wilcox, 1959, 1960; Tocher, 1962; Joel, 1963; Garaycochea and Wittke, 1964). In every one of these methods the wave-normals giving rise to the extinction curves have, for instrumental convenience and simplicity, been coplanar and perpendicular to P_0, the axis of rotation of the crystal. Nevertheless, it is clear that continuous closed extinction curves are in general obtainable provided that the contributory wave-normals themselves lie on any continuous closed cone; whether that cone be circular, elliptical, or, for that matter, completely irregular in its principal cross-section is of no importance. However, of the infinite number of possibilities within this range, there is one, the circular cone, which is immediately obvious as being instrumentally more practicable. Within this limitation to circular cones only, there is now potentially available for every axis of rotation, P_0 (the cone axis), an infinite number of wave-normal loci since the angle

[1] This technique was evolved independently and almost simultaneously by the two authors who have agreed on a joint presentation.

$P_0 N_n$ (where N_n is any wave-normal of the cone) may have any value between 0° and 90°. The special case where $P_0 N_n = 90°$ gives rise to the extinction curves that have been used until now; as in this case the wave-normals are coplanar and perpendicular to P_0, these particular extinction curves may be called *coplanar extinction curves.* The curves whose use is proposed here, i.e. those where $0° < P_0 N_n < 90°$, are of a more generalized type, and will be called *conical extinction curves.*

The present paper is concerned only with the introduction of the new conical extinction curves and with the presentation of an immediately obvious practical application. A more detailed description of the entire range of such curves is in course of preparation.

The conical extinction curves

General. Representative examples of the new conical extinction curves may be seen in figs. 2, 3, 4, 5, 8, 9. They are obviously different from the more specialized coplanar variety, and are apparently more complicated. However, both types do have some properties in common: to begin with, they consist of two centro-symmetrical curves. In the coplanar case, these two curves, the polar and equatorial curves (Joel and Muir, 1958, p. 864), are, in general, quite separate from each other and are free from mutual- or self-intersections. This is not, however, true in the conical case where, depending on circumstances, the two curves may show mutual- or self-intersections, or both, or neither (figs. 2, 3, 4, 5, 8, 9). Nevertheless, this apparent complexity is, in reality, very useful for, should intersections occur, they do so only at one or more of the axes, α, β, γ, of the indicatrix.

Any general radius vector to the indicatrix, although lying in an infinite number of central elliptic sections of the indicatrix, can be a principal axis, major or minor, of only one such elliptic section. In this case only will the general radius vector define a vibration direction (cf. Tocher, 1962, p. 53). Thus, any one general vibration direction can only be associated with one wave-normal, or to be more precise, with two wave-normals 180° apart. However, in conical extinction curves there can be no wave-normals 180° apart, so that neither of the two vibration directions associated with any general wave-normal of the cone can coincide with a vibration direction associated with any other wave-normal of the same cone: on the conical extinction curves there can therefore be no repetition of any general vibration direction, i.e. no general vibration direction can be defined by an intersection or point of coincidence.

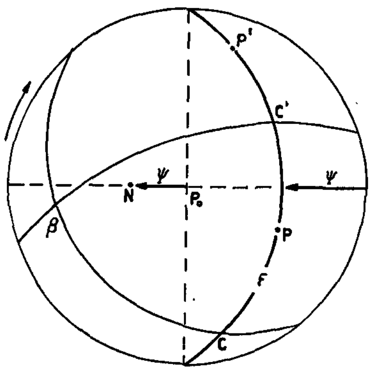
Special vibrations, however (those in the α, β, γ directions), can each be associated with an infinite number of special wave-normals (with those lying in the $\beta\gamma$, $\alpha\gamma$, and $\alpha\beta$ planes respectively). If the wave-normal cone includes two such special wave-normals lying in any one of these planes, i.e. if the wave-normal cone crosses a principal plane of the indicatrix, then the principal vibration normal to this plane must appear twice on the resultant conical extinction curve (four times if both ends of the wave-normal cone, both ends of the vibration vectors, and both stereographic hemispheres are considered). Thus, the principal vibration direction concerned must be defined by an intersection or point of coincidence. The number of such intersections or points of coincidence is, of course, equal to the number of principal planes of the indicatrix that are crossed by the wave-normal cone.

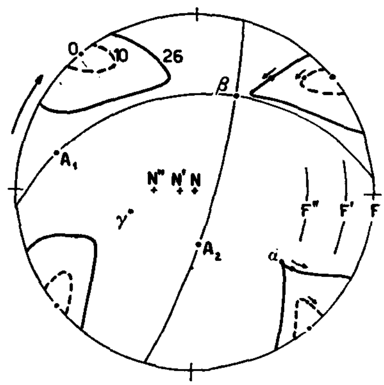
The wave-normal cone (plane) associated with the coplanar extinction curves crosses all three principal planes of the indicatrix (cf. Joel, 1950, pp. 208–209), but, despite this, the three special points, α, β, γ, on the resultant coplanar extinction curves are not self-evident at first sight. This is because they are superficially undifferentiable from the infinite number of general points of coincidence associated with the infinite number of general wave-normals of the plane: every general vibration direction lying in a coplanar extinction curve is associated with two general wave-normals 180° apart, so that each of the polar and equatorial extinction curves, apparently simple and free of intersections, is, in reality, double with an infinite number of points of coincidence (the term 'intersection' would here be invalid).

From the practical viewpoint there is an interesting consequence of this major difference between the coplanar and conical extinction curves: whereas a 180° rotation about P_0 suffices for the plotting of the former in its entirety, a full 360° rotation is necessary in the case of the latter.

Biaxial examples. In order to demonstrate some of the properties of conical extinction curves a few theoretical examples have been plotted. These theoretical curves were constructed as shown in fig. 1. For any wave-normal N of the cone, where the angle $P_0 N = \psi$, the vibration P in the associated wave-front F is determined as the bisector of angle CC'. The points, C, C', are the points of intersection of the wave-front F with the circular sections βC and $\beta C'$. The second vibration, P', is then plotted on the wave-front F at 90° from P. Vibrations associated with successive wave-normals chosen at suitable intervals around the cone are then plotted in the same way, the paper being rotated about its centre P_0 in order that the selected wave-normals of the cone fall successively on the diameter normal to the great circles of the net.

It must be noted here that, since the wave-normals lie on a small circle, centre P_0 and radius ψ, the associated wave-fronts must all be tangent to another small circle, centre P_0 and radius $90°-\psi$ (speaking



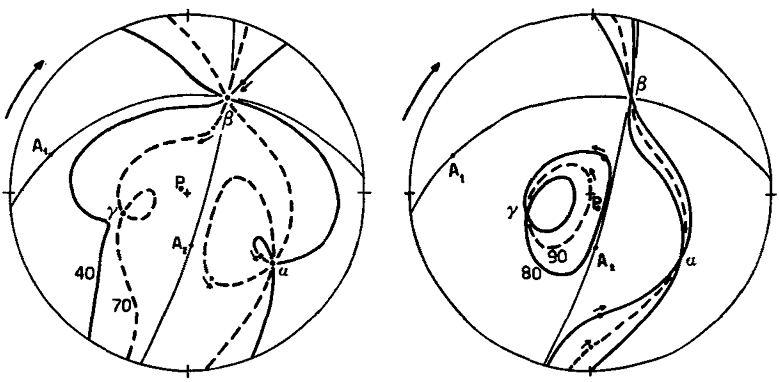


FIGS. 1 and 2: FIG. 1 (left). Stereogram illustrating the plotting of theoretical conical extinction curves. The rotation axis P_0 is in the centre of the stereogram. Also shown: biaxial indicatrix axis β; circular sections βC and $\beta C'$; wave-normal N (angle $P_0 N = \psi$) and its associated wave-front F; vibration P, located in wave-front F, and bisecting angle CC'; vibration P', located in wave-front F at 90° from P. FIG. 2 (right). Stereogram illustrating theoretical conical extinction curves in a biaxial crystal for $\psi = 0°$ (4 points on primitive, one of which is labelled 0), 10° (broken line), and 26° (continuous line). Also shown: biaxial indicatrix axes, α, β, γ; optic axes, A_1, A_2 ($A_1 A_2 = 2V = 90°$); circular sections, βA_1, βA_2; initial ($\alpha_1 = 0°$) wave-normals, N ($= P_0$), N', N'', of the wave-normal cones for which α_2 ($= \psi$) $= 0°$, 10° L, 26° L, respectively; the wave-fronts, F, F', F'', associated with N, N', N'' respectively. The vibrations in the initial wave-fronts F' and F'' are marked by dots on the corresponding extinction curves (accompanying arrows indicate direction of movement along the extinction curves as the paper turns clockwise).

three-dimensionally, the wave-fronts are tangent to a circular cone that is polar to the wave-normal cone). Moreover, all the wave-fronts must lie wholly within the belt or girdle between this second small circle and the primitive circle (or the great circle normal to P_0 if the latter is not plotted centrally). Hence, the extinction curves themselves must lie wholly within the same belt or girdle: their maximum departure from the primitive circle (or the great circle normal to P_0) is thus ψ.

Figs. 2, 3, and 4 show some conical extinction curves that were plotted as explained above. In all of these the indicatrix, shown in terms of the principal vibrations, α, β, γ, the optic axes, A_1, A_2, and the circular sections, has 2V = 90° and is in a fixed orientation with respect to the cone axis, P_0, which is plotted centrally. The only variable parameter is ψ, the cone angle; and the conical extinction curves associated with several values of ψ are shown.

When $\psi = 0°$, the special case where the cone degenerates into a single wave-normal N in coincidence with the cone-axis P_0, the extinction curves consist of the two vibrations associated with the wave-



FIGS. 3 and 4: FIG. 3 (left). Theoretical conical extinction curves for $\psi = 40°$ (continuous line) and 70° (broken line). Otherwise as in fig. 2 with the omission of initial wave-normals and associated wave-fronts. FIG. 4 (right). Theoretical conical extinction curves for $\psi = 80°$ (continuous line) and 90° (broken line). The latter is the well-known coplanar extinction curve. Otherwise as in fig. 2 with the omission of initial wave-normals and associated wave-fronts.

normal $N = P_0$. These are shown as four points, one of which is labelled 0, on the primitive circle (fig. 2).

With $\psi = 10°$ (broken lines, labelled 10, fig. 2) the curves are two closed ovals elongated in the directions of α, β, γ.

The value $\psi = 26°$ (continuous lines, labelled 26, fig. 2) was then chosen since α is 26° from the primitive circle: the curves have the same general shape as in the last case except that one of them has a cuspuate elongation terminating in the point α.

In fig. 2 the positions of the wave-normals, N ($= P_0$), N', and N'', where $\psi = 0°$, 10°, and 26° respectively, and of the corresponding wave-fronts, F, F', and F'', are shown, to indicate the initial position of the net in each case. In figs. 2, 3, 4, the vibrations, P and P', for these initial positions have been marked as unlabelled dots on each of the extinction curves, and arrows have been placed alongside them to indicate the direction of movement along the extinction curves as the paper is rotated clockwise.

With $\psi = 40°$ (continuous lines, labelled 40, fig. 3) the two curves show a mutual intersection at β and one of them a self-intersection at α.

With $\psi = 70°$ (broken lines, labelled 70, fig. 3) the only new feature is the addition of a self-intersection in the second curve at γ.

With $\psi = 80°$ (continuous lines, labelled 80, fig. 4) the two curves are again quite separate from each other. One, the smaller, shows a self-intersection at γ while the other shows two self-intersections at α and β.

With $\psi = 90°$ (broken lines, labelled 90, fig. 4) the two curves are the well known polar and equatorial coplanar extinction curves. The manner in which these split or become double as ψ departs from 90° can be clearly seen.

Uniaxial examples. Three theoretical examples have been plotted (fig. 5) for a uniaxial case with the optic axis A at 26° from the primitive circle. The extinction curves for wave-normal cones where $\psi = 14°$, 50°, and 70° are shown by broken, dotted, and continuous lines respectively. The circular section C contributes, in each case, the second part of the extinction curve but only that part of it lying within the extinction curve belt or girdle, i.e. only that part of it within the angle ψ from the primitive circle. Marks on the circular section, labelled 14 and 50, indicate the limit of its contribution to the extinction curves for these values of ψ. In this particular orientation of the indicatrix, the entire circular section constitutes one of the curves when $\psi = 70°$.

Equation of the conical extinction curves

Let **i**, **j**, **k** be three unit vectors, mutually perpendicular, and paralle to the three principal axes, $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$, of the ellipsoid. Let $A = \alpha^{-2}$, $B = \beta^{-2}$, $C = \gamma^{-2}$, where α, β, γ are the three principal refractive indices of the crystal.

A vibration direction will be represented by a unit vector **r**, and its associated wave-normal by a unit vector **n**. These vectors, **r** and **n**, satisfy the relations: $d\mathbf{r}.\mathbf{r} = 0$, $d\mathbf{r}.\mathbf{n} = 0$, and $d\mathbf{r}.\boldsymbol{\Phi}.\mathbf{r} = 0$, where $\boldsymbol{\Phi}$ is the dyadic, $\boldsymbol{\Phi} = A\mathbf{ii}+B\mathbf{jj}+C\mathbf{kk}$. The notation used here is that of Joel and Garaycochea (1957).

It follows that the vectors **r**, **n**, and $\boldsymbol{\Phi}.\mathbf{r}$ are coplanar. That is, $\mathbf{n} = \lambda\mathbf{r}+\mu\boldsymbol{\Phi}.\mathbf{r}$.

In order to determine λ and μ we multiply this equation first by **n** and then by **r** (scalar products) and then, using the fact that $\mathbf{r}.\mathbf{n} = 0$

and $\mathbf{n}^2 = \mathbf{r}^2 = 1$, we get $1 = \mu\mathbf{n}.\mathbf{\Phi}.\mathbf{r}$ and $0 = \lambda+\mu\mathbf{r}.\mathbf{\Phi}.\mathbf{r}$. Solving for λ and μ, and substituting,

$$(\mathbf{n}.\mathbf{\Phi}.\mathbf{r})\mathbf{n} = \mathbf{\Phi}.\mathbf{r}-(\mathbf{r}.\mathbf{\Phi}.\mathbf{r})\mathbf{r}. \tag{1}$$

Now, in the conical extinction curves, the wave-normals $\mathbf{n}$ are all on a circular cone around a given unit vector $\mathbf{r}_0$, i.e. they all form an angle ψ with $\mathbf{r}_0$ ($\mathbf{r}_0$ is parallel to the rotation axis of the crystal—one of the universal stage axes or the axis of a tilted spindle stage), so that $\mathbf{n}.\mathbf{r}_0 = \cos\psi$.

Therefore, if we multiply equation (1) by $\mathbf{r}_0$ (scalar product), it follows that

$$(\mathbf{n}.\mathbf{\Phi}.\mathbf{r})\cos\psi = \mathbf{r}_0.\mathbf{\Phi}.\mathbf{r}-(\mathbf{r}_0.\mathbf{r})(\mathbf{r}.\mathbf{\Phi}.\mathbf{r}). \tag{2}$$

The vector $\mathbf{n}$, which still appears in equation (2), must be eliminated in order to have the equation of the extinction curve corresponding to the angle ψ around $\mathbf{r}_0$. This elimination follows quite simply if it is remembered that $\mathbf{n}$, $\mathbf{r}$, and $\mathbf{\Phi}.\mathbf{r}$ are coplanar, that $\mathbf{n}$ and $\mathbf{r}$ are mutually perpendicular, and that $\mathbf{n}$ and $\mathbf{r}$ (but not $\mathbf{\Phi}.\mathbf{r}$) are of unit length. Therefore

$$(\mathbf{n}.\mathbf{\Phi}.\mathbf{r})^2 = (\mathbf{\Phi}.\mathbf{r})^2-(\mathbf{r}.\mathbf{\Phi}.\mathbf{r})^2, \tag{3}$$

so that, by squaring equation (2) and substituting for $\mathbf{n}.\mathbf{\Phi}.\mathbf{r}$, we have

$$[\mathbf{r}_0.\mathbf{\Phi}.\mathbf{r}-(\mathbf{r}_0.\mathbf{r})(\mathbf{r}.\mathbf{\Phi}.\mathbf{r})]^2 = [(\mathbf{\Phi}.\mathbf{r})^2-(\mathbf{r}.\mathbf{\Phi}.\mathbf{r})^2]\cos^2\psi, \tag{4}$$

which is the equation of the conical extinction curve. It is immediately apparent that when $\cos\psi = 0$ ($\psi = 90°$) the right-hand side of this equation vanishes and the equation reduces to that of the coplanar extinction curves (Joel and Garaycochea, 1957, p. 405, equation (5)).

In equation (4), $\mathbf{\Phi}$ is the dyadic $A\mathbf{ii}+B\mathbf{jj}+C\mathbf{kk}$; but it can also be replaced in this equation by another dyadic $\boldsymbol{\varphi}$ which is defined in terms of two vectors only, both of them in the optic axial plane of the indicatrix. There are two expressions for $\boldsymbol{\varphi}$: $\boldsymbol{\varphi} = \mathbf{a}_1\,\mathbf{a}_2+\mathbf{a}_2\,\mathbf{a}_1$, where $\mathbf{a}_1$ and $\mathbf{a}_2$ are unit vectors parallel to the two optic axes; and $\boldsymbol{\varphi} = 2(\mathbf{kk}\cos^2 V_\gamma - \mathbf{ii}\sin^2 V_\gamma)$, where V_γ is the angle between one optic axis and the $\boldsymbol{\gamma}$ axis of the ellipsoid (Garaycochea and Wittke, 1964).

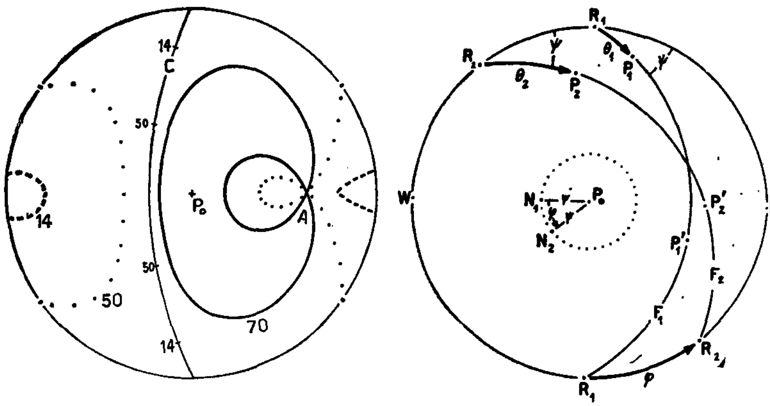
$\boldsymbol{\varphi}$ expressed in terms of $\mathbf{a}_1$ and $\mathbf{a}_2$ is particularly useful in the case of uniaxial crystals where one can put $\mathbf{a}_1 = \mathbf{a}_2 = \mathbf{a}$. After some manipulation of equation (4) it emerges that one portion of the conical extinction curve for a uniaxial crystal is the curve

$$[(\mathbf{r}_0.\mathbf{r})(\mathbf{a}.\mathbf{r})-(\mathbf{a}.\mathbf{r}_0)]^2 = [1-(\mathbf{a}.\mathbf{r})^2]\cos^2\psi \tag{5}$$

which goes through $\mathbf{a}$, but not through $\mathbf{r}_0$ unless $\cos\psi = 0$ ($\psi = 90°$). The other portion is all or part of the circular section—in fact, those parts of it within an angular distance of ψ from the plane perpendicular to $\mathbf{r}_0$.

Universal stage application

General. Conical extinction curves are usable with the universal stage in the solution of problems similar to those for which Joel and Muir (1958) advocate the use of coplanar extinction curves—for refining the positions of orthoscopically or otherwise determined indicatrix axes and for other specialized problems—and for the same reasons: '. . . the



FIGS. 5 and 6: FIG. 5 (left). Uniaxial case. Theoretical conical extinction curves for $\psi = 14°$ (broken line), 50° (dotted line), and 70° (continuous line) in the case where the angle $P_0 A = 64°$ (A = optic axis). For $\psi = 70°$, one portion of the extinction curve is the circular section C. For $\psi = 50°$ and 14°, only those parts of the circular section between the primitive circle and the points labelled 50 and 14 respectively contribute to the extinction curve. FIG. 6 (right). Stereographic plotting of universal stage observations with P_0 central. The initial wave-normal, N_1 ($\alpha_1 = 0°$), of the cone and the associated wave-front, F_1, are defined by $P_0 N_1 = \psi = \alpha_2 = 30°$ L. Vibration P_1 is plotted on wave-front F_1 with reference to R_1 ($= A_2$) as zero point in terms of $R_1 P_1 = \theta_1 = \alpha_5 = -17°$, and P_1' is plotted 90° from P_1.* Wave-normal N_2 of the same cone and its associated wave-front F_2 are plotted on the basis of $N_1 P_0 N_2 = \phi = \alpha_1 = +40°$. Vibrations P_2, P_2' are plotted on wave-front F_2 with reference to R_2, the new position of A_2, and in terms of the new extinction angle $\theta_2 = -36°$.

* Readings on axes A_1, A_3, A_5 are indexed + and − according as they are measured by clockwise or anticlockwise rotation from the zero position.

extinction position . . . can always be determined with much greater precision than that with which an optic symmetry plane can be located by the orthoscopic method' (loc. cit., p. 861). The universal stage manipulation in the case of conical extinction curves is, however, radically different from, but no less simple than, that for coplanar extinction curves.

Wave-normal cones of small ψ. There are two alternative and equally effective methods of generating these cones and of determining the associated extinction curves with the 4-axis universal stage. In one, the axes A_3 and A_4 are set at zero (in fact, simpler 2- or 3-axis stages may be used), the stage is tilted about A_2 through some suitable angle ψ $(= \alpha_2)$[1] to right or left of the zero point and locked, and all subsequent rotations are made on A_1 and A_5 only (or A_1 and A_3 if preferred). With A_2 tilted through the angle ψ $(= \alpha_2)$, the axis A_1 makes the angle ψ $(= \alpha_2)$ with the microscope axis A_5. Rotation on A_1 by suitable stages through 360° then permits examination of the specimen via a series of wave-normals whose locus is a circular cone, axis A_1 $(= P_0)$ and semi-angle α_2 $(= \psi)$. For every setting about A_1 extinction is obtained by rotation about A_5 (or A_3) and the angles, α_1 and α_5 (or α_3) respectively, are noted and used in plotting.

In the alternative method, axes A_3 (or A_1) and A_4 are used instead of A_1 and A_2 respectively, while extinction angles are measured on A_5. With the 5-axis universal stage the number of possible combinations of axes is, of course, greatly increased.

In special cases (Joel and Muir, 1958, p. 870) preliminary rotations may have to be made about the inner axes, A_1, A_2; after these are made, the only procedure that can be used with the 4-axis stage is the second outlined above, using axes A_3, A_4, and A_5. In fact, manipulation of axes A_3 and A_4 is in general more precise than that of A_1 and A_2.

Wave-normal cones of large ψ. A simple method of generating these cones and of determining the associated extinction curves with the 4-axis universal stage is as follows: axis A_3 is set and locked in its zero position so that A_2 is perpendicular to A_4; the stage is then tilted about A_4 through some suitable angle α_4 to north or south of the zero position and locked; all subsequent rotations associated with any individual wave-normal cone are then made on A_2 and A_5. With A_4 rotated through the angle α_4, the axis A_2 makes the angle $90°-\alpha_4$ with the microscope axis A_5. Rotation about A_2 $(= P_0)$ by suitable stages, up to the instrumental limit, permits examination of the specimen via a series of wave-normals whose locus is part of a circular cone, axis A_2 $(= P_0)$ and semi-angle $90°-\alpha_4$ $(= \psi)$. For every setting about A_2, extinction is obtained by rotation on A_5. The angles, α_2 and α_5 respectively, are noted for use in plotting. Should the need arise, a further part of this cone of wave-

[1] Following the terminology of Joel and Muir (1958, p. 867) the angles $\alpha_1, \alpha_2, \ldots, \alpha_5$ are the angular rotations about axes $A_1, A_2, \ldots, A_5$ respectively from their zero positions.

normals may be obtained by rotating about A_2 until the lower hemisphere is uppermost and repeating the process to the instrumental limit in this position. Alternatively, this may also be achieved by interchanging N for S or vice versa in the direction of rotation about A_4 and ensuring that α_4 N $= \alpha_4$ S. Moreover, the specimen may be rotated about an unlimited number of axes of rotation, P_0, in the plane of the inner stage, by varying the setting about A_1.

With the 4-axis stage, no provision, other than choice of the most suitable position about A_1, can be made for preliminary setting of the inner stage to accommodate special cases (Joel and Muir, loc. cit.). The 5-axis stage, however, permits preliminary setting of both A_1 and A'_4 (inner E.-W. axis), followed by use of the other axes as outlined above.

It may be noted here that wave-normal cones of large ψ may also be generated by use of the simple spindle stage: the spindle axis is simply set at some convenient angle other than 90° to the microscope axis.

Plotting

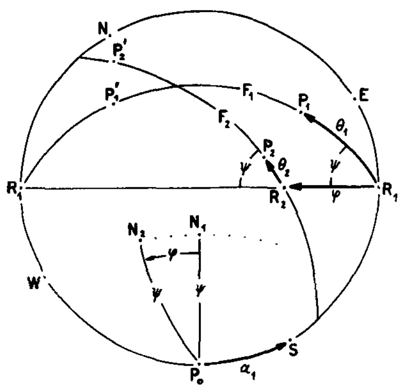
General. P_0, the axis of rotation of the specimen, may be plotted at any point in the stereogram; but for simplicity and in order that it may have some simple spacial relationship to the specimen and to the universal stage arrangements it is convenient to plot P_0 in the centre when represented by A_1 or A_3 and on the primitive circle when represented by A_2. Of the two methods of plotting, the former, with P_0 central, is the less time-consuming.

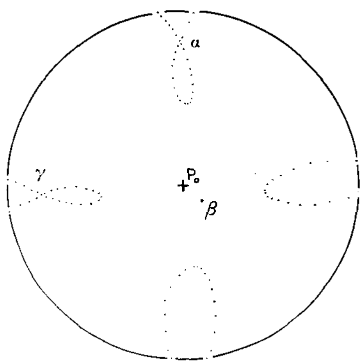
If not more than one extinction curve is to be plotted on the stereogram, then it is also simpler to plot P_0 in the centre even in the case where P_0 is represented by A_2; if, however, this curve is to be plotted in association with other conical extinction curves based on wave-normal cones of large or small ψ or with one or more coplanar extinction curves (see *Refinements*, below), then the second method of plotting, with P_0 on the primitive circle, provides the only simple approach.

P_0 ($= A_1$) *central.*[1] The wave-normals N_1, N_2,..., N_n of the cone lie on the circumference of a small circle, centre P_0 and radius ψ ($= \alpha_2$) (fig. 6). In the case of any wave-normal of the cone, say N_1, the associated wave-front F_1 makes the angle ψ ($= \alpha_2$) with the primitive circle. For this particular observation, N_1 also represents, of course, the microscope

[1] This plotting technique is described on the basis of the manipulation of axes A_1 ($= P_0$), A_2, and A_5 only. If any of the other axial combinations outlined above are employed, then the plotting instructions must be suitably modified.

axis A_5, and plane F_1 the microscope stage. A_2, perpendicular to both A_1 ($= P_0$) and A_5 ($= N_1$), lies, therefore, on the primitive circle at its intersection (R_1 in fig. 6) with plane F_1. This point R_1 marks the zero point for plotting the extinction angle θ_1 ($= \alpha_5$) on plane F_1. The vibration P_1 will be plotted on plane F_1 at the angular distance θ_1 from R_1, clockwise or anticlockwise according as the extinction angle α_5





FIGS. 7 and 8: FIG. 7 (left). Stereographic plotting with P_0 on the primitive circle. P_0 is located with respect to the south point of the stereogram in terms of the angle $\alpha_1 = -30°$. The initial wave-normal N_1 ($\alpha_2 = 0°$) of the cone and the associated wave-front F_1, are defined by $P_0 N_1 = \psi = 90° - \alpha_4$ ($\alpha_4 = 30°$ S). Vibration P_1 is plotted on wave-front F_1 with respect to R_1 ($= A_4$) as zero point, in terms of $R_1 P_1 = \theta_1 = \alpha_5 = +41°$, and P_1' is plotted 90° from P_1. Wave-normal N_2 of the same cone and its associated wave-front F_2 are plotted on the basis of $N_1 P_0 N_2 = \phi = \alpha_2 = 40°$ L. R_2, the new position of A_4, at the intersection of wave-front F_2 and the plane polar to P_0, marks the zero point for plotting vibrations P_2 and P_2' in terms of the new extinction angle $\theta_2 = +19°$. FIG. 8 (right). Points of an experimental conical extinction curve ($\psi = \alpha_4 = 40°$) for a crystal of andalusite showing determination of the indicatrix axes α and γ.

has been measured anticlockwise or clockwise respectively from the zero position. The second vibration, P_1', associated with the same wave-normal N_1, is of course plotted 90° from P_1 in the same wave-front F_1. If rotating nicols are used with a stationary stage, then the plotting of θ_1 will be in the same sense as the associated nicol rotation from the zero position.

As rotations, clockwise or anticlockwise, are made about A_1 to provide new wave-normals, then A_2 simply migrates around the primitive circle in the opposite sense and by the same amount. For example, in fig. 6, N_2 is related to N_1 by a clockwise rotation of 40° about A_1: R_2 is normal to the plane $P_0 N_2$ and 40° anticlockwise from R_1 on the primitive circle. In order to use a great circle of the net for wave-front F_2 the paper is, of course, rotated clockwise by 40° and R_2 plotted at the termination of the great circles. The extinction angle θ_2 and the vibrations P_2 and P_2' are plotted on plane F_2 in the same way as for θ_1, P_1, P_1' on plane F_1. This process is repeated for every wave-normal $N_1, N_2, \ldots, N_n$ of the cone, the paper being rotated progressively in the same sense and by the same amount as the successive rotations about A_1 until the full rotation of 360° has been completed.

In practice, of course, the wave-normals themselves need not be plotted. A mark is simply made on the primitive circle to represent the original position of A_2 ($= R_1$) when A_1 is in the starting position. The paper is then rotated progressively about the centre of the net in sympathy with rotations about A_1, and the extinction angles θ are plotted on the particular great circle of the net which makes the requisite angle ψ ($= \alpha_2$) with the primitive circle.

Should preliminary fixed settings other than zero be required on axes A_1 or A_2 or both to accommodate special cases, then the axial combination A_3 ($= P_0$), A_4, A_5 will be used. Plotting should, in the first instance, as suggested by Joel and Muir (1958, pp. 873–874), proceed as outlined above, with due allowances for the new axial combination; the positions of the indicatrix axes deduced thereby are eventually rotated to their proper orientations with respect to the plane and zero point of the inner stage.

P_0 ($= A_2$) on the primitive circle. Plotting in this orientation, although slightly less convenient and involving more manipulation of the net, is no less simple than with P_0 central. As a preliminary to plotting, P_0 must be correctly located on the primitive circle in terms of the angle α_1. With the great circles of the net running N–S, a mark is made on the paper at the south point to represent the A_1 zero mark. If the paper is then rotated in sympathy with the rotation of the inner stage about A_1, the points of termination of the great circles at the N and S points of the net represent axis A_2 ($= P_0$). Fig. 7 shows the case where $\alpha_1 = 330°$, i.e. 30° anticlockwise from zero.

P_0 having been located, the wave-normals $N_1, N_2, \ldots, N_n$ of the cone lie on the circumference of a small circle, centre P_0 and radius ψ ($= 90° - \alpha_4$). In the case of zero rotation about A_2, i.e. when $\alpha_2 = 0°$, the wave-normal lies in the plane through A_2 ($= P_0$) and perpendicular to the plane of the inner stage. It is thus most conveniently plotted on the vertical great circle through A_2 ($= P_0$). It is shown as N_1 in fig. 7. The associated wave-front, F_1, as in fig. 6, makes the angle ψ ($= 90° - \alpha_4$) with the plane polar to P_0—the paper must, of course, be rotated as necessary to construct this wave-front or to plot points within it. For this particular observation, N_1 also represents, of course, the microscope axis A_5, and plane F_1 the microscope stage. A_4, perpendicular to both A_2 ($= P_0$) and A_5 ($= N_1$), is therefore the pole of great circle $P_0\,N_1$: it is the point of intersection (R_1 in fig. 7) of wave-front F_1 and the plane polar to P_0 and, in this particular case, since plane $P_0\,N_1$ is vertical, it also lies on the primitive circle. Thus, point R_1 marks the zero point for plotting the extinction angle θ_1 ($= \alpha_5$) on plane F_1. Plotting of the vibrations P_1 and P'_1 within the wave-front F_1 follows the same directional rules with respect to R_1 as when P_0 is central (see above).

As the specimen is tilted to left or right about A_2 to provide new wave-normals, then A_4 simply migrates along the plane polar to P_0, to the left or right of R_1 for α_2 L or α_2 R readings respectively (Joel and Muir, 1958, p. 867). For example, in fig. 7, N_2 is related to N_1 by a tilt of 40° L (upwards to the left and read on the left arc) about A_2; R_2 is polar to the plane $P_0\,N_2$ and 40° to the left of R_1. R_2 lies in wave-front F_2 whose pole is N_2, and it is the zero point for plotting the vibrations P_2 and P'_2 on the basis of the extinction angle θ_2.

In practice, the wave-normals N_1, N_2,..., N_n must be plotted, but the points R_1, R_2,..., R_n and the wave-fronts F_1, F_2,..., F_n need not be marked on the paper. After P_0 and the small circle of wave-normals are located, the plane polar to P_0 (plane $R_1\,R_2$ in fig. 7) is drawn. Then the point R_n associated with the wave-normal N_n is simply the point of intersection of plane $R_1\,R_2$ with the plane normal to N_n.

If, to obtain as much of the extinction curve as possible, the inner stage is inverted by rotation about A_2, then the related plotting can be carried out in the following way. Invert the tracing paper on the net by rotation of 180° about the horizontal axis P_0 ($= A_2$) and plot exactly as outlined above: the resultant points on the extinction curve are on the lower hemisphere. Having plotted all that is possible in this orientation, return the tracing paper to its original orientation and transfer the lower hemisphere points to the upper hemisphere by simple reflection through the centre. These two steps can be done in one simply by inverting the tracing paper around the line $R_1\,R_2$ perpendicular to P_0, bearing in mind that the resultant points, although plotted on the underside of the paper, are actually upper hemisphere points.

Refinements

In general, a complete conical extinction curve will not be required to locate any particular indicatrix axis: the position of the axis can be located approximately by normal orthoscopic methods and the selected conical extinction curve plotted only in its immediate vicinity—the intersection is all that is required. In any particular case, the value of ψ should be carefully selected with certain considerations in mind. If the indicatrix symmetry plane normal to one of the axes, α, β, or γ, has been found, orthoscopically, to make an angle ψ' with the universal stage axis A_3 (making allowance for any fixed preliminary settings about A_1 or A_2 or A_4'), then ψ' is the minimum value of ψ that permits a conical

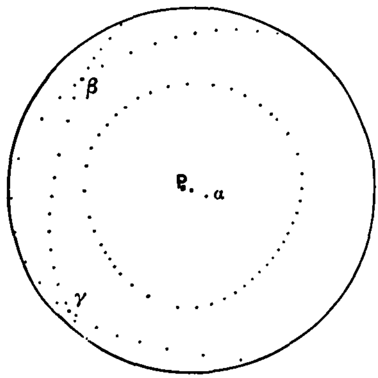


FIG. 9. Points of an experimental extinction curve ($\psi = \alpha_4 = 30°$) for a crystal of muscovite showing determination of the indicatrix axes β and γ.

extinction curve to include that particular indicatrix axis (in this case $A_3 = P_0$). For minimum accuracy, ψ should be at least a few degrees larger than the appropriate minimum value in order that the wave-normal cone intersect the plane and that, as a result, an intersection may appear on the extinction curve. In general, and within obvious limits, the greater the margin by which ψ exceeds the appropriate minimum, the greater the accuracy of the intersection, provided that, at the same time, errors due to excessive tilt of the stage are not introduced. Experiment will show, in any individual case, the best balance between the attainment of maximum accuracy of intersection and minimum tilt errors. Moreover, the ideal value of ψ will not, in general, be the same for defining all of the available indicatrix axes (at least two, and in favourable cases all three, of them are determinable).

Greater accuracy can, of course, be achieved by plotting partial conical extinction curves for more than one value of ψ, large or small, in the case of each determinable indicatrix axis. The standard of accuracy achieved in each case would then be given by the spread of the points of intersection. Further, if desired, conical extinction curves using values of ψ over the practicable range attainable may be combined with coplanar extinction curves produced by rotation about A_2 or A_4 following Joel and Muir (1958). The combined curves may all be plotted on the same stereogram, the small ψ conical set with P_0 central and both the large ψ conical sets and the coplanar sets with the various points P_0 on the primitive circle. All these curves should intersect at the determinable indicatrix axes.

Examples

Two examples of the use of this technique are presented. Fig. 8 shows the plotting of an experimental conical extinction curve ($\psi = \alpha_4 = 40°$) for a crystal of andalusite and the direct determination thereby of the indicatrix axes, α and γ. In this case, the inner horizontal axes were also tilted to bring the crystal into a suitable orientation. Fig. 9 shows an experimental extinction curve ($\psi = \alpha_4 = 30°$) for a crystal of muscovite with its cleavage nearly parallel to the plane of the thin section: the indicatrix axes, β and γ, are directly determined.

Acknowledgement. We are grateful to Mr. H. Villarroel for his help in preparing the drawings in a way suitable for publication.

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