Determinism in strontium isotope record of seawater

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Introduction

The reconstruction of marine Sr isotope record during the Phanerozoic (cf. Burke *et al.*, 1982; Jones 1991; McArthur 1994; and references therein) has been the objective of numerous studies in the last two decades. This experimental effort has been complemented by model studies, with some authors concentrating on the $^{87}Sr/^{86}Sr$ record itself (cf. Brass 1976; Berner and Rye 1992), others testing its sensitivity to various parameters (cf. Kump 1989), and still others utilizing it as a proxy for variables such as climate change (cf. Edmond and Palmer 1990; Francois and Walker 1992; Clemens *et al.*, 1993). All these studies were based on quantification of possible fluxes and their rates. Our approach differs fundamentally from those above in that it builds on a theoretical base that is independent of the quantitative state and flux parameters.

Methods

An iterative function system (IFS) has been employed as an 'amplifier' for development of a database for our modeling. The original algorithm of Barnsley (1988) was slightly modified and tested with deterministic and random datasets from literature (Fowler and Roach 1991). This confirmed that nonlinear interpolation by IFS, while enhancing the number of points in a dataset, did not influence its primary deterministic/non-deterministic character. A total of 13 different, and independent, mathematical techniques were utilized as 'test-tools' in order to obtain as much information about the character of the $^{87}Sr/^{86}Sr$ system as possible. These include difference plot, yard-stick dimension, box-counting dimension, density distribution, natural measure, information dimension, Hölder, correlation dimension, embedding dimension, Ljapunov exponent (cf. Peitgen *et al.*, 1993) and others.

Discussion of results

The similarities between short term (cf. Jones 1991; Diener *et al.*, 1994) and long term (e.g. the entire Phanerozoic) fluctuations in the $^{87}Sr/^{86}Sr$ trend indicate that this record may be of deterministic nature. Our results confirm this assertion. Both, simple (difference plot, box-counting etc.) and complex (embedding dimension, Ljapunov exponent) techniques point always towards determinism and self affinity within the system. The box-counting approach yields a fractal dimension of 1.53, while the information dimension gives a value of 1.48, the latter smaller by definition. Four independent variables define the degrees of freedom of the system. This is in accord with the postulate that, in a deterministic system, fractal dimension relates to the embedding dimension as:

$$2D_b < E \leqslant 2D_b + 1 \qquad (1),$$

where D_b is the fractal dimension and E the embedding dimension (Fowler and Roach 1991).

This theoretical approach suggests that the system is controlled by four parameters. Presumably, the riverine Sr flux and its $^{87}Sr/^{86}Sr$ ratio and the oceanic 'hydrothermal' Sr flux represent three of these parameters. The fourth one is somewhat enigmatic, since the 'hydrothermal' $^{87}Sr/^{86}Sr$ is relatively constant and therefore an unlikely candidate.

The same approach applied to marine carbon and oxygen isotope records suggests that these systems are too complex to be constrained by the above techniques, with the number of controlling variables (degrees of freedom in the system) too large to be of utility. This may reflect the overall variability in chemical composition and thermal structure of the atmosphere-hydrosphere system throughout the Phanerozoic (cf. Berner 1987; Berner 1990; Clemens *et al.*, 1993), larger numbers of natural controlling parameters, and laboratory-induced artifacts.

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