On the Use of the Gnomonic Projection; with a Projection of the Forms of Red Silver Ore.

By H. A. Miers, M.A.

[Read January 11th, 1887.]

If the planes of a crystal be replaced by lines drawn perpendicular to them through a point the intersection of the sheaf of lines so formed with the plane of the paper is the Gnomonic Projection of the crystal; each face being represented by a point and each zone by a line joining two such points. This projection, which has been frequently used and fully described, has generally been employed to represent the symmetrical disposition of the faces, the plane of the figure being a plane of symmetry.

But the practical value of the projection is best seen when it is used either as a graphic representation of the forms of a mineral, or as a method of drawing a perspective figure of any part of a crystal.

I.—To represent all the forms of a mineral (which does not belong to either of the two oblique systems) the plane of the figure must cut the whole group of normals enclosed within three adjacent planes of symmetry.

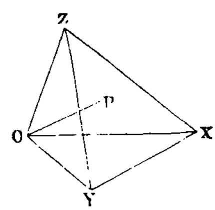

Fig. 1.

Let OX, OY, OZ (fig. 1), be any three normals, the angles between which are

$$YOZ=\xi$$
$$ZOX=\eta$$
$$XOY=\zeta$$

Let XYZ be the plane of the figure of which the normal OP is inclined to the first three normals at the angles

$$POX=\alpha$$
$$POY=\beta$$
$$POZ=\gamma$$

Let $OP = p$. Then

$$YZ = K.\ p\cos\alpha\ \sqrt{\cos^2\beta + \cos^2\gamma - 2\cos\beta.\ \cos\gamma.\ \cos\xi}.$$

$$ZH = K.\ p\cos\beta\ \sqrt{\cos^2\gamma + \cos^2\alpha - 2\cos\gamma.\ \cos\alpha.\ \cos\eta}.$$

$$XY = K.\ p\cos\gamma\ \sqrt{\cos^2\alpha + \cos^2\beta - 2\cos\alpha.\ \cos\beta.\ \cos\zeta}.$$

where $$K = \frac{1}{\cos\alpha.\ \cos\beta.\ \cos\gamma}.$$

And
$$PX = p \tan\alpha$$
$$PY = p \tan\beta$$
$$PZ = p \tan\gamma$$

Thus the triangle XYZ may be drawn, and the position of P may be laid down for any position of the plane of the figure at any required distance (p) from the origin.

Then any other point of the system being laid down from its known angles, the whole system of points corresponding to possible planes of the crystal may be mapped out by the intersections of lines joining these points.

The dihedral angles are measured on the projection by means of the "Circle of Projection" described from centre P with radius equal to p.

The following constructions are given in all descriptions of the gnomonic projection :—

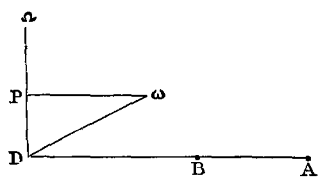

Fig. 2.

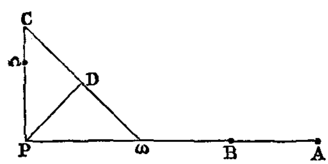

Fig. 3.

(1.) To find the angle between two normals whose projections are AB (fig. 2). Through P draw PD perpendicular to AB, meeting AB in D; draw $P\omega$ parallel to AB, meeting the circle of projection in ω. Along PD take $D\Omega = D\omega$. Then the angle between any pair of normals whose projections AB lie in the zone line ABD is the angle subtended by AB at Ω.

(2.) To find the angle between two zones whose projections are CA, CB (fig. 3). Join CP; draw $P\omega$ perpendicular to CP, meeting the circle of projection in ω. Join $C\omega$; draw PD perpendicular to $C\omega$, and in PC take $P\Omega = PD$.

Then the angle between any pair of zones whose projections CA, CB intersect in the point C is the angle subtended at Ω by their intersections with $P\omega$.

With the help of these constructions any crystal may be easily projected.

In the systems with rectangular axes and in the Hexagonal and Rhombohedral system the construction of the figure is much simplified by a convenient choice of the plane of projection.

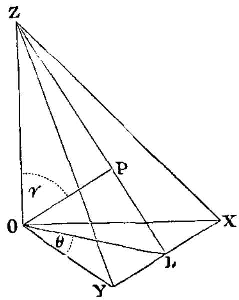

FIG. 4.

If OX, OY, OZ (fig. 4) are the rectangular axes in the former, or two lateral axes and the vertical axis in the latter,

$$\cos\alpha=\cos\beta=\sin\gamma\cos\frac{\zeta}{2}=\sin\gamma\cos\theta \text{ where } 2\theta=\zeta.$$

Let ZP meet XY in L ; then ZLX=90°.

$$ZL=\frac{p}{\sin\gamma\cos\gamma} \quad \text{and} \quad XL=\frac{p\tan\theta}{\sin\gamma}$$

$$\therefore \frac{LX}{LZ}=\tan\theta.\cos\gamma \quad \dots \quad \dots \quad \dots \quad (a)$$

$$PL=ZL\cos^2\gamma \quad \dots \quad \dots \quad \dots \quad (b)$$

$$p=\frac{PZ}{\tan\gamma}=\frac{PX}{\tan\alpha} \quad \dots \quad \dots \quad \dots \quad (c)$$

The triangle XYZ may be drawn from (*a*).
The point P may be projected from (*b*).
The circle of projection may be described from (*c*).
In the Cubic, Tetragonal, and Orthorhombic systems $\theta=45°$.

Then if $\gamma=45°$, $\cos\alpha=\frac{1}{2}$.

$$\frac{LX}{LZ}=\frac{1}{\sqrt{2}}, \quad PL=\frac{1}{2}ZL, \quad p=PZ.$$

In the Rhombohedral and Hexagonal system $\theta=30°$.

Then if $\cos\gamma = \frac{1}{\sqrt{3}}$, $\cos\alpha = \frac{1}{\sqrt{2}}$, $\alpha = 45°$.

$$\frac{LX}{LZ} = \frac{1}{3}, \quad PL = \frac{1}{3} ZL, \quad p = PX.$$

As an example of this mode of projection, Plate IV. gives a figure of all the forms of Pyrargyrite and Proustite which have been recorded, together with a number recently observed by myself. This figure is the intersection of the face-normals by a plane inclined at 54° 44½′ to the vertical axis, and at 45° to two adjacent lateral axes, and at a distance of $b* = 84{\cdot}85$ mm. from the origin, where * is the projection of its own normal on the plane of the figure.

The forms marked with an asterisk are new; h' and λ', which do not appear as separate points on the projection, refer without doubt to the same form. *Vid.* explanation of Plate. (p. 149.)

II.—To draw a perspective figure of any part of a crystal from the projection, it is only necessary to represent the edge between any two faces by a perpendicular to the line joining their projections.

Although it is impossible to project more than a limited number of the faces, the complete crystal may be drawn without difficulty when once the intersections of the planes of symmetry with the plane of the figure have been drawn, by the repetition of the zone-lines in the figure across one or other of these intersections.

The gnomonic projection has the advantages that (like the linear projection) it is a figure actually formed by the crystal and not a representation; the constructions are all made by straight lines; each face is represented by a point; the anharmonic ratio of four faces is the ratio of their intercepts measured along a straight line; and it is by far the easiest graphic method of determining the position of any face. In forming the perspective figures from it no construction lines are required.

EXPLANATION OF PLATE.

a	101		*a'*	$72\bar{1}$		*A*	$15\ 3\ \bar{4}$		*A'*	$31\bar{1}$		α	$42\bar{3}$		α'	10 1 1		Γ	$33\bar{1}$	*	Γ'	730
b	$2\bar{1}\bar{1}$		*b'*	411	*	*B*	$17\ 5\ \bar{4}$		*B'*	$73\bar{7}$		β	$5\bar{1}\bar{4}$		β'	$11\ \bar{4}\ \bar{4}$		Δ	$19\ 0\ \overline{13}$	*	Δ'	$17\ 0\ \overline{13}$
c	$40\bar{3}$		*c'*	$15\ 2\ \bar{1}$	*	*C*	$12\ 1\ \bar{3}$		*C'*	$6\bar{1}\bar{3}$		γ	$50\bar{3}$		γ'	$17\ \bar{7}\ \bar{7}$		Θ	$17\ \bar{8}\ \bar{9}$	*	Π'	13 6 0
d	$21\bar{1}$		*d'*	$55\bar{9}$	*	*D*	$54\bar{8}$		*D'*	$13\ 6\ \overline{11}$		δ	$32\bar{1}$		δ'	$11\ \bar{5}\ \bar{5}$		Λ	540	*	Ω'	$74\bar{8}$
e	110		*e'*	10 3 0		*E*	$21\bar{2}$		*E'*	$33\bar{2}$		ε	$20\ 11\ \bar{7}$		ε'	772	*	Ξ	$17\ 6\ \bar{7}$			
f	$22\bar{3}$	*	*f'*	$7\bar{2}\bar{5}$		*F*	$9\ 5\ \overline{10}$	*	*F'*	$40\ 7\ \overline{11}$		ζ	$90\bar{5}$		ζ'	771	*	Π				
g	$71\bar{2}$		*g'*	$12\ 1\ \bar{4}$		*G*	$43\bar{6}$	*	*G'*	810		η	$28\ 13\ \overline{17}$		η'	$84\bar{3}$		Σ	$8\bar{1}\bar{1}$			
h	$55\bar{4}$		*h'*	$51\bar{3}$		*H*	$8\ 5\ \overline{10}$		*I'*	$17\ 0\ \overline{11}$	*	θ	$97\bar{8}$	*	θ'	720		Υ	13 7 0			
i	$51\bar{1}$		*i'*	$52\bar{6}$		*I*	611		*K'*	$13\ 7\ \overline{14}$		κ	$70\bar{5}$				*	Φ	520			
k	$11\ 1\ \bar{4}$		*k'*	$62\bar{7}$	*	*K*	$21\ 6\ \bar{5}$	*	*N'*	$17\ 0\ \overline{15}$		λ	11 4 0		λ'	$41\ 8\ \overline{25}$	*	Ψ	$13\ 2\ \bar{3}$			
l	$62\bar{3}$		*l'*	$13\ \bar{1}\ \bar{4}$	*	*L*	$19\ 4\ \bar{5}$		*P'*	$61\bar{2}$		μ	$13\ 4\ \bar{2}$		μ'	$85\bar{3}$		Ω	$62\bar{1}$			
m	$3\bar{1}\bar{1}$	*	*m'*	$82\bar{3}$		*M*	$63\bar{2}$		*W'*	332		ν	$16\ 4\ \bar{5}$		ν'	67 22 7			$10\ 0\ \bar{7}$			
n	$40\bar{1}$	*	*n'*	$7\bar{1}\bar{4}$		*N*	$53\bar{6}$		*X'*	221		ξ	610		ξ'	$25\ 1\ \overline{11}$						
o	111		*o'*	$12\ 2\ \bar{5}$		*P*	$32\bar{3}$		*Y'*	$22\bar{1}$	*	π	$19\ 11\ \overline{12}$		π'	$90\bar{7}$						
p	210	*	*p'*	$12\ \bar{5}\ \bar{6}$	*	*Q*	$15\ 1\ \bar{3}$	*	*Z'*	$13\ 9\ \overline{11}$	*	ρ	$64\bar{3}$		ρ'	$8\bar{1}\bar{4}$						
q	$32\bar{4}$		*q'*	$73\bar{5}$		*R*	811					σ	$41\bar{1}$		σ'	$9\bar{1}\bar{6}$						
r	100		*r'*	16 1 1	*	*S*	$43\bar{1}$					τ	$3\bar{1}\bar{2}$		τ'	830						
s	$11\bar{1}$	*	*s'*	$70\bar{3}$		*T*	$4\bar{1}\bar{1}$					υ	320		υ'	$9\ 4\ \overline{11}$						
t	310	*	*t'*	$18\ 4\ \bar{7}$		*U*	$91\bar{2}$				*	φ	510		φ'	$73\bar{9}$						
u	211		*u'*	$17\ 16\ \overline{25}$	*	*V*	$13\ 1\ \bar{4}$					χ	$82\bar{1}$		χ'	$73\bar{3}$						
v	$20\bar{1}$		*v'*	$51\bar{3}$		*W*	$92\bar{3}$				*	ψ	$30\bar{1}$		ψ'	$41\bar{2}$						
w	410		*w'*	$50\bar{1}$		*X*	$8\bar{3}\bar{4}$					ω	530		ω'	$95\bar{9}$						
x	$31\bar{3}$	*	*x'*	$26\ 5\ \bar{7}$		*Y*	$81\bar{3}$															
y	$30\bar{2}$		*y'*	$15\ \bar{2}\ \bar{8}$	*	*Z*	$50\bar{4}$															
z	$51\bar{2}$		*z'*	430																		