THE DETERMINATION OF OPTIC ANGLE WITH THE UNIVERSAL STAGE

THEODORE A. DODGE, *University of Wisconsin.*

ABSTRACT

With the universal stage direct measurement of the optic angle of a biaxial crystal by rotation from a bisectrix to one or both optic axes is frequently not possible. Berek's graphical method of determination for this case is outlined here and modified to give greater flexibility and usefulness by the addition of a more comprehensive group of curves. These permit the attainment of the maximum accuracy possible under any given set of conditions. The method involves only a few simple rotations on the universal stage and is equally applicable to the old or new models. Used with the modified instrument having two east-west axes, the new curves form an integral part of universal stage procedure involving no graphical construction on the part of the operator and no corrections during experimental procedure.

In the determination of optic angle with the Fedorov Stage they afford a most valuable check whenever the complete angle, 2V, cannot be measured directly.

INTRODUCTION

Berek[1] has developed a procedure for thin-section study of biaxial minerals on the Leitz model universal stage, an essential part of which is the determination of the optic orientation and optic angle. This is performed as follows: After locating the position of the first and second optic symmetry planes, an optic symmetry axis is made vertical with the aid of a stereographic projection by rotation on the north-south horizontal and the inner vertical axes. The crystal is then turned to extinction on the outer vertical axis. From this oriented position in which two of the optic symmetry planes are parallel to the two nicols (*i.e.* north-south and east-west) and an optic symmetry axis is parallel to the axis of the microscope, the crystal is turned 45° on the outer vertical axis (*rotation* $\bar{\varphi}$) to what may be called *the reference position* and then 54.7° on the outer east-west horizontal axis (*rotation* x). Finally, it is turned to extinction on the microscope stage (*rotation* ϕ). For the given value of the angle ϕ a "Normaldiagramm" (Fig. 1) indicates the orientation in the reference position of the optic plane and acute bisectrix, and the approximate size of the optic angle. To determine the latter more accurately different rotation angles $\bar{\varphi}$ and x of size depending upon the orientation of the optic symmetry elements and the value of 2V just obtained are used in conjunction with a particular curve of a "Spezialdiagramm."

[1] M. Berek, Neue Wege zur Universalmethode: *Neues Jahrb. f. Min., Geol., und Pal., Beilage Band,* **1923.**

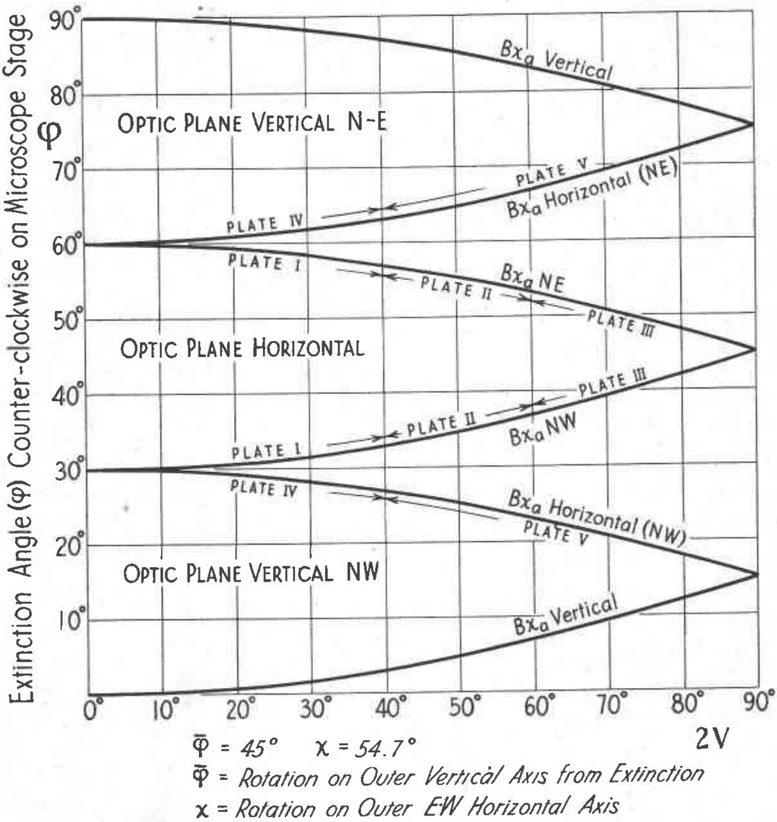

FIG. 1. Graph for determination of optic orientation in the reference position and of approximate size of 2V. Rotations $\bar{\varphi}$, x, and ϕ are made in this order. It is used prior to determination of optic sign and accurate determination of optic angle with plate indicated.

It is to this last named step that attention is called here. For a particular value of 2V and a given orientation there is one curve which will give maximum accuracy. It is represented by rotations $\bar{\varphi}$ and x of definite size. For any other value of the optic angle this will not be the most accurate curve. Theoretically, therefore, it would be desirable to have a different curve for every value of 2V, assuming a given orientation. Practically, of course, this is out of the question, and also the curve of maximum accuracy often involves so large a rotation x as to be impractical from a mechanical and optical standpoint. Berek has chosen, therefore, three ranges of 2V, and for each of these he has constructed three curves for the three possible orientations. The accuracy of each curve is as great as can be obtained with a rotation value of x which under normal conditions can always be made.

There are three limitations, however, attendant upon so restricted a choice. First, it is often possible in thin-section work to make a rotation x larger than that demanded by a given curve of Berek's "Spezialdiagramm." Now in general, accuracy increases with increase in x up to large values, so that it is frequently the case that a more accurate result than is possible with the "Spezialdiagramm" could be obtained were a suitable curve available. Secondly, in immersion work the converse is true. Thus it is often not possible to rotate through the angle x necessary for the given curve. In such an event one is limited to the approximate result of the "Normaldiagramm." With a curve for the maximum obtainable value of x under these conditions, a more accurate result would be obtained. Thirdly, with the "Spezialdiagramm" the value of x must be corrected for the difference in index between the hemisphere and mineral before the rotation is made. The index of the mineral, however, is not ordinarily obtained until after determining the optic orientation while the determination of 2V is most conveniently made at the same time as that of the orientation. It would be more logical, therefore, for the determination of optic angle to precede index determination rather than to follow it.

The purpose here is to present a procedure and accompanying curves intended to minimize these limitations on convenience and accuracy. Five groups of curves have been plotted, of which three apply to the orientation in which the optic plane is horizontal, and two, to that in which the optic plane is vertical and acute bisectrix horizontal. For the third possible orientation with acute bisectrix vertical 2V is measured by direct reading, and no curves have been calculated. In each of the five groups $\bar{\varphi}$ has a fixed value while x varies in two degree steps from 54° to 82°, approximately. In this way all three of the above mentioned limitations to the "Spezialdiagramm" are at once removed since these two values of x are the practical limits of the range of rotation possible on the outer east-west axis, and the individual curves are close enough together to allow interpolation. Thus the result of an optic angle determination is not restricted to the accuracy obtainable with the "Normaldiagramm" or "Spezialdiagramm," and no corrections are needed during experimental work. Furthermore, since the relative care with which different parts of universal stage procedure are carried out and differences in optical properties of various minerals influence the relative accuracy of different curves of any one of the five groups, individual differences in minerals and in experimental work can be compensated by correct choice of curves.

Since the introduction of the modified instrument having two east-west axes universal stage procedure has become much more simple and rapid. The present method of optic angle determination is designed to conform to these recent improvements, to increase the accuracy of 2V determinations, and to make them possible under all conditions of experimental work. Its field of application is not limited, however, to the modified instrument. With the Leitz model (Fedorov) stage this method of optic angle determination forms a supplementary procedure of use whenever an optic axis cannot be brought into the field of view, and it affords a most valuable check whenever the complete angle, 2V, cannot be measured directly.

The author wishes to express acknowledgement to Professor R. C. Emmons for suggesting the problem and giving the greatest help in its pursual.

Procedure

The procedure recommended for use with the curves accompanying this paper is similar to that outlined above, but for those not familiar with Berek's method the following detailed description will be of use. There are two principal steps: first, the determination with the aid of Fig. 1. of the orientation of the optic elements of the crystal in the reference position and the approximate size of the optic angle, and, second, the accurate determination of the optic angle by direct reading or by use of Plates I–V.

The first step consists of the three rotations: $\bar{\varphi}=45°$, $x=54.7°$, and ϕ. Rotation $\bar{\phi}$ is made in either direction on the outer vertical axis from the oriented position in which an optic symmetry axis is parallel to the microscope and two optic symmetry planes are north-south and east-west. After this rotation of forty-five degrees the crystal is in *the reference position.* Rotation x is made in either direction on the outer east-west horizontal axis. The rotation angle of 54.7° is crystallographic, so that the actual rotation will be less

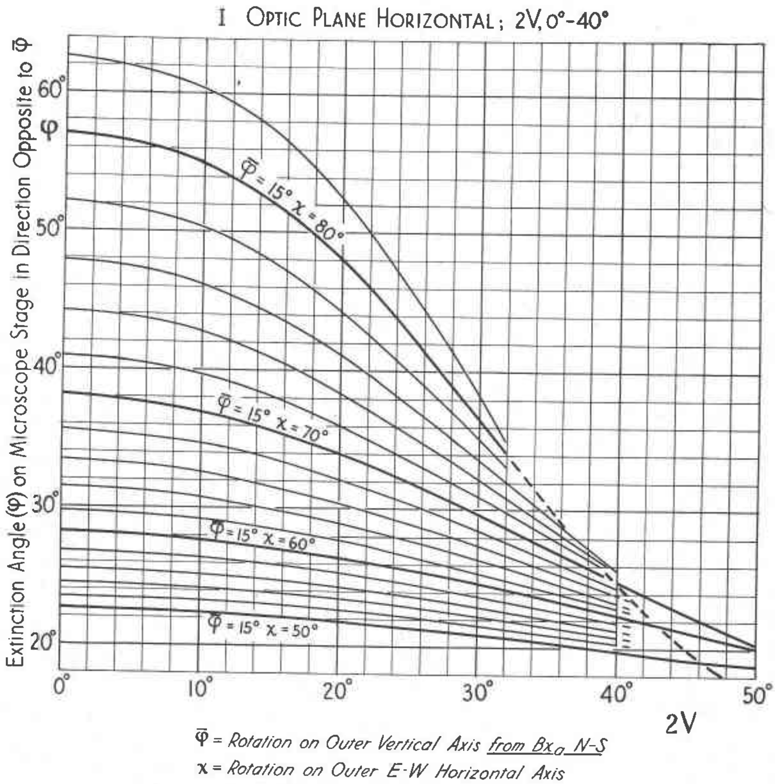

PLATE I. Graph for the determination of 2V. Rotations $\bar{\varphi}$, x, and ϕ are made in this order. Make rotation x as large as possible up to 82°. Do not interpolate between curves for x greater than 70° beyond where these curves end. If exact rotation $x=80°$ is made, the dotted portion of this curve may be used. For 2V less than 15° the curves give inaccurate results.

or more than this value depending upon whether the index of the hemisphere is smaller or greater than that of the mineral, but the correction can usually be estimated with sufficient accuracy for this procedure since ordinarily the approximate index of the mineral is known. Lastly, rotation ϕ is made counter-clockwise to extinction on the microscope stage.

Using this value of ϕ as ordinate, a point on one of the six curves of Fig. 1. is located. This gives the orientation of the optic plane and acute bisectrix in the reference position and the approximate size of the optic angle. It also indicates the plate to be used in the more accurate determination of 2V.

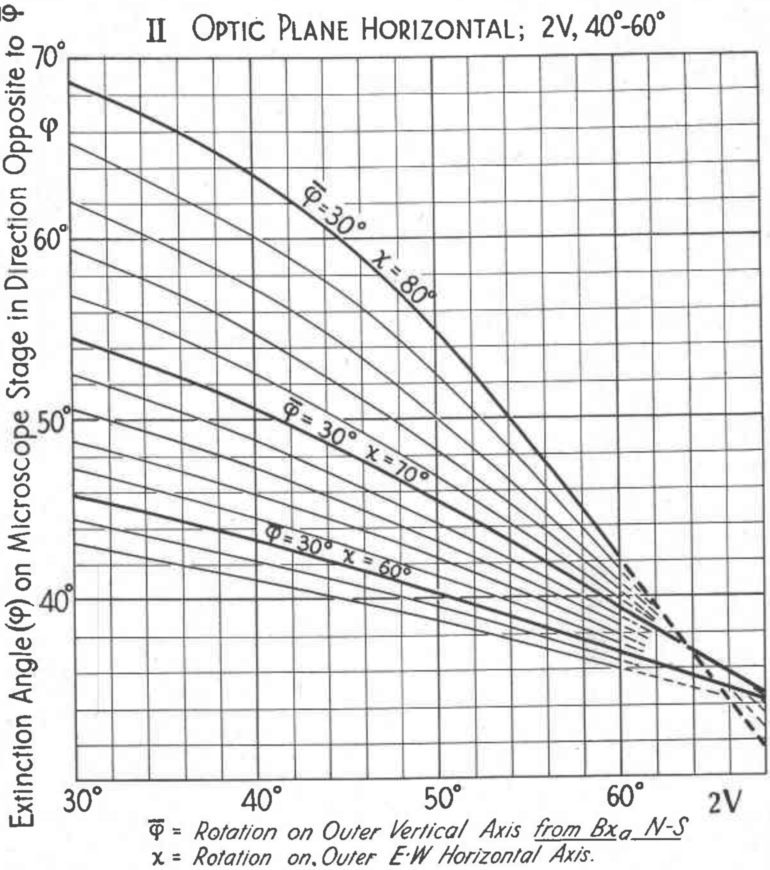

PLATE II. Graph for the determination of 2V. Rotations $\bar{\varphi}$, x, and ϕ are made in this order. Make rotation x as large as possible up to 80°. For 2V greater than 64° the curves from $x=70°$ to $x=80°$ are reversed.

It is well to notice that the extinction angle ϕ must be read carefully. If the value of ϕ is near the borderline between two different optic orientations, the following precautions are necessary. If ϕ is near 15° or 75°, in order to be sure whether the acute bisectrix is vertical or horizontal, direct reading should be made as described below. If ϕ is near 45°, in order to determine whether the acute bisectrix is northwest or northeast, rotation x on the outer east-west axis should be made as great as possible. An extinction value ϕ greater than 45° will prove that Bx_a is northeast,

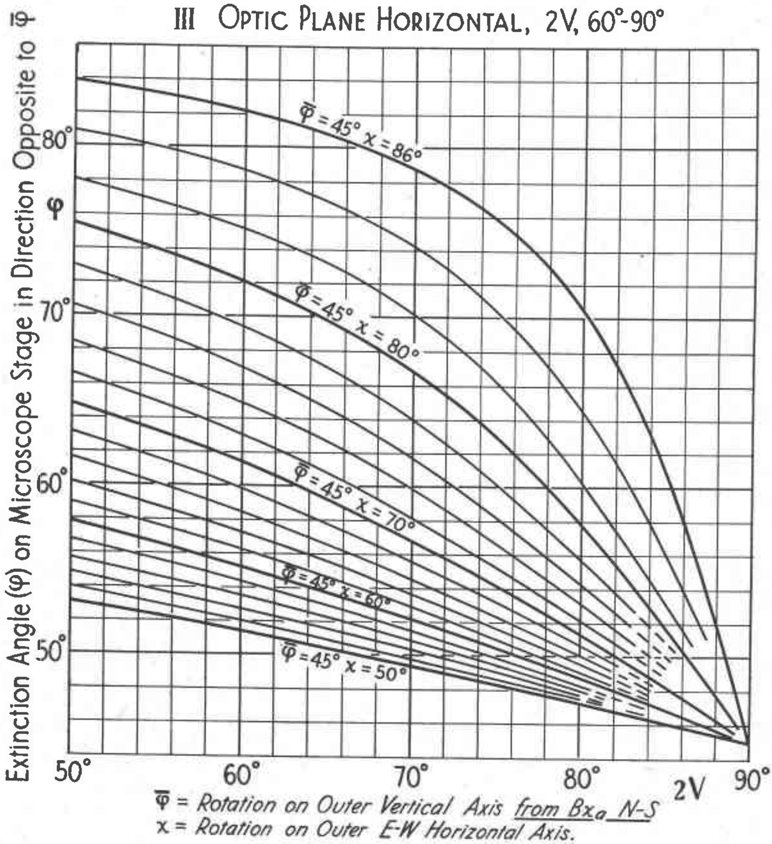

PLATE III. Graph for the determination of 2V. Rotations $\bar{\varphi}$, x, and ϕ are made in this order. In ordinary work make rotation x as large as possible up to 74°. If crystal is very carefully oriented and extinction is sharp, make rotation x as large as possible up to 86°.

and one less than 45°, that it is northwest. Unless these precautions are followed in these special cases, a wrong determination of sign may result, and in the first instance, a wrong determination of 2V. If ϕ is near 30° or 60°, there is no precautionary measure other than great care exercised in the performance of the regular procedure.

Before proceeding to the second step the optic sign is determined in the reference position, in which the orientation of the optic plane and acute bisectrix is known. The outer east-west axis

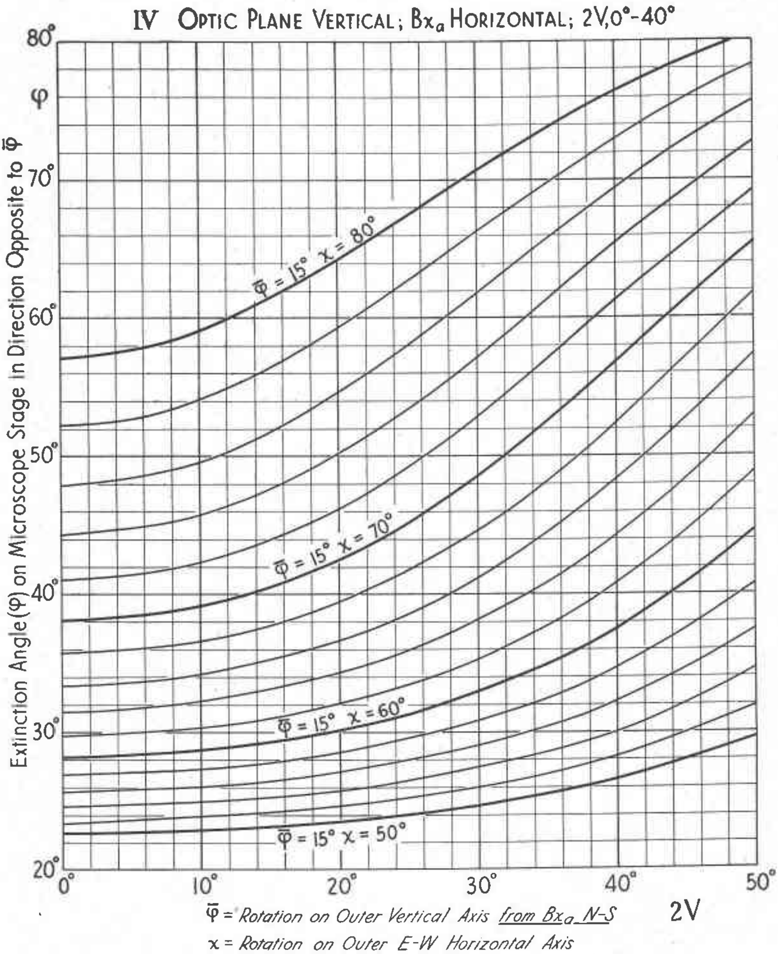

PLATE IV. Graph for the determination of 2V. Rotations $\bar{\varphi}$, x, and ϕ are made in this order. For 2V less than 30° make rotation x as large as possible up to 78°. For 2V greater than 30° make rotation x as large as possible up to 70°. For 2V less than 15° the curves give inaccurate results.

and the microscope stage are returned to the zero reading, and an accessory plate is used. If there is any doubt concerning the result obtained from Fig. 1, the precautionary measures above described should be taken before determining the sign of the mineral.

After the determination of sign *the acute bisectrix or optic plane*

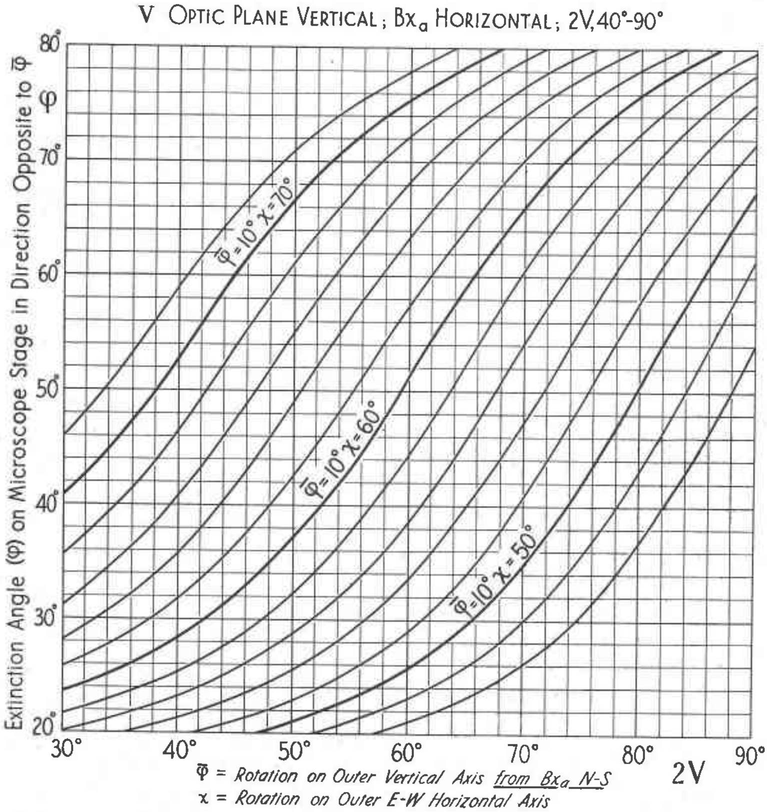

PLATE V. Graph for the determination of 2V. Rotations $\bar{\varphi}$, x, and ϕ are made in this order. The following rotations of x should be made for the various ranges of 2V.

2V=80°–90°, x=48°;
2V=70°–80°, x=52°;
2V=60°–70°, x=58°;
2V=50°–60°, x=62°;
2V=40°–50°, x=68°;

is made north-south by suitable rotation on the outer vertical axis. If, in step one, the acute bisectrix was found to be vertical, or horizontal provided 2V is large, the optic angle can be read directly by turning to the forty-five degree position on the microscope stage and measuring the angle between extinction positions reached by rotation from zero in one or both directions on the outer east-west axis.

If the direct reading is impossible, which will be the case when the optic plane is horizontal, or when it is vertical provided Bx_a

is horizontal and 2V is small, the particular plate indicated by Fig. 1 is used for the accurate determination of 2V. As in step one there are three rotations: $\bar{\varphi}$ on the outer vertical axis *from the north-south position*, x on the outer east-west axis, and ϕ to extinction on the microscope stage *in the direction opposite to* $\bar{\varphi}$. The values of $\bar{\varphi}$ and x to be used are given in the directions accompanying each plate. The extinction angle ϕ must be measured carefully, and it is well to average readings over 360°. With the given value of ϕ as ordinate a point is obtained on the curve corresponding to the value of x corrected for hemisphere index. Ordinarily it will not coincide with any of the curves given, but its position is readily found by interpolation. The size of the optic angle is the abscissa value of the point.

Example

The procedure used in the determination of 2V in a particular topaz section will serve as a specific example and will indicate the order of accuracy commonly obtainable. A crystal of topaz was cut parallel to the base, which is normal to the acute bisectrix, and the optic angle accurately determined as 66° by means of a Fuess rotation apparatus. A section of the same crystal cut parallel of a prism face was oriented on the universal stage and turned to the reference position, that is $\bar{\varphi}=45°$. Knowing the index of topaz to be a little less than that of the hemisphere (1.649), a rotation x of 54° (instead of 54.7°) was made on the outer east-west axis. The crystal was then turned counterclockwise to extinction through an angle of 22°. With this value it is found on Fig. 1 that the optic plane is vertical and the acute bisectrix horizontal and northwest in the reference position, that 2V is approximately 65°, and that Plate V should be used for the second step. Returning to the reference position by rotation to the zero readings on the outer east-west axis and the microscope stage, the sign of the mineral was determined by insertion of the gypsum plate with slow ray northwest. A rise in color showed the mineral to be positive since the acute bisectrix was northwest.

By a rotation of forty-five degrees clockwise on the outer vertical axis, the optic plane and acute bisectrix became north-south. Referring to Plate V it is found that for an optic angle of about 65° rotations $\bar{\varphi}=10°$ and $x=58°$ should be made. The crystal was turned, therefore, ten degrees (*counter-clockwise*) on the outer vertical axis and fifty-eight degrees (south) on the outer east-west axis. A value for ϕ of 58° was obtained by rotating *clockwise* to extinction on the microscope stage. Before using Plate V the value $x=58°$ must be corrected. This is done by use of a Fedorov net[2] here represented by Fig. 2. Point *a* is the location of the normal to the inner stage in the oriented position (inner east-west axis 3.5°N; north-south axis 22°E.). Point *b* is its location after the rotation $\varphi=10°$. Point *c* is its location after rotating *x* fifty-eight degrees south. The path followed is an arc convex toward the center

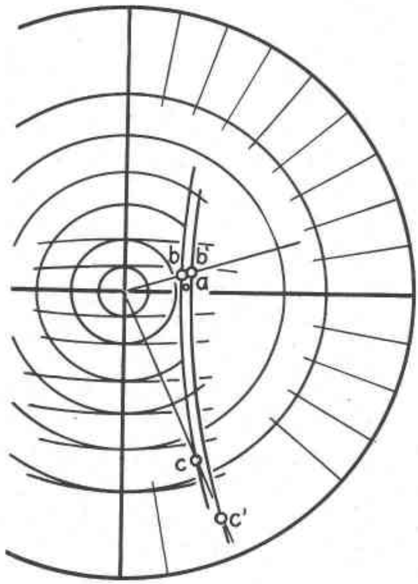

FIG. 2. Key to the use of the Fedorov net in correcting value of rotation *x*.

of the figure and the 58° is measured on the arcs of the great circles intersecting this path. Point *b* is seen to be 22° (as measured on the circles in the plane of the diagram) from the center along a radial, and point *c*, 57°. These two values are corrected for difference in index between the hemisphere and mineral by means of von Fedorov's diagram.[3] The index of the hemisphere is 1.649. The index to use for the mineral will be different in the two cases.

[2] Albert Johannsen, *Manual of Petrographic Methods*, pages 15 and 17, sec. ed., McGraw-Hill Book Co., **1918.**

[3] R. C. Emmons, A Modified Universal Stage: *Am. Mineral.*, Vol. **14,** pp. 441–461, 1929.

For point b the high index (1.625) is used because for that position of the normal to the inner stage Z is approximately north-south and horizontal. For point c the mean index (1.618) is used because in that position an optic axis is not far from vertical. Thus the values 22.5° and 58.5° are obtained for points b' and c', respectively. Measuring now the angle between b' and c', the true value, $x=60°$, is obtained (In Fig. 2 the distance $b'c'$ is greatly exaggerated).

With 58° as ordinate a point on the curve $x=60°$ gives a value of 64° for the optic angle. Other determinations varied an equal amount on either side of the correct value of 66°.

Accuracy

Any analysis of the accuracy of this method of determining 2V, aside from empirical observation, is complicated by the variability of different minerals as regards sharpness of extinction in different directions. However, this factor can to a certain extent be taken into consideration, and a reasonably good estimate of accuracy can be made without regard to it. Such an estimate is indispensable to an intelligent use of the procedure.

Estimation of the effect of sharpness of extinction upon accuracy can at best be only qualitative but is of some use. Thus, if the extinction angle ϕ is determined for an orientation in which an optic axis is nearly vertical, the result will not be as accurate as if ϕ were determined for an orientation with the optic plane nearly horizontal. In curves where errors in ϕ produce large errors in 2V, the orientation of the optic plane and the optic axes with reference to the axis of the microscope will have, therefore, great effect upon the accuracy of the determination of 2V.

From an examination of the curves themselves two important sources of error are apparent: inaccurate determination of ϕ, and inaccurate rotation x. With reference to the first of these it is apparent that for a curve making an angle of forty-five degrees with the 2V axis an error of one degree in ϕ results in an error of one degree in 2V. If this angle is greater or less than forty-five degrees, the corresponding error in 2V will be less or greater than unity. As to the second source of error, inaccuracies in x may arise from imperfect orientation of the crystal. To illustrate, if the east-west optic symmetry plane were tilted one degree north or south in the oriented position instead of being vertical, there would be an error in rotation x of almost one degree if rotation $\bar{\varphi}$ were small. Where the curves of Plates I–V are far apart along lines parallel to the 2V axis, small error in x gives a large error in determining 2V.

It is necessary, therefore, in using this procedure to keep in mind these three factors: sharpness of extinction, relation of error in ϕ to error in 2V, and relation of error in x to error in 2V. An example will serve best to clarify this point. If Fig. 1 indicates that the optic plane is horizontal and 2V is about 80°, Plate III is used with a value for $\bar{\varphi}$ of 45° while x may have any value from 50° to 86°. Obviously 50° is a poor choice since an error in ϕ of one degree gives an error in 2V of four degrees. For the curve $x=86°$ a one degree error in ϕ gives a half a degree error in 2V. However, there are the two other factors to consider. First, for $x=86°$ and $2V=80°$, an optic axis is very nearly vertical, so that it will be difficult to determine ϕ accurately. More important is the fact that an error in x of one degree gives an error in 2V of two degrees. Unless, therefore, the orientation of the crystal is very good and the extinction is very sharp, less accuracy will be obtained by a rotation of 86° than by one of 74° where an error in x of two degrees gives an error in 2V of one degree, and where an optic axis is less nearly vertical, even though an error of one degree in ϕ gives an error in 2V of one and one-half degrees. The relative importance of these factors in the choice of x is not easy to determine and varies with different minerals and different workers, but a general comprehension of their significance greatly enhances the accuracy of a 2V determination by this procedure.

The actual accuracy usually obtained varies principally with the size of the optic angle. In general the larger the value of 2V, the greater the accuracy. For an optic angle less than twenty degrees an error of ten degrees is not excessive. For values of 2V above twenty degrees careful work yields results within two to three degrees and occasionally within one degree.

By direct reading an error less than two degrees is usually possible. Berek's "Spezialdiagramm" contains curves for the orientation in which the acute bisectrix is vertical, but for this case direct reading is considerably more accurate, and no such curves are presented here. However, if only the half angle, V, can be measured directly as is ordinarily the case with the standard Fedorov instrument, it is doubtful whether direct reading is as accurate as indirect except when the acute bisectrix is vertical.

The loci of the curves themselves are more accurate than experimental determinations require. Points have been plotted every four to ten degrees, and errors in single points are not important.

Derivation of the Curves

The above outlined method for the graphical determination of 2V is based upon the fact, derived from trigonometric relationships and the Biot-Fresnel rule of crystal optics, that for a given random position of the optic plane of a biaxial crystal the size of the extinction angle in a given direction is a function of the size of the optic angle. If this random position of the crystal is known, it is possible, therefore, to determine the optic angle from the extinction angle about a given axis. With the universal stage where rotations are made upon axes lying in planes of optic symmetry, the problem of obtaining an expression for this relationship is greatly simplified.

Thus for the case in which the optic plane is horizontal, the relation between ϕ and 2V is the following:

$$\phi=\frac{K+K'}{2}$$

where

$$\tan K=\tan\ (\bar{\varphi}+V)\cos x$$
$$\tan K'=\tan\ (\bar{\varphi}-V)\cos x$$

These equations were used in plotting the curves of Fig. 1 and Plates I–III.

For vertical optic plane (Plates IV &V) the following equation employed by Berek was used.

$$\sec V=\sqrt{\sin^2\bar{\varphi}+\frac{\cos\ 2\bar{\varphi}}{\sin^2 x}-\frac{\cos\ x}{\sin^2 x}\sin\ 2\bar{\phi}\ \mathrm{cotg}\ 2\phi}$$

The choice of curves was made with a view to combining accuracy and simplicity. It was based mainly upon graphical trials and the curves used by Berek.