CRYSTAL CLASSIFICATION AND SYMBOLISM

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Abstract

In Figure 1 crystals are classified according to increasing symmetry into two phyla (axial and axihedral), three divisions (trimetric, dimetric, monometric), six systems, seven systems and subsystems, seven families (monaxial, polyaxial, anastrephaxial, orthaxihedral, monaxihedral, mesaxihedral, and polyaxihedral), fourteen orders (rows), and thirty-two point groups or classes. This would seem to offer a wide range of choice which might reasonably satisfy all or at least most wants outside of those concerned with crystal structure itself. In addition attention is called to three diagonal rows, which however embrace only 20 of the crystal classes. The Mauguin system of symbolism for both space and point groups is briefly recapitulated, as is the Schoenflies method. The former is recommended.

Introduction

This paper has as its main object the presentation of a classification of crystals in which these solids are so arranged that existing symbolism and nomenclature are more readily comprehended, especially by the beginning student. Apparently nearly every crystallographer (mineralogist, physicist, chemist, metallographer, etc.) who has given much thought to the matter, has had some unique ideas as to the arrangement and designation of crystal classes. The former is of relatively minor significance; the latter is unfortunate, as it tends to cause real confusion.

Present Classification

The chart (Fig. 1) is divided into seven columns on the basis of total symmetry, and into fourteen rows based on the type of symmetry of the (unique) crystal axis. The columns are in two major groupings **(phyla),** depending on whether axes only **(axial)** or both axes and planes **(axihedral)** are present.[1] The rows are in three major groupings **(divisions),** giving the most useful classification of crystals from the optical point of view.

Each column includes three to six crystal classes; this grouping of classes is of morphological value, and so names are given to these **families,** as shown at the top of each column in Fig. 1. The columns are in accord with Schoenflies' symbolism (11, 148–149)[2] as is indi-

[1] The monoclinic clinohedral class (Cs = m) is in the former, since its symmetry can be regarded either as a two-fold inversion axis or as a plane (but it can hardly be considered to be axihedral).

[2] Numbers in parenthesis refer to titles (and page numbers where appropriate) listed at the conclusion of this article.

cated. The names are similar to those applied by Swartz (12); the significance of each is given in the chart.

Each crystal **system** (or **subsystem**) embraces two rows **(orders)**,[3] except there is but one for the orthorhombic and there are three in the isometric. Four rows have but one class each, and six have but two; two rows have three classes each, and two have five. The characteristic symmetry marking each row is shown on the left; it is the same for rows 4 and 5, which are separated because of systematic differences. Omitting the first three rows, each of which has but one class, the number of classes in each pair of rows for 2-, 4-, 3-, and 6-fold axes is 5, 7, 5, 7, 5; the last figure is for the three isometric rows.

This results in 89[4] "boxes" (rectangles), 32 of which represent crystal **classes** or **point groups.** The data given in each of the rectangles are explained at the base of the chart on the left. Excepting the isometric system, five classes may be considered to belong in each family (one for each of a 1-, 2-, 3-, 4-, and 6-fold simple or inversion axis), but duplication and non-crystallographic symmetry reduce this number of 35 to 27, as is explained below.

Schoenflies (11) has three non-isometric families: cyclic (C), dihedral (D), and sphenoidal (S). The first with a single (simple) symmetry axis has three subdivisions which include those classes without planes (C_n), those with a single (horizontal) plane (C_n^h), and those with several (vertical) planes (C_n^v). The dihedral family with symmetry one principal n-fold axis and n 2-fold axes normal to it also has three subdivisions: those classes without planes (D_n), those with diagonal planes (D_n^d), and those with both vertical and horizontal planes (D_n^h). Moreover D gives way to T (tetrahedral) or O (octahedral) in the isometric, the only system with more than one axis of greater than 2-fold symmetry[5] [and D may be shown as

[3] To save space, and since combining rows 1 and 2 makes no difference in the sequence of numbering the crystal classes, these are shown as a single row numbered 1–2 in Fig. 1. Class 1 is in row 1 and class 2 is in row 2. Names instead of numbers may be used for the rows as is done for the columns (families); thus row 10 becomes the hexagonal inversion order, row 8 the rhombohedral order, etc.

[4] Boxes III–10 and IV–10 are combined into a single one for reasons pointed out later. Since boxes IV–1 and V–1 are similarly combined, what is referred to as rows 1 and 2 in Fig. 1 contains but 6 boxes.

[5] Since only families with multiple symmetry elements (at least three each of at least two types of symmetry) can be represented in the isometric, it is clear why Cn, Sn, and C_n^h are not so represented. Similarly C_n^v is not represented because while it

V (vierer) or Q (quadratic) in the orthorhombic and in the scalenohedral class of the tetragonal system]. The sphenoidal family (S refers to an alternating axis) has two analogous subdivisions (see footnote 14). The writer's families thus agree with Schoenflies' sub-divisions, but his two major groupings (axial and axihedral) contrast with those of Schoenflies.

Monaxial family (column I of Fig. 1) includes those classes with but a single (simple) rotation axis of symmetry and with no other symmetry. Of necessity such axes are polar, and the crystals may be regarded as hemimorphic.[6] Rotatory polarization as well as pyro-and pïezo-electric phenomena are possible in the crystals of this family. Since the only simple axes occurring in crystals are 2-, 3-, 4-, and 6-fold, these four classes and the one with no symmetry at all (which may be regarded as having a simple 1-fold rotation axis) complete this simplest family. The Hermann-Mauguin symbol[7] (7) for each of these classes is the same as the expression of the total symmetry by this system of notation, the principles of which are explained in the lower right portion of Fig. 1.[8]

Polyaxial family including those classes with several symmetry axes, but lacking planes or center, contains all enantiomorphous[9]

may be considered as polysymmetric, its symmetry is "one-dimensional" (all elements parallel one direction), not "three-dimensional" as required by the isometric. Thus only three families can have isometric representatives, and since there are only three types of "principal axis" (Fig. 1) present in this system, and one of these is an inversion axis, it is clear that at most two isometric classes can occur in any one family, and but one can occur in that polysymmetric family (mesaxihedral) characterized by the presence of an inversion axis.

[6] Hilton (5, 92) includes only those crystals in class no. 4 ($C_2=2$) of Fig. 1 of the monaxial family as hemimorphic, although the symmetry axis in each of the monaxial classes is polar. If hemimorphic forms are limited to those which may be regarded as but half (one end) of the corresponding holohedral forms, then monaxial classes of the dimetric division are not hemimorphic; Dana's *Textbook of Mineralogy* (1932 ed. by W. E. Ford, pp. 103, 118, 131) states that they are hemimorphic.

[7] This type of symbol, first proposed by C. Hermann, was greatly simplified and owes its present form to the efforts of Ch. Mauguin.

[8] W. Soller (*Am. Mineral.*, vol. **19**, p. 412, 1934) unfortunately suggests that the Mauguin symbol for the even-numbered inversion axes be used for alternating axes instead; the rule of priority, if no other, prohibits the acceptance of this suggestion.

[9] Hilton (5, 92) does not list class no. 28 (T=23) of Fig. 1 as containing enantiomorphous forms. Tutton (14, 131) lists the eleven classes of the monaxial and polyaxial families as enantiomorphous, as does Jaeger (6, 79), who links this property with optical rotatory power, at least in most cases (pp. 256, 261–262, 268). If screw axes are essential for optical rotatory power, representatives of class 1 and of certain space groups (see Table 3) of all other monaxial and polyaxial classes

forms. Rotatory polarization is also possible in the crystals of this family, as it is among those of the monaxial family, though difficult to observe in crystals of those classes of these families belonging in the trimetric division. Representatives of classes Nos. 17 and 28 may show pyro-and piezo-electric effects. The non-isometric crystals of the polyaxial family are characterized by one n-fold axis with n 2-fold axes normal to it. If $n=1$, the symmetry is that of the monoclinic sphenoidal class (C2 = 2); thus only four non-isometric classes appear in this family.

The **Hermann-Mauguin symbol** for each of the polyaxial classes (except D2 = 222) is a simplification of the expression for the total symmetry, as is indicated in Fig. 1. Mauguin (7, 545) does not use superscripts as shown in parentheses in certain rectangles of Fig. 1 to indicate the total number or amount of any given kind of symmetry element present. In the dimetric division his symmetry elements are listed in this order: vertical axis, horizontal axis and/or (horizontal or vertical) planes, if any; where there are 2-folds normal to the planes (and therefore centers of symmetry) this is not specifically indicated by Mauguin's slightly abbreviated symbolism which drops the 2; this is also true in the orthorhombic and isometric systems. In the isometric Mauguin lists the symmetry elements in this order: axes parallel cube edge, cube diagonal, and cube-face diagonal; 4-folds normal to planes (class No. 32 = Oh = m 3 m) or 2-folds in the third place (class no. 31 = 0 = 43) are not indicated in the abbreviated point-group symbolism. Each isometric point-group has the numeral **3** as the *second* unit of the point-group symbol, whereas **3** is the *first* unit in the symbol of each of the five rhombohedral classes, none of which has a horizontal plane of symmetry.

Anastrephaxial[10] **family** (sphenoidal of Schoenflies) includes those classes with but a single inversion axis of symmetry (rotary inversion); these classes can also be derived in terms of single alternating axes of symmetry (rotary reflections). In fact an inversion 4-fold gives the same results as an alternating 4-fold; otherwise the correspondence is less obvious, as shown by Table 1.[11] The

must lack this property. Screw axes also occur in various space groups of D2d = $\bar{4}$2m and of all orthaxihedral classes except C3h = $\bar{6}$. See F. Bernauer: Gedrillte Kristalle, 1929.

[10] Term derived from the Greek *anastrepho* (turn upside down, invert) and *axon* (axis), with the kind assistance of Professor G. E. Smith.

[11] Classes developed from only the even-numbered inversion axes form the

TABLE 1. CORRESPONDENCE BETWEEN ALTERNATING AXES AND INVERSION AXES

Alternating Axes (Sn) (rotary reflections)	Inversion Axes (C_n^i) or (nc) or ($\bar{n}$) (rotary inversions)
S2	$Ci=C_1^i=\bar{1}=i$ (inversion or center)
S=S1	$2c=\bar{2}=m$ (plane)
S6	$C_3^i=\bar{3}$
C'_4=S4	$4c=\bar{4}$
S3	$6c=\bar{6}=3/m$(3-fold with normal plane)

writer prefers the inversion axes because $\bar{3}$ (3-fold inversion axis) is found in the rhombohedral subsystem, $\bar{6}$ in the hexagonal subsystem, whereas the reverse is true if alternating axes are used, the tendency then being to hide the true symmetry relationships as is shown by the fact that some writers include hexagonal classes in the rhombohedral and vice versa. Moreover, if alternating axes are used, putting what corresponds to the anastrephaxial family in the axial phylum might seem inconsistent.[12] The Herman-Mauguin symbol for each of the anastrephaxial classes is the same as the expression of the total symmetry by this system of notation, as is true for the monaxial family. Pyro- and piezo-electric phenomena are possible in the crystals of class no. 3 (Cs=m) of this family. As those other classes in this family in which n is odd have a center of symmetry, their representatives cannot be expected to exhibit such phenomena (6, 101).

Orthaxihedral family, each class of which has but one axis normal to one plane, has but four classes, as where $n=1$ the symmetry is the same as that of the monoclinic clinohedral class (Cs=m), placed by preference in the anastrephaxial family. In the classes of this family in the dimetric division, as in those of other families that have both axes and planes of symmetry, are the di-forms, except disphenoids. Orthaxihedral classes except

groups nc (17, 15) and lack a center of symmetry. Classes derived from only the odd-numbered inversion axes have a center of symmetry, and Rogers (10, 172) prefers to refer to these only as alternating axes ("rotoflections"). Only the latter (anastrephaxial) classes are given the symbol C_n^i, which may also be used for orthaxihedral classes nos. 5, 13, and 25 of Fig. 1 (17, 15). All these symbols have been conveniently summarized by Davey (3, 218–221).

[12] Of course the "alternating planes" are present, whether the symmetry is regarded as alternating axes or inversion axes. Classification is primarily for purposes of convenience in bringing out certain relationships which one wishes to emphasize. Therefore it is considered permissible to put this family where it is in Fig. 1. Swartz (12) chose to stress alternating axes; therefore used the term *amebaxial.*

class no. 5 ($C_2^h = 2/m$), the holohedral of the monoclinic system, are paramorphic, as is also class no. 29 (Th = m3) of the isometric (5, 92). Orthaxihedral classes in which *n* is an even number have a center of symmetry.[13]

Monaxihedral family which contains classes with several planes of symmetry meeting in one axis of symmetry, has but four classes, as where $n = 1$ there could be but one plane, which leads to the monoclinic clinohedral class (Cs = m) of the anastrephaxial family. The axis is polar and the representatives, which may show pyro- and piezo-electric effects, are hemimorphic (cf. monaxial family), as are those of class no. 30 (Td = $\bar{4}$3m) of the mesaxihedral family, according to Hilton (5, 92). Ordinary polar axes are not confined to the monaxial and monaxihedral families, but are also found in classes nos. 17 (quartz), 22 (benitoite), 28 (cobaltite), and 30 (sphalerite) of Fig. 1. Representatives of all these (as well as those of class no. 3 = Cs = m) may exhibit pyro- and piezo-electric phenomena. The Mauguin symbol (mm) for class no. 7 (C_2^v) of the monaxihedral family does not directly show the 2-fold axis present, since if a crystal has but two perpendicular planes of symmetry, their line of intersection must be a 2-fold axis.

Mesaxihedral family, including those classes with both planes and symmetry axes, but with no plane containing more than one symmetry axis, and all planes lying midway between (Greek *mesos*) symmetry axes, embraces but two non-isometric classes, as is indicated in Fig. 1. Where $n = 1$, this family produces class no. 5 ($C_2^h = 2/m$), already placed with the orthaxihedral family. Where $n = 2$ and 3, this results in classes nos. 10 and 20 of the mesaxihedral family, whose principal axes have symmetries of $\bar{4}$ and $\bar{3}$ respectively, higher than those of the starting symmetry because of the demands of the planes and the other axes. Similarly where $n = 4$ and 6, this leads to principal axes of alternating 8- and 12-fold symmetry respectively, axes non-existent in crystals.

The mesaxihedral family thus bears a close relation to the anastrephaxial family; in each class in either family the principal axis is an inversion axis, or may also be regarded as an alternating axis.[14]

[13] It should be pointed out that class no. 21 ($C_3^h = \bar{6}$) may be placed equally well with either the anastrephaxial or orthaxihedral families as is indicated in Fig. 1. Schoenflies preferred it in the latter as shown by his choice of symbol (C_3^h instead of S3). Mauguin's choice of symbol ($\bar{6}$ in place of 3/m) indicates the reverse, probably because he wished to emphasize its hexagonal symmetry. It is the only non-Triclinic class represented by but a single space group.

[14] Thus Schoenflies gave S_4^u as alternative for D_2^d and S_6^u as alternative for D_3^d,

In no other classes are inversion axes present as crystal axes, except in class no. 22 (benitoite)[15] of the polyaxihedral family [and also excepting class no. 21 ($C_3^h=\bar{6}$), included in both orthaxihedral and anastrephaxial families of Fig. 1]. The classes of the anastrephaxial and mesaxihedral families having n an odd number contain a center of symmetry. Representatives of those lacking a center of symmetry (excepting the tetragonal classes) may exhibit pyro- and piezo-electric phenomena.

Polyaxihedral family with several planes of symmetry each of which includes at least two axes of symmetry (four in class no. $32=Oh=m3m$) embraces but four non-isometric classes, as where $n=1$, this leads to class no. 7 ($C_2^v=m\ m$) in the monaxihedral family. The family includes four holohedral classes. As is true of the classes of the orthaxihedral family, all polyaxihedral classes with n an even number have a center of symmetry, as do the two isometric classes of this group. The latter are therefore the only isometric classes having inversion 3-folds. Representatives of class no. 22 ($D_3^h=\bar{6}m$) are the only ones which may exhibit pyro- and piezo-electric phenomena.[15a]

General Discussion

It will be noted that twenty of the classes lie on three **diagonal rows** trending northwest-southeast (map parlance) in Fig. 1. These diagonal rows consist of classes numbered 1,[16] blank, 3, 5, 7, 10,

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S_n signifying a rotary reflection and the u (Umklappung) referring to rotation about the 2-fold axes.

[15] Class no. 22 ($D_3^h=\bar{6}m$) is here put in the polyaxihedral family because each of its planes *contains* two symmetry axes; i.e., it is *not* mesaxihedral (see footnote 18). Mauguin (7, 545) chose $\bar{6}\ 2\ m$ as its symbol, which is analogous to the symbols of the mesaxihedral classes. Bernal et al (1, 529) have since abbreviated this to $\bar{6}$ m. Had the symbol 3/m 2 m (or 3/m m) been chosen, this would have tended to hide the hexagonal nature of the class, but the present symbol serves to mask its true family relations. Bragg (2, 86) puts $\bar{6}$ m with $\bar{4}$ 2 m (no. 10 of Fig. 1) forming the groups n d (17, 15) and with $\bar{4}$ 3 m (no. 30 of Fig. 1), and places $\bar{3}$ m (no. 20 of Fig. 1) in his last column, which thus consists of five holohedral classes (all except monoclinic and triclinic, assuming a holohedral rhombohedral class) forming the groups D_n^i (17, 15) plus $Oi=Oh$.

[15a] Willi Kleber has recently (*Centr. Min., Geol. u. Pal.* A(9), pp. 241–250, 1934) derived the 32 classes by using the stereographic projection. He puts them in six families: cyclic (5 classes), dihedral (4-none isometric), gyroidal (3 plus $C_3^h=\bar{6}$), spiegel (14 plus $C_3^h=\bar{6}$), tetrahedral (3), and octahedral (2). Cyclic is monaxial; dihedral is polyaxial lacking isometric representatives; gyroidal is anastrephaxial less $Cs=m$; tetrahedral and octahedral embrace the isometric classes; and spiegel includes all the rest.

15–4, 6, 9, 13, 18, 20, 22–11, 17, 19, 21, 25, blank, 30, 32. Each diagonal row is separated from its neighbor by two boxes (going vertically), except the right hand part of the lowest diagonal row is shifted one box lower because of the intermediate nature of class no. 21 ($C_3^h = \bar{6}$). These three diagonal rows sweep across the whole chart from class no. 1 to class no. 32. The major significance of these diagonal rows seems to be that they indicate a proper sequence of class arrangment; that is, as one proceeds downward across the rows (orders), thus in general increasing the symmetry of the "Principal Axis" (Fig. 1), one also goes with equal regularity to the right from one family (column) to the next, thus more or less automatically adding on other (consequential) elements of symmetry.

While rows 3, 4, and 5 can be collapsed into one, as can rows 6 and 7, 8 and 9, and 13 with 12 or 14, the diagonal rows are then destroyed. Suppose the tetragonal classes (rows 6 and 7) are put intermediate between the rhombohedral and hexagonal classes, as has been done by some;[17] then the diagonal row symmetry is almost completely removed. The polyaxial family can be bodily interchanged with the anastrephaxial family without even altering the class numbers; in fact the writer's first charts did this. But then the diagonal symmetry is broken.[18] Although several simple axes (polyaxial family) may seem to some to be more complex

[16] What is shown as the top row in Fig. 1 in reality consists of two rows, as is indicated by the numbering on the right.

[17] The basis for this is presumably the fact that the 3-fold type of axis is regarded as having a lower grade of symmetry than does the 4-fold. This is incorrect, as can be seen by comparing an alternating 4-fold with the simple and inversion 3-folds. By analogy then the inversion 6-fold (row 10) should be separated from the 6-fold (row 11) by the 4-fold (row 7), thus splitting the hexagonal subsystem itself. If rows are arranged in the order of 3, $\bar{4}$, $\bar{3}$, $\bar{6}$, 4, 6, there is one partial diagonal row, which however does not join with either trimetric or monometric classes. If put in the order 3, $\bar{4}$, $\bar{6}$, $\bar{3}$, 4, 6 there are three diagonal rows (two of them partial), two of which are "hanging," but the third and major one (which however is partial, having two blanks) joins class no. 29 (Th = m 3) of the isometric. Neither of these arrangements compares favorably with that of Fig. 1 from the point of view of diagonal rows. Moreover the very close relationship between hexagonal and rhombohedral subsystems is sufficient to preclude the desirability of separating them by the tetragonal system.

[18] A further note regarding the position in Fig. 1 of class 22 ($= D_3^h = \bar{6}m$) is here justified, as it will be noted that were this class put as straddling the mesaxihedral and polyaxihedral families, it would add one more class (no. 27) to the intermediate diagonal row; moreover both (all) the classes of the hexagonal inversion order would then be of this duplex nature. If the definition of the mesaxihedral family be changed to read "with all planes lying midway between *crystal* axes (which are *not*

than a single inversion axis (anastrephaxial family), this is not true, since the latter involves two kinds of symmetry, the former but one. In reality the anastrephaxial family is intermediate between families I and II and the families of the axihedral phylum; this is indicated by the intermediate position of class no. 21 ($C_3^h = \bar{6}$); by the fact that most morphologists think of class no. 3 (Cs = m) as having symmetry of but one plane, no axis;[19] and by consideration of the two kinds of symmetry present in the symmetry unit of each class of the anastrephaxial family. It may also be added that the eleven classes of the monaxial and polyaxial families which are in juxtaposition in Fig. 1 are the ones containing only symmetry elements of the first sort (involving nothing but simple rotations), and so correspond to those derived first by Schoenflies (11, 74).

Space Lattices and Space Groups

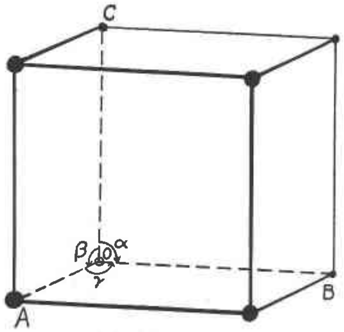
The advantages of the Hermann-Mauguin over the Schoenflies symbolism are not apparent from Fig. 1 or from any study limited to the point-groups (crystal classes). For that reason there is here added Tables 2 and 3 which show the extension of this symbolism to the 230 space groups. Table 2 summarizes the data regarding the fourteen Bravais space lattices and their five variants. In space group terminology no subscripts indicating the system as shown in the table are necessary, since the system of the lattice in question is apparent from the other symmetry indicated by the Mauguin symbolism.[20] There are taken to be but six primitive lattices, as is shown in Table 2, since the hexagonal lattice *Ch*

symmetry axes in class 22)" and if the qualification of "no plane containing more than one symmetry axis" be omitted, then class 22 would properly be regarded as belonging to both families. The position shown in Fig. 1 is however preferred, since the whole table is based on symmetry and not on crystal axes (where the two do not coincide), and it hardly seems wise to make an exception for a single class. Moreover some have made the *a*-axes coincide with the 2-folds in this class (e.g., see 18, 37; 12, **31**; and 16, II), although this is unfortunate, since it makes the first order forms hexagonal and the second order forms trigonal, contrary to the order in all other hexagonal classes. For examples of proper orientation see 10, 190; 16, III, VII; and Ford: Dana's *Textbook of Mineralogy*, 1932, p. 119. In addition with class 22 left as it is in Fig. 1 there is the normal number of five non-isometric classes in the polyaxihedral family (allowing for duplication), as is also true for the mesaxihedral family (remembering that two classes are missing from this family because of the non-occurrence in crystals of 8- and 12-fold alternating axes).

[19] Thus it has been called the anaxial class (*Am. Mineral.*, vol. **12**, p. 219, 1927).

[20] *Pa* (anorthic) is preferred for brevity to *Ptr* (triclinic). While *R* is the standard

(which is also primitive in the ordinary sense of the word) may be regarded as a special case of the (001)-centered orthorhombic (three pinacoids) lattice (*Co*) where the edges $a:b=1:\sqrt{3}$. The rotation of the *a*-axis of this lattice 30° produces the larger variant designated *H*.[21]



Primitive Lattice

TABLE 2. THE FOURTEEN BRAVAIS SPACE LATTICES

Six primitive lattices (*P*)

[Call OA = a, OB = b, OC = c]

	edges	*angles*	
Pa[*Ptr*]	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$	three pinacoids
Pm	"	$\alpha = \gamma = 90° \neq \beta$	three pinacoids
Po	"	$\alpha = \beta = \gamma = 90°$	three pinacoids
Pt	$a = b \neq c$	"	2° tetragonal prism and pinacoid
Pi	$a = b = c$	"	hexahedron
R[*Pr*]	"	$\alpha = \beta = \gamma \neq 90°$	rhombohedron

Three body-centered lattices (*I*)

Io corresponding to *Po*; *It* corresponding to *Pt*; and *Ii* corresponding to *Pi*.

Three lattices with a single centered (001) face (*C*) [plus four variants marked*]

Cm (cf. *Pm*); *Co* (cf. *Po*...variants are *Ao and *Bo which are the same except for orientation); *Ct (cf. *Pt* of which it is a variant); and *Ch* (special case of *Co* where $a:b=1:\sqrt{3}$...variant *H is similar except rotated 30°, so that $a:b=\sqrt{3}:1$; lattice *Ch* is same as that consisting of a 60° orthorhombic prism with basal pinacoid, three of which units form a hexagonal prism with centered basal pinacoid).

Two lattices with all faces centered (*F*); [plus one variant marked*]

Fo (cf. *Po*); *Ft (cf. *Pt*...variant of *It*); and *Fi* (cf. *Pi*).

. . . .

Notes: *Cm* and *Co* may be regarded as rhombic prisms with basal pinacoid. *Fo* is same as body-centered orthorhombic prism with basal pinacoid; it is also the same as an orthorhombic dipyramid. *Io* is same as orthorhombic brachy- and macrodomes. *Ct* is same as four units of *Pt*. *It* is same as a second order tegragonal dipyramid; rotating it 45° to the first order form (and translating it parallel *c* one-half the unit distance) leads to the variant *Ft*. *Fi* corresponds to the octahedron, or the rhombohedron with 60–120° face angles, four of which rhombohedra constitute a dodecahedron. *Ii* corresponds to the rhombohedron with face angle of 109° 28′.

symbol for the rhombohedral lattice, *Pr* might be preferred by some morphologists since it better indicates the analogy with the other primitive lattices.

[21] Schiebold (9, 32) uses *Ch* for this, and *Ph* for the lattice designated *Ch* by Mauguin. From the point of view of space group notation it is better to omit *P* from the designation of either hexagonal or rhombohedral lattices.

TRIMETRIC SPACE GROUPS

Space Group Number	Class No. Schoenflies Symbol	Schoenflies Number	Mauguin symbol: Normal orientation	Mauguin symbol: Other orientations
1	1–C1	1	P1	A1, B1, C1, F1, I1, . . .
2	2–Ci	1	P$\bar{1}$	A$\bar{1}$, B$\bar{1}$, C$\bar{1}$, F$\bar{1}$, I$\bar{1}$, . . .
3	3–Cs	1	Pm	Bm
4		2	Pc	Pa, Pn, Ba, Bd
5		3	Cm	Am, Im, Fm
6		4	Cc	Aa, Ia, Fd
7	4–C2	1	P2	B2
8		2	P2_1	B2_1
9		3	C2	A2, I2, F2
10	5–C2h	1	P2/m	B2/m
11		2	P2_1/m	B2_1/m
12		3	C2/m	A2/m, I2/m, F2/m
13		4	P2/c	P2/a, P2/n, B2/a, B2/d
14		5	P2_1/c	P2_1/a, P2_1/n, B2_1/a, B2_1/d
15		6	C2/c	A2/a, I2/a, F2/d
16	6–D2	1	P222	—
17		2	P222_1	P$2_1$22, P2$2_1$2
18		3	P$2_12_1$2	P2$2_12_1$, P$2_1$2$2_1$
19		4	P$2_12_12_1$	—
20		5	C222_1	A$2_1$22, B2$2_1$2
21		6	C222	A222, B222
22		7	F222	—
23		8	I222	—
24		9	I$2_12_12_1$	—
25	7–C2v	1	Pmm	—
26		2	Pmc	Pcm
27		3	Pcc	—
28		4	Pma	Pbm
29		5	Pca	Pbc
30		6	Pnc	Pcn
31		7	Pmn	Pnm
32		8	Pba	—
33		9	Pna	Pbn
34		10	Pnn	—
35		11	Cmm	—
36		12	Cmc	Ccm
37		13	Ccc	—
38		14	Amm	Bmm
39		15	Abm	Bma
40		16	Ama	Bbm
41		17	Aba	Bba

Space Group Number	Class No. Schoenflies Symbol	Schoenflies Number	Mauguin symbol: Normal orientation	Mauguin symbol: Other orientations
42		18	Fmm	—
43		19	Fdd	—
44		20	Imm	—
45		21	Iba	—
46		22	Ima	Ibm
47	8–D2h	1	Pmmm	—
48		2	Pnnn	—
49		3	Pccm	Pbmb, Pmaa
50		4	Pban	Pcna, Pncb
51		5	Pmma	Pmmb, Pmam, Pmcm, Pbmm Pcmm
52		6	Pnna	Pnnb, Pnan, Pncn, Pbnn Pcnn
53		7	Pmna	Pnmb, Pman, Pncm, Pbmn Pcnm
54		8	Pcca	Pccb, Pbab, Pbcb, Pbaa Pcaa
55		9	Pbam	Pcma, Pmcb
56		10	Pccn	Pbnb, Pnaa
57		11	Pbcm	Pbma, Pcam, Pcmb, Pmab Pmca
58		12	Pnnm	Pmnn, Pnmn
59		13	Pmmn	Pmnm, Pnmm
60		14	Pbcn	Pbna, Pcan, Pcnb, Pnab Pnca
61		15	Pbca	Pcab
62		16	Pnma	Pnam, Pbnm, Pcmn, Pmnb Pmcn
63		17	Cmcm	Ccmm, Amma, Amam, Bmmb Bbmm
64		18	Cmca	Ccma, Abma, Abam, Bmab Bbam
65		19	Cmmm	Ammm, Bmmm
66		20	Cccm	Amaa, Bbmb
67		21	Cmma	Abmm, Bmam
68		22	Ccca	Abaa, Bbaa
69		23	Fmmm	—
70		24	Fddd	—
71		25	Immm	—
72		26	Ibam	Ibma, Imaa
73		27	Ibca	—
74		28	Imma	Imam, Ibmm

DIMETRIC SPACE GROUPS

Space Group Number	Class Number Schoenflies Symbol	Schoenflies Number	Mauguin symbol: Normal orientation	Mauguin symbol: Other orientation
75	9–S4	1	P$\bar{4}$	C$\bar{4}$
76		2	I$\bar{4}$	F$\bar{4}$
77	10–D2d	1	P$\bar{4}$2m	C$\bar{4}$m2
78		2	P$\bar{4}$2c	C$\bar{4}$c2
79		3	P$\bar{4}2_1$m	C$\bar{4}$m2_1
80		4	P$\bar{4}2_1$c	C$\bar{4}$c2_1
81		5	C$\bar{4}$2m	P$\bar{4}$m2
82		6	C$\bar{4}$2c	P$\bar{4}$c2
83		7	C$\bar{4}$2b	P$\bar{4}$b2
84		8	C$\bar{4}$2n	P$\bar{4}$n2
85		9	F$\bar{4}$2m	I$\bar{4}$m2
86		10	F$\bar{4}$2c	I$\bar{4}$c2
87		11	I$\bar{4}$2m	F$\bar{4}$m2
88		12	I$\bar{4}$2d	F$\bar{4}$d2
89	11–C4	1	P4	C4
90		2	P4_1	C4_1
91		3	P4_2	C4_2
92		4	P4_3	C4_3
93		5	I4	F4
94		6	I4_1	F4_1
95	12–D4	1	P42	C422
96		2	P4$2_1$	C42$2_1$
97		3	P$4_1$2	C$4_1$22
98		4	P4_12_1	C$4_1$2$2_1$
99		5	P$4_2$2	C$4_2$22
100		6	P4_22_1	C$4_2$2$2_1$
101		7	P$4_3$2	C$4_3$22
102		8	P4_32_1	C$4_3$2$2_1$
103		9	I42	F42
104		10	I$4_1$2	F$4_1$2
105	13–C4h	1	P4/m	C4/m
106		2	P4_2/m	C4_2/m
107		3	P4/n	C4/a
108		4	P4_2/n	C4_2/a
109		5	I4/m	F4/m
110		6	I4_1/a	F4_2/d

TABLE 3.—THE 230 SPACE GROUPS (first part)

DIMETRIC SPACE GROUPS (concl.)

Space Group Number	Class No. Schoenflies Symbol	Schoenflies Number	Mauguin symbol: Normal orientation	Mauguin symbol: Other orientation
111	14–C4v	1	P4mm	C4mm
112		2	P4bm	C4mb
113		3	P4cm	C4mc
114		4	P4nm	C4mn
115		5	P4nc	C4cn
116		6	P4cc	C4cc
117		7	P4mc	C4cm
118		8	P4bc	C4cb
119		9	I4mm	F4mm
120		10	I4cm	F4mc
121		11	I4md	F4dm
122		12	I4cd	F4dc
123	15–D4h	1	P4/mmm	C4/mmm
124		2	P4/mcc	C4/mcc
125		3	P4/nbm	C4/amb
126		4	P4/nnc	C4/acn
127		5	P4/mbm	C4/mmb
128		6	P4/mnc	C4/mcn
129		7	P4/nmm	C4/amm
130		8	P4/ncc	C4/acc
131		9	P4/mmc	C4/mcm
132		10	P4/mcm	C4/mmc
133		11	P4/nbc	C4/acb
134		12	P4/nnm	C4/amn
135		13	P4/mbc	C4/mcb
136		14	P4/mnm	C4/mmn
137		15	P4/nmc	C4/acm
138		16	P4/ncm	C4/amc
139		17	I4/mmm	F4/mmm
140		18	I4/mcm	F4/mmc
141		19	I4/amd	F4/ddm
142		20	I4/acd	F4/ddc
143	16–C3	1	C3	H3
144		2	C3$_1$	H3$_1$
145		3	C3$_2$	H3$_2$
146		4	R3	—
147	17–D3	1	H32	C312
148		2	C32	H312
149		3	H3$_1$2	C3$_1$12
150		4	C3$_1$2	H3$_1$12
151		5	H3$_2$2	C3$_2$12
152		6	C3$_2$2	H3$_2$12
153		7	R32	—

Space Group Number	Class No. Schoenflies Symbol	Schoenflies Number	Mauguin symbol: Normal orientation	Mauguin symbol: Other orientation
154	18–C3v	1	C3m	H31m
155		2	H3m	C31m
156		3	C3c	H31c
157		4	H3c	C31c
158		5	R3m	—
159		6	R3c	—
160	19–C3i	1	C$\bar{3}$	H$\bar{3}$
161		2	R$\bar{3}$	—
162	20–D3d	1	H$\bar{3}$m	C$\bar{3}$1m
163		2	H$\bar{3}$c	C$\bar{3}$1c
164		3	C$\bar{3}$m	H$\bar{3}$1m
165		4	C$\bar{3}$c	H$\bar{3}$1c
166		5	R$\bar{3}$m	—
167		6	R$\bar{3}$c	—
168	21–C3h	1	C$\bar{6}$	H$\bar{6}$
169	22–D3h	1	C$\bar{6}$m	H$\bar{6}$2m
170		2	C$\bar{6}$c	H$\bar{6}$2c
171		3	H$\bar{6}$m	C$\bar{6}$2m
172		4	H$\bar{6}$c	C$\bar{6}$2c
173	23–C6	1	C6	H6
174		2	C6$_1$	H6$_1$
175		3	C6$_5$	H6$_5$
176		4	C6$_2$	H6$_2$
177		5	C6$_4$	H6$_4$
178		6	C6$_3$	H6$_3$
179	24–D6	1	C62	H62
180		2	C6$_1$2	H6$_1$2
181		3	C6$_5$2	H6$_5$2
182		4	C6$_2$2	H6$_2$2
183		5	C6$_4$2	H6$_4$2
184		6	C6$_3$2	H6$_3$2
185	25–C6h	1	C6/m	H6/m
186		2	C6$_3$/m	H6$_3$/m
187	26–C6v	1	C6mm	H6mm
188		2	C6cc	H6cc
189		3	C6cm	H6mc
190		4	C6mc	H6cm
191	27–D6h	1	C6/mmm	H6/mmm
192		2	C6/mcc	H6/mcc
193		3	C6/mcm	H6/mmc
194		4	C6/mmc	H6/mcm

ISOMETRIC SPACE GROUPS

Space Group Number	Class Number Schoenflies Symbol	Schoenflies Number	Mauguin symbol
195	28–T	1	P23
196		2	F23
197		3	I23
198		4	P2$_1$3
199		5	I2$_1$3
200	29–Th	1	Pm3
201		2	Pn3
202		3	Fm3
203		4	Fd3
204		5	Im3
205		6	Pa3
206		7	Ia3
207	30–Td	1	P$\bar{4}$3m
208		2	F$\bar{4}$3m
209		3	I$\bar{4}$3m
210		4	P$\bar{4}$3n
211		5	F$\bar{4}$3c
212		6	I$\bar{4}$3d
213	31–O	1	P43
214		2	P4$_2$3
215		3	F43
216		4	F4$_1$3
217		5	I43
218		6	P4$_1$3
219		7	P4$_3$3
220		8	I4$_1$3
221	32–Oh	1	Pm3m
222		2	Pn3n
223		3	Pm3n
224		4	Pn3m
225		5	Fm3m
226		6	Fm3c
227		7	Fd3m
228		8	Fd3c
229		9	Im3m
230		10	Ia3d

TABLE 3.—THE 230 SPACE GROUPS (second part)

Table 3 showing the 230 space groups is taken from Bernal et al (1, 525–530), rearranged in sequence to conform to the development of the crystal classes as shown in Fig. 1. In the Mauguin space group symbolism the initial capital letter shows the space lattice according to the terminology of Table 2. The other symbols correspond to the Mauguin point-group symbols as shown in Fig. 1 (lower right) with the addition of screw axes (shown by a numerical subscript to the axis symbol, with different subscript numbers for different types of screw axes) and glide planes. For the latter the letters a, b, c are used if the translation is $\frac{a}{2}$, $\frac{b}{2}$, $\frac{c}{2}$; n is used if the translation is $\frac{a+b}{2}$, $\frac{b+c}{2}$ or $\frac{a+c}{2}$, or from the corner to the center of a face parallel the glide plane; and d where it is $\frac{a+b}{4}$, $\frac{b+c}{4}$, or $\frac{a+c}{4}$, or one-quarter of a face-diagonal. From the Mauguin symbol of any one of the 230 space groups, the corresponding point group (crystal class) can be obtained by dropping the space lattice designation and the subscript numeral indicating a screw axis and substituting m for any of the letters indicating a glide plane.

Concluding Remarks

Scientists should make classifications their slaves, not the reverse. One[22] would have the systems as more fundamental than the classes, while others (10, 199 and 12, 383) hold out for the opposite. Much depends on the purposes to which a classification is to be put. The optical crystallographer is satisfied with the three divisions, as is the crystallographer working in many other physical fields. The working mineralogist rarely uses more than systematic splitting as an aid in non-instrumental mineral determination. The pure morphologist may find use for the 32 classes, but this is not universally true. The crystal structure worker needs the 230 space groups. Numerous other types of groupings have been proposed, as examination of the very limited bibliography here appended will prove. Swartz (12, 385–397) has published a very satisfactory brief history of the subject up to 1902, well worthy of perusal by the present-day student.

[22] *Am. Mineral.*, vol. **16**, pp. 26, 30, 1931.

The 32 classes may be developed using as symmetry operations only rotations, inversions, and the two combined (rotary inversions); or rotations, reflections, and the two combined (rotary reflections) will lead to their derivation.[23] Normally different types of classification will come from the two methods, as is demonstrated by numerous papers. Fig. 1 presents a classification which depends less upon the type of symmetry operation used to develop the classes than it does on the order of increasing inherent symmetry as one proceeds from class to class. While the term "anastrephaxial" implies that inversion axes are stressed, this is hardly true as comparison with Wyckoff (17, 15) will show. The term as well as one set of symbols conforms to those of Mauguin; the Schoenflies symbols including those of the alternating axes (Sn) are also given; no matter which are used the same results are reached in this type of classification.

So far as known the class numbers of Fig. 1 do not agree in all details with those of any other author. History indicates that the numbers here given will not meet with universal approval. The numbering of the space groups up to 230 as is done in Table 3 can have no greater significance than do the class numbers themselves. Unless general agreement can be reached on class number—as has been done on the numbering of space groups in any one class—names or symbols are to be preferred. In any case practically the only advantage of numbers is in ease of printing. The tremendous advantages in all other ways of the Mauguin space group symbolism, which in place of a numeral of no inherent significance puts a simple set of symbols giving the essential symmetry elements of the space group in question, and the extreme ease with which the corresponding point group symbol can be derived from this, warrants the rapid adoption of this system.

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[23] Failure to recognize this accounts for such statements as "the center of symmetry . . . is manifestly secondary" (12, 384–385), "the center of symmetry is a true (i.e., essential?—D.J.F.) element of symmetry" (10, 166), "both rotatory-inversions and rotatory-reflections must be used as symmetry operations" (10, 201), and "only two-, four-, and six-fold rotary reflection axes are possible" (Soller, *op. cit.*, p. 418).

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SYMMETRY / SYSTEM	Principal Axis	AXIAL=SYMMETRY AXES ONLY: MONAXIAL One (polar) axis only Hemimorphic Cyclic=Cn	AXIAL: POLYAXIAL More than one axis—no planes Enantiomorphic Dihedral=Dn	AXIAL: ANASTREPH-AXIAL One inversion axis only Sphenoidal=Sn	AXIHEDRAL=BOTH AXES AND PLANES: ORTHAXI-HEDRAL One plane (horizontal) perpendicular to one axis. C^h_n	AXIHEDRAL: MONAXI-HEDRAL One (polar) axis only, with parallel (vertical) planes. Hemimorphic C^v_n	AXIHEDRAL: MESAXI-HEDRAL Planes (none horizontal) between (diagonal to) axes. D^d_n	AXIHEDRAL: POLYAXI-HEDRAL Planes (one horizontal) coinciding with symmetry axes. D^h_n	Row number	SYMMETRY / SYSTEM
TRIMETRIC DIVISION										
TRICLINIC	1	1(XXXII) C_1 Asymmetric 1	(Cf. 4)		(Cf. 3)			(Cf. 7)	1	TRICLINIC
TRICLINIC	$\bar{1}$			2(XXXI) $C_i(=S_2)$ Pinacoidal $\bar{1}(=i)$			(Cf. 5)		2	TRICLINIC
MONOCLINIC	$\bar{2}$			3(XXX) $C_s(=S)$** Clinohedral m(=$\bar{2}$)					3	MONOCLINIC
MONOCLINIC	2	4(XXIX) C_2 Sphenoidal 2			5(XXVIII) C_{2h}* Prismatic 2/m (i)				4	MONOCLINIC
ORTHO-RHOMBIC	2		6(XXVII) $D_2(=V)$ Orthorhombic Disphenoidal 222			7(XXVI) C_{2v} Orthorhombic Pyramidal m m (2 m m)		8(XXV) $D_{2h}(=V_h)$ Orthorhombic Dipyramidal mmm (2/m2/m2/m i)	5	ORTHO-RHOMBIC
DIMETRIC DIVISION										
TETRAG-ONAL	$\bar{4}$			9(XXIV) $S_4(=C'_4)$ Tetragonal Disphenoidal $\bar{4}$			10(XXIII) $D_{2d}(=V_d)$ Ditetragonal Scalenohedral $\bar{4}$ 2 m($\bar{4}$ 2^2 m^2)		6	TETRAG-ONAL
TETRAG-ONAL	4	11(XXII) C_4 Tetragonal Pyramidal 4	12(XXI) D_4 Tetragonal Trapezohedral 4 2(4 2^2 2^2)		13(XX) C_{4h} Tetragonal Dipyramidal 4/m (i)	14(XIX) C_{4v} Ditetragonal Pyramidal 4 m m(4 m^2 m^2)		15(XVIII) D_{4h} Ditetragonal Dipyramidal 4/mmm(4/m2/$m^2$2/m^2i)	7	TETRAG-ONAL
HEXAGONAL — Rhombohedral Subsystem	3	16(XVII) C_3 Trigonal Pyramidal 3	17(XVI) D_3 Trigonal Trapezohedral 3 2(3 2^3)			18(XV) C_{3v} Ditrigonal Pyramidal 3 m(3 m^3)			8	Rhombohedral Subsystem — HEXAGONAL
HEXAGONAL — Rhombohedral Subsystem	$\bar{3}$			19(XIV) $C_{3i}(=S_6)$ Trigonal Rhombohedral $\bar{3}$ (i)			20(XIII) D_{3d} Ditrigonal Scalenohedral $\bar{3}$ m($\bar{3}$ 2/m^3 i)		9	Rhombohedral Subsystem — HEXAGONAL
HEXAGONAL — Hexagonal Subsystem	$\bar{6}$			21(XII) $C_{3h}(=S_3)$ Trigonal Dipyramidal $\bar{6}$ (=3/m)				22(XI) D_{3h} Ditrigonal Dipyramidal $\bar{6}$ m(3/m[=$\bar{6}$]$2^3$$m^3$)	10	Hexagonal Subsystem — HEXAGONAL
HEXAGONAL — Hexagonal Subsystem	6	23(X) C_6 Hexagonal Pyramidal 6	24(IX) D_6 Hexagonal Trapezohedral 6 2(6 2^3 2^3)		25(VIII) C_{6h} Hexagonal Dipyramidal 6/m (i)	26(VII) C_{6v} Dihexagonal Pyramidal 6 m m(6 m^3 m^3)		27(VI) D_{6h} Dihexagonal Dipyramidal 6/mmm(6/m2/$m^3$2/m^3i)	11	Hexagonal Subsystem — HEXAGONAL
MONOMETRIC DIVISION										
ISOMETRIC	2^3		28(V) T Gyrotris-tetrahedral 2 3(2^3 3^4)					29(IV) T_h Diploidal m 3(2/m^3 $\bar{3}^4$ i)	12	ISOMETRIC
ISOMETRIC	$\bar{4}^3$						30(III) T_d Hextetrahedral $\bar{4}$ 3 m($\bar{4}^3$ 3^4 m^6)		13	ISOMETRIC
ISOMETRIC	4^3		31(II) O Gyricosi-tetrahedral 4 3(4^3 3^4 2^6)					32(I) O_h Hexoctahedral m 3 m(4/$m^3$$\bar{3}^4$2/$m^6$ i)	14	ISOMETRIC
Column Number		I	II	III	IV	V	VI	VII		Column number

EXPLANATION: Significance of position is indicated in the rectangle to the left. The class number in the upper left is shown in Arabic numerals going from minimum to maximum symmetry, and in Roman numerals for the reverse. The Schoenflies symbol is used in the upper right. With few exceptions the class name is according to Groth (but using Greek prefixes). Symbol and total symmetry are shown below with Hermann-Mauguin symbols; these are explained in the other column

number	symbol
class name	
symbol (symmetry)	

Hermann-Mauguin symbols:
- 1, 2, 3, 4, 6,—(ordinary) symmetry axes.
- $\bar{1}$(=i), $\bar{2}$(=m), $\bar{3}$, $\bar{4}$, $\bar{6}$(=3/m),—inversion axes.
- m(=$\bar{2}$)—plane of symmetry; i(=$\bar{1}$)—center of symmetry (inversion).
- 2/m, 3/m(=$\bar{6}$), 4/m, 6/m,—axes with normal plane of symmetry.
- Superscripts indicate more than one of a given element of symmetry.

REMARKS: Crystals of classes in the monaxial and monaxihedral families, as well as those of classes 3, 17, 22, 28, and 30, may show pyro- and piezo-electric phenomena. Rotatory polarization may occur in monaxial and polyaxial crystals.

FOOTNOTES:
* In the monoclinic morphologists generally make the symmetry axis=b; thus the Schoenflies symbol for Class 5 may not seem appropriate;** nor would the symbol C1h which has been used for Class 3.
In Classes 6, 8, and 10, Q (quadratic) may replace V (vierer).

Fig. 1.—CRYSTAL CLASSES (point groups) arranged by D. Jerome Fisher.