ORIENTED INCLUSIONS OF TOURMALINE IN MUSCOVITE*

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CONTENTS

SUMMARY

The paper presents a study of the habit, plane of flattening, and orientation to the mica of tourmaline crystals enclosed between the basal cleavages of muscovite. The study is largely statistical, and includes an analysis of 710 examples from Gilsum, N. H., and of 109 examples from New York City.

The following forms and planes of flattening were identified:

	Gilsum, N. H.				New York City			
Plane of Flattening	(11$\bar{2}$0)	(10$\bar{1}$0)	(0001)	Other	(11$\bar{2}$0)	(10$\bar{1}$0)	(0001)	Other
Per cent of Total	59.9	12.3	16.7	11.1	45.9	23.9	22.9	7.2
Observed Forms	ore_1r_1	or_1r	a		or_1r	or_1r	amm_1	

Although the tourmaline inclusions did not appear on casual examination to be oriented in relation to the muscovite, tendencies for arrangement in preferred crystallographic orientations were recognized on statistical study. The 18 well-defined and 8 ill-defined separate orientations established included approximately one-half of the total number of inclusions observed from each locality. The remaining inclusions are not distributed at random, but occupy regions, between the clearly defined positions of orientation, in which the statistical population is presumably insufficient to define exact positions of orientation. The number of different orientations that can be recognized appears to depend only on the extent of statistical investigation.

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The observed positions of orientation are found to correspond to calculated positions in which a direction of relatively low index in the plane of flattening of the tourmaline coincides with a direction of relatively low index in (001) of the muscovite. The frequency of orientation roughly increases with the increasing rationality of the coinciding directions. The observed orientations are as follows:

Locality	Plane of Flattening	Reference Direction	Coincidences of Reference Direction on Muscovite (001) with Per Cent of Total, at Each Position									
Gilsum	$(11\bar{2}0)$	c	110 $1\bar{1}0$ 16.6	320 $3\bar{2}0$ 4.5	130 $1\bar{3}0$ 4.1	100 2.7	340 $3\bar{4}0$ 2.4	010 2.1	120 $1\bar{2}0$ 1.8	230 $2\bar{3}0$ 1.8	310 $3\bar{1}0$ 1.7	210 $2\bar{1}0$ 1.4
	$(11\bar{2}0)$	$10\bar{1}1$	110 $1\bar{1}0$?	100 ?								
	$(11\bar{2}0)$	$\bar{1}011$	110 $1\bar{1}0$?	010 ?								
New York City	$(11\bar{2}0)$	c	110 $1\bar{1}0$ 9.2	010 3.7	320 $3\bar{2}0$ 3.7	130 $1\bar{3}0$ 2.7	100 1.8	310 $3\bar{1}0$ 1.8				
Gilsum	$(10\bar{1}0)$	c	110 $1\bar{1}0$ 1.1	130 $1\bar{3}0$ .42	010 .28	320 $3\bar{2}0$ .29	100 ?	310 $3\bar{1}0$?	340 $3\bar{4}0$?			
New York City	$(10\bar{1}0)$	c	010 3.7	110 $1\bar{1}0$ 2.7	100 1.8							

			Percussion Figure	Pressure Figure	19° posit. (Table 8)	11° posit. (Table 8)	5° posit. (Table 8)
Gilsum	(0001)	$11\bar{2}0$	7.3	2.6	1.6	?	?
New York	(0001)	$11\bar{2}0$	10.6	3.5	2.9	?	

It is suggested that, in general, orientation is not restricted to specific, limiting conditions of crystallographic coincidence, but that a degree of frequency is associated with each of an infinitude of orientations between over- or inter-growing and substrate crystals of any two species. The rate of growth of tourmaline along its polar *c* axis is noted to be greater in the antilogous direction than in the analogous direction.

Introduction

Although tourmaline crystals often occur as flattened inclusions between the basal laminae of muscovite, the orientation of the crystals with respect to the muscovite has seldom been remarked. The recognized instances of orientation, described by Volger,[1] Linck[2] and Mügge[3] from a few observations, fall into three types: crystals flattened on $(11\bar{2}0)$ with *c* parallel to a ray of the percussion figure or the pressure figure of the mica, and crystals flattened on (0001) with the faces of $(11\bar{2}0)$ parallel to the rays of the pressure figure. The minute needle-like inclusions characteristic of phlogopite, oriented parallel to the pressure or percussion figures and causing asterism, have been thought to be tourmaline but have since[4] been identified as rutile.

In the present study, the habit, plane of flattening and orientation was determined of 710 tourmaline crystals included in pale yellow-brown muscovite from a pegmatite at Gilsum, N. H., and of 109 crystals included in brown muscovite from a pegmatitic zone in Manhattan schist at 172nd Street and Fort Washington Avenue, New York City. The pegmatites of the Gilsum area have been described by Megathlin.[5]

The majority of the inclusions were prismatic in habit, and ranged between 0.5–5 mm. in length, 0.1–1 mm. in width and 0.005–0.5 mm. in thickness. One doubly terminated crystal was noted that had the remarkable dimensions of $920 \times 0.2 \times 0.01$ mm. Some of the crystals were so thin as to give interference colors in ordinary light. It was observed that the flattening—that is, the relation of length and breadth to thickness—of the prismatic inclusions was greater in small crystals than in large crystals. The Gilsum crystals ranged in color from pure black in the thicker, opaque individuals to various shades of yellow-brown and smoke-gray in the thinner, transparent, individuals. The New York City crystals ranged in color from a brownish black to a light yellow-brown.

[1] Volger, O., *Jahresber. Wetterauischen Ges., Hanau,* **1861–63,** p. 67.
[2] Linck, G., *Jenaische Zeits. Naturwiss.*, vol. **33,** pp. 350–351, 1899.
[3] Mügge, O., *Neues Jahrb., Beil.-Bd.* **16,** p. 388, 1903.
[4] Pogue, J. E., *Proc. U. S. Nat. Mus.*, vol. **39,** pp. 572–576, 1911.
[5] Megathlin, G. R., *Econ. Geol.*, vol. **24,** pp. 163–181, 1929.

The angle χ, used to denote the angles of the percussion figure opposite to 010 and bisected by the optic plane, and which mark the actual position of the subsidiary, 110 and $1\bar{1}0$, rays of the figure, had a minimum value of 50° 45′ for the Gilsum muscovite, and of 52° 18′ for the New York City muscovite. Walker[6] obtained minimum values of χ ranging from 52° 53′ for muscovite from Murray Bay, Quebec, to 55° 57′ for muscovite from Utö, Sweden.

The great majority of the inclusions are either flattened in the prism zone and elongated parallel to *c*, or are flattened on (0001). A small proportion of the inclusions appeared to be flattened on planes inclined to *c*. No tendency for orientation of the inclusions to the muscovite was apparent on a cursory examination of the specimens.

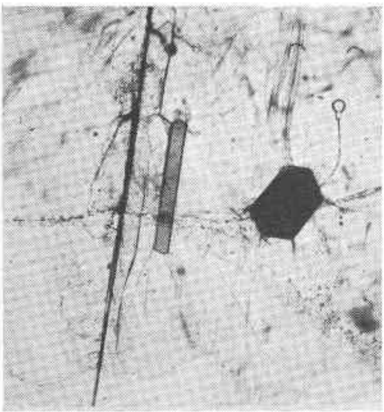


FIG. 1. The prismatic tourmaline inclusion is flattened on $(11\bar{2}0)$ and is oriented with *c* parallel to 010 of the muscovite. The 010 ray of a percussion figure can be seen in the photograph. The hexagonal inclusion is flattened on (0001) and is bounded laterally by $(11\bar{2}0)$; the crystal is oriented with the faces of $(11\bar{2}0)$ parallel to the rays of the pressure figure. Gilsum, N. H. ×30.

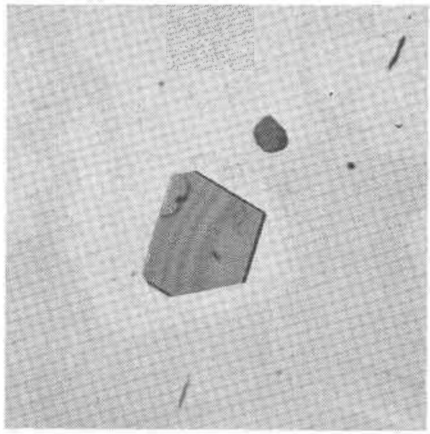


FIG. 2. The prismatic tourmaline inclusion is flattened on $(11\bar{2}0)$; the twelve-sided inclusion is flattened on (0001) and is bounded laterally by $(11\bar{2}0)$, $(10\bar{1}0)$ and $(01\bar{1}0)$. New York City. ×30.

HABIT AND FLATTENING OF PRISMATIC INCLUSIONS

Inclusions flattened in the prism zone are shown in Figs. 1 and 2. The crystals are characteristically terminated by a pair of faces at each extremity of *c*; additional faces may be present but are minute in size.

[6] Walker, T. L., *Am. Jour. Sci.*, ser. 4, vol. **2**, p. 5, 1896.

TABLE 1
HABIT AND FLATTENING OF PRISMATIC INCLUSIONS, GILSUM, N. H., AND NEW YORK CITY

Locality: Gilsum, N.H.	Number of Crystals Observed	Plane of Flattening	Measured Angles (Average)				Calculated Angles and Corresponding Forms			
			Pole X		Pole X′		Antilogous Pole		Analogous Pole	
			A	B	M	N	A	B	M	N
Habit I	96	$(11\bar{2}0)$	44°16′	62°48′	75°8′	62°49′	44°3′ $o(02\bar{2}1)$	62°40′ o'/o''	75°30′ r_1'/r_1''	62°40′ $r_1(01\bar{1}\bar{1})$
Habit II	25	$(11\bar{2}0)$	44°27′	63°9′	75°15′	62°55′	44°3′ $o(02\bar{2}1)$	62°40′ $r(10\bar{1}1)$	75°30′ $e_1(10\bar{1}\bar{2})$	62°40′ $r_1(01\bar{1}\bar{1})$
Habit III	12	$(11\bar{2}0)$	44°30′	62°26′	75°34′	62°59′	44°3′ $o(02\bar{2}1)$	62°40′ $r(10\bar{1}1)$	75°30′ r_1'/r_1''	62°40′ $r_1(01\bar{1}\bar{1})$
Habit IV	28	$(11\bar{2}0)$	75°54′	62°39′	75°27′	63°0′	75°30′ r'/r''	62°40′ $r(10\bar{1}1)$	75°30′ r_1'/r_1''	62°40′ $r_1(01\bar{1}\bar{1})$
Habit V	22	$(10\bar{1}0)$	48°40′	49°4′	66°49′	66°8′	48°9′ $o(02\bar{2}1)$	48°9′ o''	65°53′ $r_1(01\bar{1}\bar{1})$	65°53′ r_1''
Habit VI	11	$(10\bar{1}0)$	66°36′	65°20′	65°42′	65°30′	65°53′ $r(10\bar{1}1)$	65°53′ r''	65°53′ $r_1(01\bar{1}\bar{1})$	65°53′ r_1''

Inclusions otherwise flattened: 30 Inclusions unidentified as to flattening: 367

New York City										
Habit I	28	$(11\bar{2}0)$	44°17′	63°8′	74°56′	62°32′	44°3′ $o(02\bar{2}1)$	62°40′ o'/o''	75°30′ r_1'/r_1''	62°40′ $r_1(01\bar{1}\bar{1})$
Habit II	16	$(11\bar{2}0)$	75°19′	62°50′	75°27′	62°18′	75°30′ r'/r''	62°40′ $r(10\bar{1}1)$	75°30′ r_1'/r_1''	62°40′ $r_1(01\bar{1}\bar{1})$
Habit III	6	$(10\bar{1}0)$	48°36′	48°23′	66°37′	66°21′	48°9′ $o(02\bar{2}1)$	48°9′ o''	65°53′ $r_1(01\bar{1}\bar{1})$	65°53′ r_1''
Habit IV	17	$(10\bar{1}0)$	65°44′	66°9′	66°17′	65°21′	65°53′ $r(10\bar{1}1)$	65°53′ r''	65°53′ $r_1(01\bar{1}\bar{1})$	65°53′ r_1''

Inclusions otherwise flattened: 7 Inclusions unidentified as to flattening: 10

The exact plane of flattening of the inclusions was determined by first measuring the angles made by the terminating forms with *c*, measured in the plane of flattening. A tabulation of the measurements showed that most of the inclusions fell into habit groups characterized by a close similarity of terminal angles. By trial, assuming various planes of flattening in the prism zone, rational indices for the terminal faces were obtained for some of the habit groups in the case of $(11\bar{2}0)$, and for the remaining habit groups in the case of $(10\bar{1}0)$. This data is tabulated for the two occurrences in Table 1. The terminal angles of the remaining crystals differed from those of all of the habit groups and varied among themselves; some of these crystals were flattened in the prism zone on unidentified planes, but most of them appeared to be flattened on planes inclined to *c*. A proportion of the inclusions from both localities had rounded or irregular terminations and their plane of flattening could not be identified.

In identifying the forms from the measurements it is important to note whether the terminating faces are perpendicular to the plane of flattening or are inclined to it, since faces that bevel or truncate a given termination make the same plane angle on *c*. The form *o* $(02\bar{2}1)$ occurred at one end of the polar *c* axis only and, in accordance with the general rule, this end was taken as the antilogous pole. The habit differs in forms present, their combinations and their relative frequencies, both with the plane of flattening and with the locality.

Polar growth of tourmaline. Many of the crystals from both localities showed internal zones of growth. The spacing of the growth zones varies at opposite ends of the polar axis, being invariably wider at the antilogous pole, even in crystals terminated at opposite poles by geometrically like forms. This relation indicates that the rate of growth is greater in the antilogous direction of the polar axis than in the opposite, analogous, direction. The rate of solution in tourmaline, on the other hand, is greater in the analogous direction.[7] Other substances[8] show a similar variability in rate of growth at opposite ends of a polar axis.

Orientation of Prismatic Inclusions

Inclusions Flattened on $(11\bar{2}0)$. The orientation of the prismatic inclusions to the muscovite can be defined by stating the position of a direction in the plane of flattening of the inclusions relative to a direction in (001) of the muscovite. The observed positions of the inclusions flattened on $(11\bar{2}0)$, obtained by direct measurement of the angle made by the *c* direction of tourmaline to 010 of muscovite are graphed to the

[7] Frondel, C., *Am. Mineral.*, vol. **20**, pp. 855–856, 1935.
[8] Bentivoglio, M., *Proc. Royal Soc. London*, vol. **115A**, pp. 81–83, 1927.

nearest degree in Tables 2 (Gilsum) and 3 (New York City). The positive and negative inclinations about 010, the plane of symmetry, are not distinguished.

TABLE 2. OBSERVED ORIENTATIONS OF TOURMALINE (11$\bar{2}$0) UPON MUSCOVITE (001).
GILSUM, N. H.
Reference directions: $c \wedge 010$

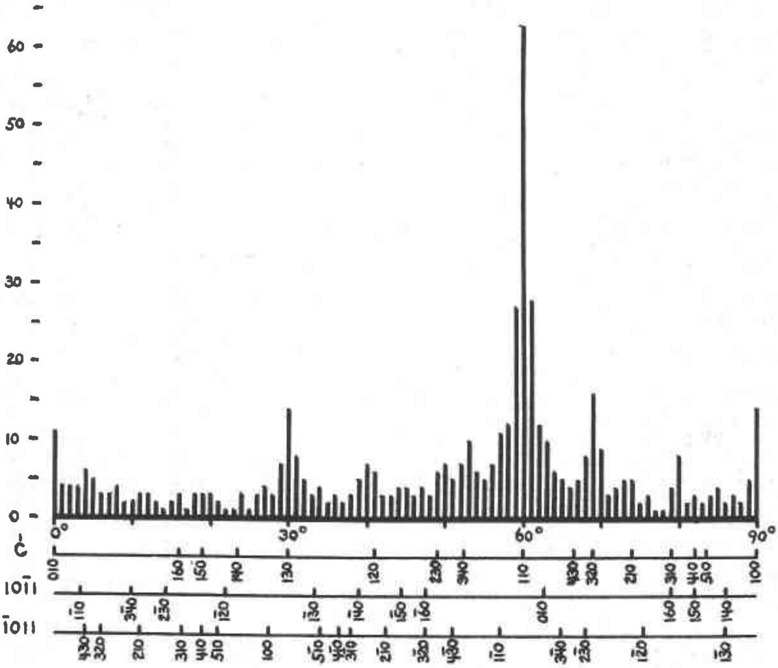


TABLE 3. OBSERVED ORIENTATIONS OF TOURMALINE (11$\bar{2}$0) UPON MUSCOVITE (001).
NEW YORK CITY
Reference directions: $c \wedge 010$

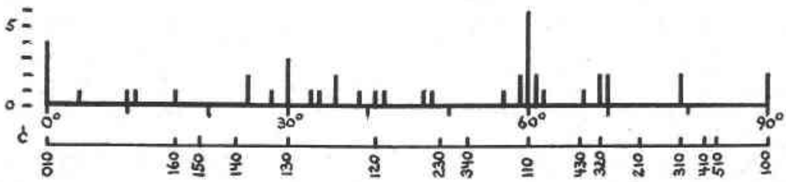


Table 2 (Gilsum), which comprises 528 inclusions,[9] shows maxima at particular values of $c \wedge 010$, of which those at 0°, 30°, 40°, 50°, 53°, 60°,

[9] This graph is composite, and contains a proportion of inclusions flattened on (10$\bar{1}$0) and other, unidentified, planes. The majority of the inclusions from Gilsum had rounded or irregular terminations and the plane of flattening could not be determined by the method

69°, 74°, 80° and 90° can be considered as well-defined. These maxima represent preferred orientations of the inclusions in the contact surface tourmaline $(11\bar{2}0)$–(001) muscovite.

In Table 4 the calculated coincidences[10] for c and a few other directions of relatively low indices in tourmaline $(11\bar{2}0)$ and muscovite (001) are tabulated for values of $c\wedge 010$. The table cites the successive positions in which crystallographic directions in the plane of flattening of the tourmaline coincide with crystallographic directions in (001) of the muscovite as the tourmaline crystal is rotated through 90° on the contact surface. For instance, tourmaline $10\bar{1}4$ coincides with muscovite $4\bar{1}0$ when c is inclined 1° 09′ to 010, and with muscovite $\bar{7}20$ when c is inclined 2° 00′ to 010. The details of coincidence within the table could be extended indefinitely by including directions of higher indices in the two minerals. Similar coincidence tables can be calculated for any plane of contact between two species of crystals.

By reference to the table of calculated coincidences, it is found that the various observed positions of orientation mark major calculated coincidences in the contact surface. All appear to correspond to calculated coincidences made by c with directions of relatively low indices in muscovite (001), as follows: 010, 130, 120, 230, 340, 110, 320, 210, 310 and 100. These positions fall into a N_3 complication series, with an extra term at 3/4, but the percentages of orientation at each position (see Summary) have no analogous relationship. It should be understood that the fact that the most rational coincidences are made by c is not a consequence of using c as a reference direction for measurement; the same coincidences would have been found if any other direction in tourmaline $(11\bar{2}0)$ had been chosen as the basis of measurement.

Considerations of crystallographic coincidence made by tourmaline c upon muscovite (001), therefore, have influenced the crystallization of the tourmaline to a greater extent than any of the innumerable other coincidences in the respective contact planes. As will be seen, however, there is evidence that other coincidences in the contact surface, between directions of higher indices, are working in conjunction with c to control orientation at the various observed positions. Also,

described, although the position of c could be identified in every instance. In preparing this graph, all observations were included except those of crystals previously identified by measurement (Table 1) as being flattened on some plane other than $(11\bar{2}0)$. Assuming that the proportion of crystals flattened on $(11\bar{2}0)$ out of the total number of prismatic inclusions noted is the same as that of crystals whose plane of flattening was determinable, the composite graph will contain 80% of $(11\bar{2}0)$ crystals. Separate plotting of the crystals definitely identified as being flattened on $(11\bar{2}0)$ gave a graph that closely paralleled that of Table 2.

[10] Tourmaline $c=.44767$; muscovite $a:b:.=57735:1$.

TABLE 4

CALCULATED PRINCIPAL COINCIDENCES BETWEEN MUSCOVITE (001) AND TOURMALINE $(11\bar{2}0)$
Divergence from exact coincidence as given. Observed coincidences indicated by asterisk.

Tourm. $c\wedge$ muscov. 010	$10\bar{1}4$ / $\bar{1}01\bar{4}$	$\bar{1}014$ / $10\bar{1}\bar{4}$	$10\bar{1}2$ / $\bar{1}01\bar{2}$	$\bar{1}012$ / $10\bar{1}\bar{2}$	$10\bar{1}1$ / $\bar{1}01\bar{1}$	$\bar{1}011$ / $10\bar{1}\bar{1}$	$20\bar{2}1$ / $\bar{2}02\bar{1}$	$\bar{2}021$ / $20\bar{2}\bar{1}$	$40\bar{4}1$ / $\bar{4}04\bar{1}$	$\bar{4}041$ / $40\bar{4}\bar{1}$	$000\bar{1}$ / 0001	c
* 0											100 0′	010 0′
1	$4\bar{1}0$ 9′	510 −13′		730 −24′		760 0′		470 −20′		270 −28′		
2	$7\bar{2}0$ 0′	610 −8′	$2\bar{1}0$ 24′	520 −30′		650 −26′		350 3′	$1\bar{4}0$ −23′			
? 3		710 −21′			$1\bar{1}0$ 20′	540 −27′	$1\bar{2}0$ −9′					
4	$3\bar{1}0$ 28′		$7\bar{4}0$ 15′	310 −24′		430 −5′				130 12′		
5			$5\bar{3}0$ 24′	720 8′		750 −5′		230 4′			$7\bar{1}0$ 17′	
6	$5\bar{2}0$ 22′			410 17′		320 17′					$6\bar{1}0$ 30′	
7	$7\bar{3}0$ 28′	100 22′	$3\bar{2}0$ 27′		$5\bar{6}0$ −23′		$3\bar{7}0$ −28′	570 0′	$1\bar{5}0$ 18′		$5\bar{1}0$ 25′	
8			$7\bar{5}0$ 5′	510 −5′	$4\bar{5}0$ −29′	530 14′		340 22′			$4\bar{1}0$ −13′	
9	$2\bar{1}0$ 16′		$4\bar{3}0$ 5′	610 0′		740 5′	$2\bar{5}0$ −20′			250 −5′	$7\bar{2}0$ −22′	
10			$5\bar{4}0$ −17′	710 −13′	$3\bar{4}0$ −15′			450 8′	$1\bar{6}0$ 18′			
11	$7\bar{4}0$ 7′		$6\bar{5}0$ −16′			210 14′		560 14′		370 −13′	$3\bar{1}0$ 6′	
12	$5\bar{3}0$ 16′	$7\bar{1}0$ −5′	$7\bar{6}0$ 10′		$5\bar{7}0$ 23′			670 −1′	$1\bar{7}0$ 6′			
13		$6\bar{1}0$ 8′				730 26′					$5\bar{2}0$ 0′	
14	$3\bar{2}0$ 19′	$5\bar{1}0$ 3′		100 30′	$2\bar{3}0$ 27′	520 20′	$1\bar{3}0$ −3′				$7\bar{3}0$ 6′	170 −6′
15	$7\bar{5}0$ −3′									120 6′		
16	$4\bar{3}0$ −3′	$4\bar{1}0$ 25′	$1\bar{1}0$ 30′			310 26′		110 −3′			$2\bar{1}0$ −6′	160 6′
17	$5\bar{4}0$ −25′	$7\bar{2}0$ 16′			$3\bar{5}0$ 26′							
18	$6\bar{5}0$ −24′	$3\bar{1}0$ −16′			$4\bar{7}0$ 3′	720 −2′	$2\bar{7}0$ 17′				$7\bar{4}0$ −15′	
19	$7\bar{6}0$ 2′		$6\bar{7}0$ −28′	$7\bar{1}0$ −13′		410 7′				470 −5′	$5\bar{3}0$ −6′	150 6′
20		$5\bar{2}0$ −22′	$5\bar{6}0$ −13′	$6\bar{1}0$ 0′				650 11′		350 18′		
21		$7\bar{3}0$ −16′	$4\bar{5}0$ −19′	$5\bar{1}0$ −5′		510 −15′	$1\bar{4}0$ 22′	540 10′			$3\bar{2}0$ −3′	
22					$1\bar{2}0$ 14′	610 −10′					$7\bar{5}0$ −25′	
23	$1\bar{1}0$ 22′	$2\bar{1}0$ −28′	$3\bar{4}0$ −5′	$4\bar{1}0$ 17′		710 −23′		430 −28′		230 19′	$4\bar{3}0$ −25′	140 25′
24			$5\bar{7}0$ −27′	$7\bar{2}0$ 8′				750 −28′				
25				$3\bar{1}0$ −24′			$1\bar{5}0$ 3′	320 −6′		570 15′	$5\bar{4}0$ 13′	
26		$7\bar{4}0$ 23′	$2\bar{3}0$ −23′		$3\bar{7}0$ −5′				010 −12′		$6\bar{5}0$ 14′	270 20′
? 27	$5\bar{6}0$ −21′			$5\bar{2}0$ −30′		100 20′		530 −9′		340 −23′		
28	$4\bar{5}0$ −27′	$3\bar{2}0$ −25′		$7\bar{3}0$ −24′	250 3′		$1\bar{6}0$ 3′	740 −18′		450 23′		
29			$3\bar{5}0$ −24′							560 29′		

TABLE 4.—(*Continued*)

CALCULATED PRINCIPAL COINCIDENCES BETWEEN MUSCOVITE (001) AND TOURMALINE ($11\bar{2}0$)
Divergence from exact coincidence as given. Observed coincidences indicated by asterisk.

Tourm. $c\wedge$ muscov. 010	$10\bar{1}4$ $\bar{1}01\bar{4}$		$\bar{1}014$ $10\bar{1}\bar{4}$		$10\bar{1}2$ $\bar{1}01\bar{2}$		$10\bar{1}2$ $\bar{1}01\bar{2}$		$10\bar{1}1$ $\bar{1}01\bar{1}$		$\bar{1}011$ $10\bar{1}\bar{1}$		$20\bar{2}1$ $\bar{2}02\bar{1}$		$\bar{2}021$ $20\bar{2}\bar{1}$		$40\bar{4}1$ $\bar{4}04\bar{1}$		$\bar{4}041$ $40\bar{4}\bar{1}$		$000\bar{1}$ 0001		c	
* 30	$3\bar{4}0$	−13′	$7\bar{5}0$	13′									$1\bar{7}0$	−9′	210	−9′			670	14′	$1\bar{1}0$	0′	130	0′
31			$4\bar{3}0$	13′	$4\bar{7}0$	13′	$2\bar{1}0$	24′																
32	$5\bar{7}0$	25′	$5\bar{4}0$	−9′							$7\bar{1}0$	−3′			730	3′								
33			$6\bar{5}0$	−8′			$7\bar{4}0$	15′	$1\bar{3}0$	20′	$6\bar{1}0$	10′			520	−3′								
34	$2\bar{3}0$	29′	$7\bar{6}0$	18′			$5\bar{3}0$	24′			$5\bar{1}0$	5′							110	12′	$6\bar{7}0$	2′		
35					$1\bar{2}0$	24′									310	3′					$5\bar{6}0$	17′	250	−17′
36							$3\bar{2}0$	27′	$2\bar{7}0$	−20′	$4\bar{1}0$	27′									$4\bar{5}0$	11′		
37	$3\bar{5}0$	28′	$1\bar{1}0$	−22′			$7\bar{5}0$	5′			$7\bar{2}0$	18′			720	−25′							370	−25′
38	$4\bar{7}0$	5′					$4\bar{3}0$	5′			$3\bar{1}0$	−14′			410	−16′			650	26′	$3\bar{4}0$	25′		
39					$3\bar{7}0$	5′	$5\bar{4}0$	−17′	$1\bar{4}0$	−15′					510	22′			540	25′	$5\bar{7}0$	3′		
40							$6\bar{5}0$	−16′			$5\bar{2}0$	−20′			610	27′	170	−18′						
* 41			$6\bar{7}0$	−20′	$2\bar{5}0$	13′	$7\bar{6}0$	10′			$7\bar{3}0$	−14′			710	14′			430	−13′	$2\bar{3}0$	7′	120	−6′
42	$1\bar{2}0$	16′	$5\bar{6}0$	−5′													160	−6′	750	−13′				
43			$4\bar{5}0$	−11′							$2\bar{1}0$	−26′							320	9′				
44									$1\bar{5}0$	26′			010	−3′							$3\bar{5}0$	6′		
45			$3\bar{4}0$	3′	$1\bar{3}0$	−30′	$1\bar{1}0$	30′			$7\bar{4}0$	−35′					150	−6′	530	6′	$4\bar{7}0$	−17′	470	−17′
46	$3\bar{7}0$	−3′	570	−19′							$5\bar{3}0$	−26′			100	3′			740	−3′			350	6′
47									$1\bar{6}0$	26′														
48	$2\bar{5}0$	5′	$2\bar{3}0$	−15′			670	−28′			$3\bar{2}0$	−23′							210	6′				
* 49					$2\bar{7}0$	−10′	$5\bar{6}0$	−13′	170	14′							140	13′			$1\bar{2}0$	−6′	230	7′
50							$4\bar{5}0$	−19′			$7\bar{5}0$	15′							730	18′				
51			$3\bar{5}0$	−16′							$4\bar{3}0$	15′			$6\bar{1}0$	−27′			520	12′			570	3′
* 52					$1\bar{4}0$	−5′	$3\bar{4}0$	−5′			$5\bar{4}0$	−7′					270	8′					340	25′
53	$1\bar{3}0$	22′	470	21′			570	−27′			$6\bar{5}0$	−6′			$5\bar{1}0$	28′			310	18′	$3\bar{7}0$	−25′		
54											$7\bar{6}0$	20′			$4\bar{1}0$	−10′							450	11′
55							$2\bar{3}0$	−23′							$7\bar{2}0$	−19′			720	−10′	$2\bar{5}0$	−17′	560	17′
56	$2\bar{7}0$	−18′	$1\bar{2}0$	−28′	$1\bar{5}0$	−24′											130	−12′	410	−1′			670	2′
? 57											$1\bar{1}0$	−20′			$3\bar{1}0$	9′								
58							$3\bar{5}0$	−24′					170	−3′					510	−23′				
59	$1\bar{4}0$	−13′			$1\bar{6}0$	−24′									$5\bar{2}0$	3′			610	−18′				

TABLE 4.—(*Continued*)

CALCULATED PRINCIPAL COINCIDENCES BETWEEN MUSCOVITE (001) AND TOURMALINE ($11\bar{2}0$)
Divergence from exact coincidence as given. Observed coincidences indicated by asterisk.

Tourm. $c\wedge$ muscov. 010	$10\bar{1}4$ $\bar{1}01\bar{4}$	$\bar{1}014$ $10\bar{1}\bar{4}$	$10\bar{1}2$ $\bar{1}01\bar{2}$	$\bar{1}012$ $10\bar{1}\bar{2}$	$10\bar{1}1$ $\bar{1}01\bar{1}$	$\bar{1}011$ $10\bar{1}\bar{1}$	$20\bar{2}1$ $\bar{2}02\bar{1}$	$\bar{2}021$ $20\bar{2}\bar{1}$	$40\bar{4}1$ $\bar{4}04\bar{1}$	$\bar{4}041$ $40\bar{4}\bar{1}$	$000\bar{1}$ 0001	c
* 60				$4\bar{7}0$ 13′			160 9′	$7\bar{3}0$ 9′			$1\bar{3}0$ 0′	110 0′
61		$3\bar{7}0$ 13′				670 −18′			250 −29′			
62			170 24′			$5\bar{6}0$ −3′		$2\bar{1}0$ −3′	370 23′			
* 63		$2\bar{5}0$ 21′			010 −20′	$4\bar{5}0$ −9′	150 9′					
64	$1\bar{5}0$ 28′			$1\bar{2}0$ 24′				$7\bar{4}0$ −12′		100 12′	$2\bar{7}0$ 20′	650 14′
65						$3\bar{4}0$ 5′		$5\bar{3}0$ −3′				540 13′
66						$5\bar{7}0$ −17′						
67	$1\bar{6}0$ 28′	$1\bar{3}0$ −22′					140 28′	$3\bar{2}0$ 0′	120 −18′		$1\bar{4}0$ 25′	430 −25′
68				$3\bar{7}0$ 5′		$2\bar{3}0$ −13′		$7\bar{5}0$ −22′				750 −25′
* 69	$1\bar{7}0$ 16′							$4\bar{3}0$ −22′		$7\bar{1}0$ 5′		320 −3′
70				$2\bar{5}0$ 13′			270 23′			$6\bar{1}0$ 18′		
71		$2\bar{7}0$ −2′				$3\bar{5}0$ −14′		$5\bar{4}0$ 16′	470 −29′	$5\bar{1}0$ 13′	$1\bar{5}0$ 6′	530 −6′
72								$6\bar{5}0$ 17′	350 −6′	$4\bar{1}0$ −25′		740 −15′
73						$4\bar{7}0$ 23′						
* 74		$1\bar{4}0$ 3′		$1\bar{3}0$ −30′			130 3′			$7\bar{2}0$ 26′	$1\bar{6}0$ 6′	210 −6′
75									230 −5′	$3\bar{1}0$		
76			010 −30′			$1\bar{2}0$ −26′		$1\bar{1}0$ 3′			$1\bar{7}0$ −6′	730 6′
77					170 −26′				570 −9′	$5\bar{2}0$ −12′		520 0′
78		$1\bar{5}0$ −16′		$2\bar{7}0$ −10′					340 13′	$7\bar{3}0$ −6′		
* 79					160 −14′		250 −14′					310 6′
80								$6\bar{7}0$ 5′	450 −1′	$2\bar{1}0$ −18′		
81		$1\bar{6}0$ −16′		$1\bar{4}0$ −5′		$3\bar{7}0$ 15′	370 −22′	$5\bar{6}0$ 20′	560 5′			720 −22′
82					150 −14′			$4\bar{5}0$ 14′	670 −10′	$7\bar{4}0$ −27′		410 −13′
83	010 −22′	$1\bar{7}0$ −28′				$2\bar{5}0$ 23′				$5\bar{3}0$ −18′		510 25′
84								$3\bar{4}0$ 28′				610 30′
85				$1\bar{5}0$ −24′			120 −3′	$5\bar{7}0$ 6′		$3\bar{2}0$ −15′		710 17′
86					140 5′				110 −12′			
87						$1\bar{3}0$ −20′		$2\bar{3}0$ 10′		$7\bar{5}0$ 23′		
88				$1\bar{6}0$ −24′						$4\bar{3}0$ 23′		
89			170 24′		270 0′		470 −14′		760 28′	$5\bar{4}0$ 1′		
* 90							350 9′	$3\bar{5}0$ 9′	650 2′	$6\bar{5}0$ 2′	010 0′	100 0′

there is no reason to suppose that lower coincidences in the contact surface are not acting independently at other values of $c\wedge 010$, to control orientation. The observations may not be sufficiently numerous, as a whole, to define such positions in the graphs. As a matter of fact, evidence can be found of controlling effect of coincidences made by tourmaline $10\bar{1}1$ and $\bar{1}011$. The shoulders at the 57° and 63° positions on the 60° maximum are too well-defined to be caused by errors of measurement, and seem to correspond to major coincidences made by $\bar{1}011$ with $1\bar{1}0$ and of $10\bar{1}1$ with 010. An ill-defined maximum at 4° also apparently corresponds to a coincidence made by $10\bar{1}1$ with 110, and an ill-defined maximum at 27° apparently corresponds to a coincidence made by $\bar{1}011$ with 100. The coincidences made by $\bar{1}011$, $10\bar{1}1$ and c are indicated on the graph.

The broadness of the 60° maximum may also have been caused in other ways. Possibly the nearly oriented inclusions were exactly oriented in their nuclear stage of development, but subsequently diverged from this position due to disturbances of growth. On the other hand, it may be that the inclusions are exactly oriented on minute blocks of the muscovite crystal that are in sub-parallel position with the whole crystal, or vice versa. Rayleigh,[11] however, has shown that the (001) surfaces of some muscovite crystals are uniform in macrostructure to an extraordinarily high degree.

The observed positions of orientation of the inclusions flattened on $(11\bar{2}0)$ from the New York City occurrence are graphed for the measured values of $c\wedge 010$ in Table 3. Only those crystals known (Table 1) to be flattened on $(11\bar{2}0)$ are included. As with the Gilsum occurrence, definite maxima are present which identify positions of preferred orientation of the inclusions. The maxima correspond in angular position to calculated coincidences (Table 4) made by c with 010, 130, 110, 320, 310 and 100. The relative frequency of the various orientations, however, is slightly different from that of the Gilsum occurrence. Such a variation may be caused by a difference in lattice dimensions arising from a difference in composition, or they may reflect different temperatures of formation if significant relative changes in the dimensions of the two structures take place with changes in temperature.

Inclusions flattened on $(\bar{1}010)$. The observed positions of orientation of the inclusions definitely identified (Table 1) as being flattened on $(10\bar{1}0)$ are graphed for the measured values of $c\wedge 010$ in Tables 5 (Gilsum) and 6 (New York City). The calculated coincidences for c and a few other directions of relatively low indices in tourmaline $(10\bar{1}0)$ and muscovite (001) are tabulated for values of $c\wedge 010$ in Table 7.

[11] Rayleigh, *Phil. Mag.*, vol. **19**, pp. 96–99, 1910.

TABLE 7

CALCULATED PRINCIPAL COINCIDENCES BETWEEN MUSCOVITE (001) AND TOURMALINE (10$\bar{1}$0)
Divergence from exact coincidence as given. Observed coincidences indicated by asterisk.

Tourm. $c\wedge$ muscov. 010	$\bar{1}2\bar{1}8$ / $1\bar{2}1\bar{8}$	$1\bar{2}18$ / $\bar{1}2\bar{1}8$	$\bar{1}2\bar{1}4$ / $1\bar{2}1\bar{4}$	$1\bar{2}14$ / $\bar{1}2\bar{1}4$	$\bar{1}2\bar{1}2$ / $1\bar{2}1\bar{2}$	$1\bar{2}12$ / $\bar{1}2\bar{1}\bar{2}$	$\bar{1}2\bar{1}1$ / $1\bar{2}1\bar{1}$	$1\bar{2}11$ / $\bar{1}2\bar{1}\bar{1}$	$\bar{2}4\bar{2}1$ / $2\bar{4}2\bar{1}$	$2\bar{4}21$ / $\bar{2}4\bar{2}\bar{1}$	$000\bar{1}$ / 0001	c
* 0	5$\bar{1}$0 −12′	510 −12′	5$\bar{2}$0 −23′	520 −23′							100 0′	010 0′
1		610 −7′	7$\bar{3}$0 −17′		5$\bar{4}$0 20′	430 −18′		230 −2′		130 −11′		
2	4$\bar{1}$0 10′	710 −20′		310 −17′	6$\bar{5}$0 21′	750 −18′	3$\bar{5}$0 −3′					
3	7$\bar{2}$0 1′		2$\bar{1}$0 −29′	720 15′		320 4′	4$\bar{7}$0 −26′	570 −6′	2$\bar{7}$0 9′			
4				410 24′				340 16′				
5	3$\bar{1}$0 29′		7$\bar{4}$0 −38′			530 1′					7$\bar{1}$0 17′	
6		100 23′	5$\bar{3}$0 −29′	510 2′	1$\bar{1}$0 7′	740 −8′		450 2′	1$\bar{4}$0 14′	250 −28′	6$\bar{1}$0 30′	
7	5$\bar{2}$0 23′			610 7′			1$\bar{2}$0 −15′	560 8′		370 24′	5$\bar{1}$0 25′	
8	7$\bar{3}$0 29′		3$\bar{2}$0 −26′	710 −6′		210 1′		670 −7′			4$\bar{1}$0 −13′	
9											7$\bar{2}$0 −22′	
10	2$\bar{1}$0 17′		7$\bar{5}$0 12′		6$\bar{7}$0 9′	730 13′			1$\bar{5}$0 −5′			
11		7$\bar{1}$0 −6′	4$\bar{3}$0 12′		5$\bar{6}$0 24′	520 7′					3$\bar{1}$0 6′	
12	7$\bar{4}$0 8′	6$\bar{1}$0 7′	5$\bar{4}$0 −10′		4$\bar{5}$0 18′		3$\bar{7}$0 26′	110 −9′		120 −17′		
13	5$\bar{3}$0 17′	5$\bar{1}$0 2′	6$\bar{5}$0 − 9′	100 23′	3$\bar{4}$0 −28′	310 13′	2$\bar{5}$0 −26′		1$\bar{6}$0 −5′		5$\bar{2}$0 0′	
14			7$\bar{6}$0 17′								7$\bar{3}$0 6′	170 −6′
15	3$\bar{2}$0 20′	4$\bar{1}$0 24′			5$\bar{7}$0 10′	720 −15′			1$\bar{7}$0 −17′			
16	7$\bar{5}$0 −2′	7$\bar{2}$0 15′				410 −6′		650 5′		470 −28′	2$\bar{1}$0 −6′	160 6′
17	4$\bar{3}$0 −2′	3$\bar{1}$0 −17′	1$\bar{1}$0 −23′	7$\bar{1}$0 −20′	230 14′			540 4′		350 −5′		
18	5$\bar{4}$0 −24′			6$\bar{1}$0 −7′		510 −28′	1$\bar{3}$0 −9′	430 26′			7$\bar{4}$0 −15′	
19	6$\bar{5}$0 −23′	5$\bar{2}$0 −23′		5$\bar{1}$0 −12′		610 −23′		750 26′			5$\bar{3}$0 −6′	150 6′
20	7$\bar{6}$0 3′	7$\bar{3}$0 −17′			3$\bar{5}$0 13′					230 −4′		
21			6$\bar{7}$0 −21′	4$\bar{1}$0 10′	4$\bar{7}$0 −10′			320 −12′			3$\bar{2}$0 −3′	
22		2$\bar{1}$0 −29′	5$\bar{6}$0 −6′	7$\bar{2}$0 1′			2$\bar{7}$0 11′			570 −8′	7$\bar{5}$0 −25′	
23			4$\bar{5}$0 −12′					530 −15′		340 14′	4$\bar{3}$0 −25′	140 25′
24	1$\bar{1}$0 23′		3$\bar{4}$0 2′	3$\bar{1}$0 29′		100 7′		740 −24′				
25		7$\bar{4}$0 22′			1$\bar{2}$0 1′		1$\bar{4}$0 16′			450 0′	5$\bar{4}$0 13′	
26			5$\bar{7}$0 −20′	5$\bar{2}$0 23′				210 −15′		560 6′	6$\bar{5}$0 14′	270 20′
27		3$\bar{2}$0 −26′		7$\bar{3}$0 29′						670 −9′		
28	5$\bar{6}$0 −20′		2$\bar{3}$0 −16′					730 −3′				
29	4$\bar{5}$0 −26′	7$\bar{5}$0 12′		2$\bar{1}$0 17′	3$\bar{7}$0 −18′	7$\bar{1}$0 10′	1$\bar{5}$0 −3′	520 −9′	010 −11′			

TABLE 7 (*Continued*).

CALCULATED PRINCIPAL COINCIDENCES BETWEEN MUSCOVITE (001) AND TOURMALINE (10$\bar{1}$0)
Divergence from exact coincidence as given. Observed coincidences indicated by asterisk.

Tourm. $c\wedge$ muscov. 010	$\bar{1}2\bar{1}8$ / $1\bar{2}1\bar{8}$	$1\bar{2}18$ / $\bar{1}2\bar{1}\bar{8}$	$\bar{1}2\bar{1}4$ / $1\bar{2}1\bar{4}$	$1\bar{2}14$ / $\bar{1}2\bar{1}\bar{4}$	$\bar{1}2\bar{1}2$ / $1\bar{2}1\bar{2}$	$1\bar{2}12$ / $\bar{1}2\bar{1}\bar{2}$	$\bar{1}2\bar{1}1$ / $1\bar{2}1\bar{1}$	$1\bar{2}11$ / $\bar{1}2\bar{1}\bar{1}$	$\bar{2}4\bar{2}1$ / $2\bar{4}2\bar{1}$	$2\bar{4}21$ / $\bar{2}4\bar{2}\bar{1}$	$000\bar{1}$ / 0001	c
* 30		4$\bar{3}$0 12′				6$\bar{1}$0 23′					1$\bar{1}$0 0′	130 0′
31	3$\bar{4}$0 −12′	5$\bar{4}$0 −10′	3$\bar{5}$0 −17′	7$\bar{4}$0 8′	2$\bar{5}$0 −10′	5$\bar{1}$0 18′		310 −3′		110 −11′		
32		6$\bar{5}$0 −9′		5$\bar{3}$0 17′		4$\bar{1}$0 −20′	1$\bar{6}$0 −3′	720 29′				
33	5$\bar{7}$0 26′	7$\bar{6}$0 17′	4$\bar{7}$0 20′			7$\bar{2}$0 −29′						
34	2$\bar{3}$0 −30′			3$\bar{2}$0 20′			170 −15′	410 −22′		760 29′	6$\bar{7}$0 2′	
35				7$\bar{5}$0 −2′		3$\bar{1}$0 −1′		510 16′		650 3′	5$\bar{6}$0 17′	250 −17′
36		1$\bar{1}$0 −23′	1$\bar{2}$0 −29′	4$\bar{3}$0 −2′	1$\bar{3}$0 7′			610 21′		540 2′	4$\bar{5}$0 11′	
37				5$\bar{4}$0 −24′		5$\bar{2}$0 −7′		710 8′		430 24′		370 −25′
38	3$\bar{5}$0 29′			6$\bar{5}$0 −23′		7$\bar{3}$0 −1′				750 24′	3$\bar{4}$0 25′	
39	4$\bar{7}$0 6′			7$\bar{6}$0 3′							5$\bar{7}$0 3′	
40		6$\bar{7}$0 −21′			2$\bar{7}$0 27′	2$\bar{1}$0 −13′				320 −14′		
41		5$\bar{6}$0 −6′	3$\bar{7}$0 12′								2$\bar{3}$0 7′	20 −6′
42		4$\bar{5}$0 −12′			1$\bar{4}$0 −28′	7$\bar{4}$0 −22′		100 9′		530 −17′		
43	1$\bar{2}$0 17′		2$\bar{5}$0 20′	1$\bar{1}$0 23′		5$\bar{3}$0 −13′			170 5′	740 −26′		
44		3$\bar{4}$0 2′									3$\bar{5}$0 6′	
45		5$\bar{7}$0 −20′				3$\bar{2}$0 −10′			160 17′	210 −17′	4$\bar{7}$0 −17′	470 −17′
46												350 6′
47	3$\bar{7}$0 −2′	2$\bar{3}$0 −16′	1$\bar{3}$0 −23′	5$\bar{6}$0 −20′	1$\bar{5}$0 13′	7$\bar{5}$0 28′		6$\bar{1}$0 −21′		730 −5′		
48				4$\bar{5}$0 −26′		4$\bar{3}$0 28′	010 −9′	5$\bar{1}$0 −26′	150 17′	520 −11′		
49	2$\bar{5}$0 6′					5$\bar{4}$0 6′					1$\bar{2}$0 −6′	230 7′
50		3$\bar{5}$0 −17′		3$\bar{4}$0 −12′	1$\bar{6}$0 13′	6$\bar{5}$0 7′		4$\bar{1}$0 −4′		310 −5′		
51			2$\bar{7}$0 −3′					7$\bar{2}$0 −13′		720 27′		570 3′
? 52		4$\bar{7}$0 20′		5$\bar{7}$0 26′	1$\bar{7}$0 1′							340 25′
53								3$\bar{1}$0 15′	140 −24′	410 −24′	3$\bar{7}$0 −25′	
54	1$\bar{3}$0 23′		1$\bar{4}$0 2′	2$\bar{3}$0 30′		1$\bar{1}$0 −7′				510 14′		450 11′
55		1$\bar{2}$0 −29′						5$\bar{2}$0 9′		610 19′	2$\bar{5}$0 −17′	560 17′
56								7$\bar{3}$0 15′	270 −29′	710 6′		670 2′
57	2$\bar{7}$0 −17′			3$\bar{5}$0 29′								
58			1$\bar{5}$0 −17′	4$\bar{7}$0 6′		6$\bar{7}$0 −5′		2$\bar{1}$0 3′				
59						5$\bar{6}$0 10′			130 11′			
* 60	1$\bar{4}$0 −12′	3$\bar{7}$0 12′				4$\bar{5}$0 4′		7$\bar{4}$0 −6′			1$\bar{3}$0 0′	110 0′

TABLE 7 (*Continued*)

CALCULATED PRINCIPAL COINCIDENCES BETWEEN MUSCOVITE (001) AND TOURMALINE ($10\bar{1}0$)
Divergence from exact coincidence as given. Observed coincidences indicated by asterisk.

	Tourm. $c\wedge$ muscov. 010	$\bar{1}2\bar{1}8$ $1\bar{2}18$		$1\bar{2}18$ $\bar{1}2\bar{1}\bar{8}$		$\bar{1}2\bar{1}4$ $1\bar{2}1\bar{4}$		$1\bar{2}14$ $\bar{1}2\bar{1}\bar{4}$		$\bar{1}2\bar{1}2$ $1\bar{2}1\bar{2}$		$1\bar{2}12$ $\bar{1}2\bar{1}\bar{2}$		$\bar{1}2\bar{1}1$ $1\bar{2}1\bar{1}$		$1\bar{2}11$ $\bar{1}2\bar{1}\bar{1}$		$\bar{2}4\bar{2}1$ $2\bar{4}21$		$2\bar{4}21$ $\bar{2}4\bar{2}\bar{1}$		$000\bar{1}$ 0001		c	
	61					$1\bar{6}0$	−17′									$5\bar{3}0$	3′			100	11′				
	62			$2\bar{5}0$	20′			$1\bar{2}0$	17′			$3\bar{4}0$	18′	170	3′										
	63					$1\bar{7}0$	−29′					$5\bar{7}0$	−4′			$3\bar{2}0$	6′								
	64													160	15′	$7\bar{5}0$	−16′	250	−6′			$2\bar{7}0$	20′	650	14′
	65	$1\bar{5}0$	29′									$2\bar{3}0$	0′			$4\bar{3}0$	−16′							540	13′
	66			$1\bar{3}0$	−23′			$3\bar{7}0$	−2′	010	−7′							370	−14′	$6\bar{1}0$	−19′				
	67													150	15′	$5\bar{4}0$	22′			$5\bar{1}0$	−24′	$1\bar{4}0$	25′	430	−25′
	68	$1\bar{6}0$	29′					$2\bar{5}0$	6′			$3\bar{5}0$	−1′			$6\bar{5}0$	23′							750	−25′
*	69											$4\bar{7}0$	−24′							$4\bar{1}0$	−2′			320	−3′
	70	$1\bar{7}0$	17′	$2\bar{7}0$	−3′													120	5′	$7\bar{2}0$	−11′				
	71																					$1\bar{5}0$	6′	530	−6′
	72													140	−26′	$1\bar{1}0$	9′			$3\bar{1}0$	17′			740	−15′
	73			$1\bar{4}0$	2′			$1\bar{3}0$	23′			$1\bar{2}0$	−13′												
	74													270	29′			470	−6′	$5\bar{2}0$	11′	$1\bar{6}0$	6′	210	−6′
	75																	350	17′	$7\bar{3}0$	17′				
	76							$2\bar{7}0$	−17′							$6\bar{7}0$	11′					$1\bar{7}0$	−6′	730	6′
	77			$1\bar{5}0$	−17′	010	−23′									$5\bar{6}0$	26′			$2\bar{1}0$	5′			520	0′
	78											$3\bar{7}0$	28′	130	9′	$4\bar{5}0$	20′	230	18′						
?	79							$1\bar{4}0$	−12′			$2\bar{5}0$	−24′			$3\bar{4}0$	−26′			$7\bar{4}0$	−4′			310	6′
	80			$1\bar{6}0$	−17′					170	−13′							570	14′	$5\bar{3}0$	5′				
	81															$5\bar{7}0$	12′							720	−22′
	82			$1\bar{7}0$	−29′					160	−1′							340	−24′	$3\bar{2}0$	8′			410	−13′
	83													250	−8′	$2\bar{3}0$	16′	450	22′	$7\bar{5}0$	−14′			510	25′
	84	010	−23′					$1\bar{5}0$	29′			$1\bar{3}0$	−7′					560	28′	$4\bar{3}0$	−14′			610	30′
	85									150	−1′			370	−16′			670	13′					710	17′
	86															$3\bar{5}0$	15′			$5\bar{4}0$	24′				
	87							$1\bar{6}0$	29′							$4\bar{7}0$	−8′			$6\bar{5}0$	25′				
	88											$2\bar{7}0$	13′												
	89							$1\bar{7}0$	17′	140	18′			120	3′			110	11′						
*	90																					010	0′	100	0′

Definite maxima, identifying positions of preferred orientation of the inclusions, can be recognized in Table 5 (Gilsum). These positions are found, from Table 7, to correspond to major calculated coincidences, made by *c* with 010, 130, 110 and 320. Furthermore, there seems to be a tendency towards orientation in positions corresponding to coincidences of *c* with 100, 310 and 340, although the maxima are not well-defined statistically.

In Table 6 (New York City), well-defined maxima are present at positions corresponding to calculated coincidences made by *c* with 010, 110 and 100.

TABLE 5. OBSERVED ORIENTATIONS OF TOURMALINE $(10\bar{1}0)$ UPON MUSCOVITE (001). GILSUM, N. H.

Reference directions: $c\wedge 010$

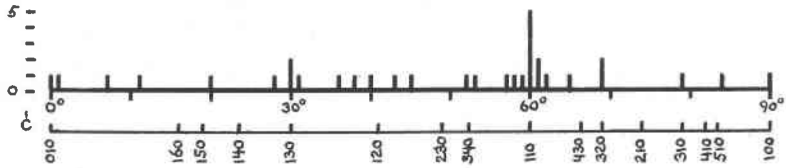
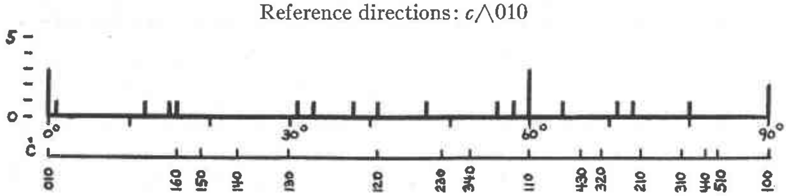


TABLE 6. OBSERVED ORIENTATIONS OF TOURMALINE $(10\bar{1}0)$ UPON MUSCOVITE (001). NEW YORK CITY

Reference directions: $c\wedge 010$



The relative frequency of the various observed orientations differs for the two localities, as in the case of the inclusions flattened on $(11\bar{2}0)$. The relative frequency of the various observed orientations also differs between inclusions flattened on $(11\bar{2}0)$ and on $(10\bar{1}0)$, and varies independently for the two localities. This fact indicates that the coincidences made by the *c* direction alone upon (001) of muscovite do not act independently, but that other coincidences in the contact planes, characteristic of the particular plane of flattening, act in conjunction with *c* in controlling orientation. The percentage of crystals flattened on $(11\bar{2}0)$ and on $(10\bar{1}0)$ is also different for the two localities.

HABIT AND ORIENTATION OF INCLUSIONS FLATTENED ON (0001)

Crystals flattened on (0001) were found to comprise 16.7% of the total number of inclusions observed from Gilsum, and 22.9% of the

TABLE 8

CALCULATED PRINCIPAL COINCIDENCES BETWEEN MUSCOVITE (001) AND TOURMALINE (0001)
Divergence from exact coincidence as given. Observed coincidences indicated by asterisk.

	Tourm. $11\bar{2}0 \wedge$ muscov. 010	$10\bar{1}0$ / $\bar{1}010$		$\bar{1}100$ / $1\bar{1}00$		$0\bar{1}10$ / $01\bar{1}0$		$11\bar{2}0$ / $\bar{1}\bar{1}20$		$1\bar{2}10$ / $\bar{1}2\bar{1}0$		$2\bar{1}\bar{1}0$ / $\bar{2}110$		$21\bar{3}0$ / $\bar{2}\bar{1}30$		$12\bar{3}0$ / $\bar{1}\bar{2}30$		$\bar{3}210$ / $3\bar{2}\bar{1}0$		$2\bar{3}10$ / $\bar{2}3\bar{1}0$		$1\bar{3}20$ / $\bar{1}3\bar{2}0$		$\bar{3}120$ / $3\bar{1}\bar{2}0$	
*	0	130	0′	100	0′	$1\bar{3}0$	0′	010	0′	$1\bar{1}0$	0′	110	0′					530	1′	$5\bar{3}0$	1′	$2\bar{3}0$	0′	230	0′
	1																	740	−8′						
	2																			$3\bar{2}0$	4′			570	−4′
	3													170	1′			210	1′	$7\bar{5}0$	−18′	$3\bar{5}0$	−1′	340	18′
	4					$2\bar{7}0$	20′			$6\bar{7}0$	2′	650	14′							$4\bar{3}0$	−18′	$4\bar{7}0$	−24′		
?	5	250	−17′	$7\bar{1}0$	17′					$5\bar{6}0$	17′	540	13′	160	13′			730	13′					450	4′
	6			$6\bar{1}0$	30′					$4\bar{5}0$	11′							520	7′	$5\bar{4}0$	20			560	10′
	7	370	−25′	$5\bar{1}0$	25′	$1\bar{4}0$	25′					430	−25′							$6\bar{5}0$	21′			670	−5′
	8			$4\bar{1}0$	−13′					$3\bar{4}0$	25′	750	−25′	150	13′			310	13′			$1\bar{2}0$	−13′		
	9			$7\bar{2}0$	−22′					$5\bar{7}0$	3′	320	−3′												
	10																	720	15′						
?	11	120	−6′	$3\bar{1}0$	6′	$1\bar{5}0$	6′			$2\bar{3}0$	7′	530	−6			010	0′	410	− 6′	$1\bar{1}0$	7′			110	−7′
	12											740	−15′												
	13			$5\bar{2}0$	0′									140	−28′			510	−28′			$3\bar{7}0$	28′		
	14			$7\bar{3}0$	6′	$1\bar{6}0$	6′	170	−6′	$3\bar{5}0$	6′	210	−6′					610	−23′			$2\bar{5}0$	−24′		
	15	470	−17′							$4\bar{7}0$	−17′			270	27′					$6\bar{7}0$	9′			650	7′
	16	350	6′	$2\bar{1}0$	−6′	170	−6′	160	6′			730	6′							$5\bar{6}0$	24′			540	6′
	17											520	0′							$4\bar{5}0$	18′			430	28′
	18			$7\bar{4}0$	−15′															$3\bar{4}0$	−28′			750	28′
*	19	230	7′	$5\bar{3}0$	−6′			150	6′	$1\bar{2}0$	−6′	310	6′	130	7′			100	7′			$1\bar{3}0$	−7′		
	20																			$5\bar{7}0$	10′			320	−10′
	21	570	3′	$3\bar{2}0$	−3′							720	−22′												
	22	340	25′	$7\bar{5}0$	−25′							410	−13′							$2\bar{3}0$	14′			530	−13′
	23			$4\bar{3}0$	−25′			140	25′	$3\bar{7}0$	−25′	510	25′									$2\bar{7}0$	13′	740	−22′
	24	450	11′									610	30′	250	−10	170	1′	$7\bar{1}0$	10′						
	25	560	17′	$5\bar{4}0$	13′					$2\bar{5}0$	−17′	710	17′					$6\bar{1}0$	23′	$3\bar{5}0$	13′			210	−13′
	26	670	2′	$6\bar{5}0$	14′			270	20′					370	−18′	160	13′	$5\bar{1}0$	18′	$4\bar{7}0$	−10′	$1\bar{4}0$	18′		
	27																	$4\bar{1}0$	−20′					730	−1′
	28																	$7\bar{2}0$	−29′					520	−7′
	29																								
*	30	110	0′	$1\bar{1}0$	0′	010	0′	130	0′	$1\bar{3}0$	0′	100	0′	120	1′	150	−1′	$3\bar{1}0$	−1′	$1\bar{2}0$	1′	150	−1′	310	−1′

total number of inclusions observed from New York City. The basal inclusions rarely exceed 1 mm. in diameter, and were readily recognized by their habit and by the absence of pleochroism and birefringence.

The basal inclusions in the Gilsum muscovite have a symmetrical hexagonal outline (Fig. 1); rarely an additional set of three or of six minor modifying faces is present. The bounding faces of the hexagonal crystals can not be identified with certainty, but must belong either to $(11\bar{2}0)$, or to $(10\bar{1}0)$ and $(01\bar{1}0)$ in combination. Since $(01\bar{1}0)$ is usually subordinate in development to $(10\bar{1}0)$ when these forms occur together, the equant outline of the crystals suggests that the bounding form is $(11\bar{2}0)$. This interpretation is supported by the fact that $(11\bar{2}0)$ is characteristic of the crystals that are flattened in the prism zone from this locality. The additional three or six modifying faces are evidently those of $(10\bar{1}0)$ and $(01\bar{1}0)$. A number of the inclusions do not possess straight sides, but are rounded or irregularly developed.

The basal inclusions in the New York City muscovite are twelve-sided (Fig. 2), with $(11\bar{2}0)$, $(10\bar{1}0)$ and $(01\bar{1}0)$ as the bounding forms. The crystals are usually distorted in such manner as to prevent the separate identification of the several forms. On a few crystals the habit was dominated by a triangular set of three faces; these faces were assumed to belong to $(10\bar{1}0)$.

The orientation of the inclusions to the muscovite can be defined, as with the prismatic inclusions, by stating the position of a direction in the plane of flattening relative to a direction in (001) of the muscovite. In Table 8 the calculated coincidences for a few directions of relatively low indices in tourmaline (001) and muscovite (001) are tabulated for values of the angle tourmaline $11\bar{2}0 \wedge 010$ muscovite. The symmetry of these basal plates restricts the range of measurement to 0°–30°.

TABLE 9. OBSERVED ORIENTATIONS OF TOURMALINE (0001) UPON MUSCOVITE (001). GILSUM, N. H., AND NEW YORK CITY

Reference directions: $11\bar{2}0 \wedge 010$

Orientation	Gilsum, N. H.	New York City
Percussion figure 010, 110, $1\bar{1}0$	39	6
Pressure figure 100, 130, $1\bar{3}0$	14	2
Perc. or press. fig.; undetermined	4	7
19° position (Table 8)	9	3
11° position (Table 8)	?	?
5° position (Table 8)	?	
Other positions	29	6
Position unknown	24	1
Total number observed	119	25

The observed orientations of the basal inclusions are cited in Table 9. The orientation can be definitely stated only when a bounding form—reference direction—can be identified. Inclusions on which the position of $(11\bar{2}0)$ is known and which make angles of 0°, 30° and 60°, without respect to sign, with muscovite 010 for the various faces of this form are oriented to the rays of the percussion figure, while inclusions in which the angles are 30°, 60° and 90° are oriented to the rays of the pressure figure (Fig. 1). However, if the actual position of a reference direction, such as $11\bar{2}0$, could be identified in the inclusions, as in the case of those flattened in the prism zone, in which the position of *c* is readily determinable, the range of measurement would be 90° and it very probably would be found that the orientation varies with respect to the several rays of the percussion and pressure figures. A number of twelve-sided crystals on which the separate forms could not be identified were noted to be oriented to both the pressure and percussion figures (the interfacial angles being 30°); these crystals were tabulated separately.

A number of inclusions occurred at values of $11\bar{2}0 \wedge 010$ (taken as the minimum angle made by a face of $(11\bar{2}0)$ with muscovite 010) between 0° and 30°, as follows:

Gilsum: 2, 3, 4, 5, 5, 5, 6, 7, 8, 9, 10, 11, 11, 12, 12, 13, 15, 16, 16, 17, 18, 18, 18, 18, 19, 19, 19, 19, 20, 20, 21, 22, 23, 25, 26, 26, 27, 28

New York City: 3, 7, 11, 12, 15, 18, 19, 19, 26

From these values a marked tendency can be recognized for orientation at an angular position of 19°, and ill-marked tendencies for orientation as 11° and 5°. As is seen from Table 8, these orientations correspond to calculated coincidences between directions of relatively low indices in the tourmaline and the muscovite, although the coincidences are not as marked as at the 0° (percussion figure) and 30° (pressure figure) positions. It is difficult, however, to identify the particular direction or directions in the tourmaline to whose coincidences the orientation can be referred.

In calculating the percentages of orientation at each position (see Summary), inclusions whose orientation was uncertain or undeterminable were distributed among the recognized orientations in proper proportion.

Since the parallel forms—pedions—(0001) and $(000\bar{1})$ of tourmaline are unlike in structure, it can be presumed that the orientation of the basally flattened inclusions also varies with respect to these forms. Hemimorphic crystals, including tourmaline, tend to attach themselves to a substrate by a particular pole of the polar axis.[12]

[12] Holzner, J., *Zeit. Krist.*, vol. **65**, pp. 175–179, 1927.

Discussion

It is generally found in oriented growths that the surface of contact is characterized by the coincidence of crystal planes with similar atomic arrangements and in which the atomic spacings are equal or small multiples. The fact of orientation in such instances is usually immediately apparent by reason of a parallelism among the overgrowing crystals, or by the parallelism of obvious crystallographic characters of the overgrowing and substrate crystals. The recognition of orientation has been ordinarily confined to growths showing such features. Growths which are not arranged in such manner and which seemingly are randomly distributed upon the substrate, or which do not appear to satisfy the particular conditions of crystallographic coincidence mentioned, have been termed unoriented.

On the other hand, it may be considered that a degree of frequency is associated with each of an infinitude of crystallographic orientations between contact growths of any two species. The term unoriented then would not apply to any over- or inter-growing crystal. The various positions of crystallographic orientation, each of different overall coincidence, may be expressed discontinuously by reference to particular directions in the contact surface. It is apparent, on such a basis, that the relative frequency of the various orientations would have to be investigated by a statistical method. It is also necessary to apply a statistical method in a theory relating orientation to specific, limiting conditions of crystallographic coincidence to establish the fact of random distribution in all but the postulated positions.

In the present study, the inclusions appeared on casual examination to be unoriented to the muscovite. Tendencies for orientation, however, became apparent on statistical study, and the number of orientations thus established increased with increasing number of measurements. Further, in the present stage of development of the graphs, the observations that occur between the established orientations, and which can not be recognized statistically as being oriented, do not appear to be distributed by chance. For instance, in Table 2 such observations are more concentrated in the regions from 1° to 8°, 42° to 48°, 54° to 58°, and 62° to 67°, than in the regions from 9° to 25°, 35° to 38°, 75° to 78° and 81° to 89°. If these observations were of chance origin, uninfluenced by considerations of crystallographic coincidence with the mica, they would be equally distributed in these regions. The fact that they are unequally distributed indicates, on the contrary, that their origin has been controlled. Presumably the statistical population or the accuracy of measurement is insufficient to define the separate positions of orientation.

A definite indication that on increasing the number of observations, additional planes of flattening, with orientations thereon, will also be established, is found in the fact that separate plotting of $c \wedge 010$ of the inclusions from both localities identified as being flattened other than on (0001), $(11\bar{2}0)$ or $(10\bar{1}0)$, the exact plane of flattening being unknown, yielded a graph with a well-defined maxima at 60°.

It can not be concluded, however, that the number of orientations established will increase indefinitely with increasing number of observations, as it may be that the observations are not sufficiently numerous to exhaust a given range of specific orientations, and beyond which only random distribution would be found. Nevertheless, a view relating orientation to a frequency basis could be regarded as being established as a general law, if it were found invariably that the recognition of orientations between two species of crystals depended only on the extent of statistical investigation. On such a basis, orientation would be more frequent in positions of relatively high coincidence, inasmuch as crystallization from solution upon a substrate tends to be so ordered that crystallographically similar planes in the overgrowing and substrate crystals coincide.[13]

Growths of low coincidence, and of low frequency, have been obtained experimentally in the limiting case of complete identity of structure of the overgrowing and substrate crystals, as with growths of K alum upon K alum.[14] Such so-called heterotwins or heterogrowths, in which the position of the overgrowing crystal can not be expressed by any operation of symmetry, as in ordinary twins, have also been observed with quartz, feldspar and other species. In general, in the case of complete identity or near identity of structure between two crystals, the frequency of orientation in positions other than that of complete conformability would be relatively small. With increasing dissimilarity in structure, the percent of occurrences oriented in any one position would decrease, and the relative frequency of the different orientations would approach each other in value. The fact of orientation would then go unrecognized on casual examination, and a large number of observations would be necessary to establish even the more frequent positions of orientation. With a close similarity of structure in a few planes and a general dissimilarity in the other planes, the like planes would be loci of orientations of relatively high frequency, and a wide interval would separate them statistically from orientations in the unlike planes.

The greatest number of orientations previously recognized for a pair of substances seem to be those for the exsolution growths of hematite

[13] Frondel, C., *Am. Jour. Sci.*, ser. 5, vol. **30**, p. 54, 1935.

[14] Schubnikov, A., and Schaskolsky, M., *Zeit. Krist.*, vol. **85**, pp. 1–16, 1933.

in feldspar, for which 10 different contact planes, with an undetermined number of orientations thereon, have been found.[15] A greater number of orientations would be expected for a given number of observations of intergrowths than for the same number of observations of overgrowths, inasmuch as the opportunity for orientation in positions of relatively high coincidence is not restricted to the coincidences of a particular substrate plane.

Associated Minerals

In addition to the tourmaline, the following minerals were observed as inclusions in the Gilsum muscovite: albite, beryl, biotite, garnet, magnetite, smoky quartz (in sheets 1 mm. or so thick and 100 sq. cms. and more in area), zoisite fibers, and abundant deep brown particles, occasionally developed into dendrite-like growths, of hematite. The magnetite and the hematite growths were readily seen to be oriented parallel to either the percussion or pressure figures of the mica.

In a few instances the tourmaline inclusions were partly or completely surrounded by a narrow irregular border of greenish biotite. Magnetite crystals were observed to abut against or to be molded on tourmaline crystals. The muscovite laminae are not bent around the tourmaline inclusions, but are intersected by and interfinger with the tourmalines. The two minerals have crystallized simultaneously, and the inclusions represent overgrowths that have been enclosed by the continued growth of the muscovite. Occasionally the tourmaline inclusions are arranged in indistinct rows outlining zones of growth in the mica.

A variety of minerals were observed as inclusions in the New York City muscovite. Primary inclusions, enclosed during the growth of the muscovite crystals, included apatite, actinolite, biotite, dumortierite, garnet, hematite, magnetite, quartz, and unidentified minerals in minute acicular and pin-point crystals, frequently surrounded by pleochroic halos. Secondary inclusions, apparently deposited in cleavage openings by meteoric solutions, included chalcedony, calcite, siderite and pyrite. Goethite, limonite, bright-red spherules of turgite (?), and a pale greenish blue unidentified mineral, possibly a sulphate containing ferrous iron, occurred as alteration products of the pyrite. The goethite crystals showed a marked tendency for orientation parallel to the percussions and the pressure figures of the mica. A parallel arrangement was also noted among some of the quartz and pyrite inclusions.

[15] Andersen, O., *Am. Jour. Sci.*, ser. 4, vol. **40,** p. 364, 1915.

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