USE OF THE WULFF NET IN MINERAL DETERMINATION WITH THE UNIVERSAL STAGE

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Abstract

Use of the universal stage in determination of rock-forming minerals, and especially of the plagioclases, is increasing. The plotting of data obtained by such measurements usually presents some difficulty. The Wulff stereographic net is a most convenient and valuable aid in such work for its use obviates much tedious construction. In this paper are given, in the simplest possible form, the necessary procedures for plotting the coordinates of optical symmetry planes, morphological reference planes and their poles. Instructions for transforming the projection to conform with standard reference charts, such as those prepared by Reinhard, are also included.

Introduction

In recent years use of the universal stage for mineral determination has greatly increased. More particularly, plagioclase determination by the Fedorov method has been recognized as a technique possessing more inherent accuracy than other thin-section procedures. The early Fedorov diagrams[1] were composite in character. They show the migration of the poles of optical symmetry axes in the plagioclase series with respect to conspicuous morphological reference planes. These diagrams are relatively difficult to use and interpret. Simplified stereographic projections of these data were later prepared by Reinhard.[2] The latter afford a somewhat easier means of plagioclase determination.[3] However, plotting and interpretation of the data necessary even for general mineral determination with the universal stage is sometimes inordinately difficult at first. It is the purpose of this paper to describe in detail the essential plotting procedures required in this type of work.

In the writer's opinion, a most essential accessory for plotting the coordinates obtained in mineral determination with the universal stage is the Wulff stereographic net.[4] With its use much construction with various protractors and scales is eliminated. The net is used directly as a plat, and the coordinates obtained by measurement are quickly and accurately located.

[1] Fedorov, E. S., Universalmethode und Feldspatstudien. Part II. Feldspathbestimmungen: *Zeits. Krist.*, **27,** 337–398 (1897).

[2] Reinhard, M., *Universal Drehtischmethoden.* 119 pp. B. Wepf & Cie., Basel, Switzerland **(1931)**.

[3] These five charts are included in Reinhard's book but may also be purchased in separate sets from B. Wepf & Cie, Basel, Switzerland, at a cost of approximately $0.75.

[4] Wulff, George, Untersuchungen im Gebiete der optischen Eigenschaften isomorpher Kristalle: *Zeits. Krist.*, **36,** 1–28 (1902).

General Concepts

In stereographic projection[5] the center of the crystal, whose faces or optical symmetry planes and vectors are projected, is imagined to be coincident with the center of a *reference sphere* (Fig. 1). By convention,

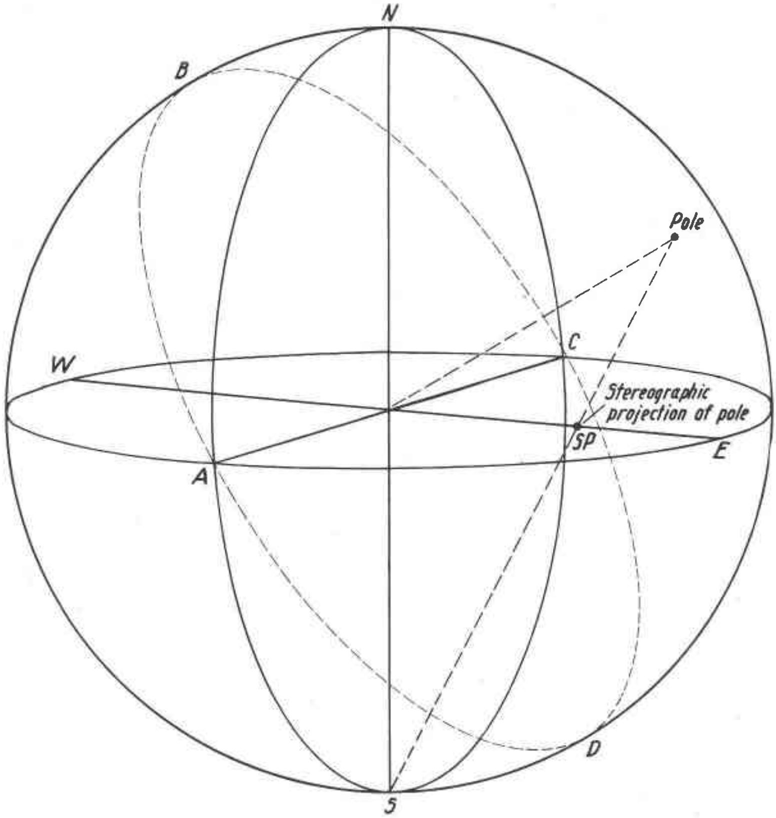

Fig. 1. Reference sphere showing equatorial plane *AWCE;* vertical great circle or meridian *ANCS;* and inclined plane *ABCD*. The pole of *ABCD* and its stereographic projection are also given.

[5] For additional information on the principles of this projection see:
Penfield, S. L., The stereographic projection and its possibilities from a graphical standpoint: *Am. Jour. Sci.*, ser. 4, **11,** 1–24, 115–144 (1901).
Idem, On the solution of problems in crystallography by means of graphical methods, based upon spherical and plane trigonometry: *Am. Jour. Sci.*, ser. 4, **14,** 249–284 (1902).
Johannsen, A., *Manual of Petrographic Methods*, Chap. II, 2nd. Ed. **(1918)**.
Boecke, H. E., *Die Anwendung der stereographischen Projection bei kristallographischen Untersuchungen*. pp. 58, Gebrüder Borntraeger, Berlin **(1911)**.
Goldschmidt, V., Über stereographischen Projection: *Zeits. Krist.*, **30,** 260–271 (1899).
Dana, J. D., *Textbook of Mineralogy*. 4th Ed. (Revised by W. E. Ford), 49–56 **(1932)**.

the crystal is oriented so that the crystallographic axis "*c*" coincides with the north-south or vertical axis of the sphere. The eye is assumed to be at the south pole of the sphere. A perpendicular, which passes through the center of the sphere, is erected from the projected face and extended until it intersects the sphere surface. The point of intersection of the face normal with the sphere surface is the *pole* of that face (Fig. 1). A line drawn from any such pole to the south pole passes through the equatorial plane of the reference sphere. The point at which this imaginary line pierces the equatorial plane is the *stereographic projection* of the face or plane in question (Fig. 1). The equatorial plane of the reference sphere is, therefore, the *projection plane* (*Grundkreis*).

In stereographic projection only poles lying in the northern hemisphere, together with those which fall on the equator of the sphere, are customarily projected. Projected poles of all faces parallel to the vertical axis of a crystal lie on the projection circumference. The projected pole of any face which is perpendicular to the vertical axis of the reference sphere, i.e., a horizontal face, lies at the center of the projection.

A stereographic projection has two important properties which make it particularly suitable for crystallographic and universal stage work. First, the projections of both great and small circles are circular arcs and not ellipses. Second, the angular relationship between any two poles on the reference sphere is preserved in the projection. That is, the projection is angle-true (*Winkeltreu*).

Definitions

A *great circle* (*Grosskreis*) is one with a diameter equal to that of the reference sphere (Fig. 1, *ABCD*). The plane of any great circle passes through the center of the sphere. The projected arcs of great circles pass through diametrically opposite points on the circumference of the projection plane. Any great circle which passes through both the north and south poles of the reference sphere is perpendicular to the equatorial plane and is projected as a straight line. Such perpendicular great circles are called *meridians* or *vertical great circles* (Fig. 1, *ANCS*). Any great circle which makes an angle with the north-south axis appears as a circular arc when projected. The equator of the reference sphere is the only possible horizontal great circle, and its projection coincides with the stereographic projection plane.

A circle with a diameter smaller than that of the reference sphere is a *small circle* (*Kleinkreis*). Horizontal small circles, which lie parallel to the equatorial plane of the reference sphere, are projected as concentric circles about the center of the projection. A vertical small circle is one whose center lies in the equatorial plane and whose plane is parallel to the vertical axis of the reference sphere. Vertical small circles are projected as circular arcs of varying radii.

Description of Wulff Net

The base of the standard Wulff net is a circle 20 cm. in diameter (Fig. 2). On this are projected a series of great circles at inclinations of every two degrees with respect to the vertical axis of the reference sphere. At right angles to these, and also every two degrees, is a series of vertical small circles. Since the projected circles are given at two-degree intervals, interpolation with considerable accuracy is possible. Only those parts of the circles lying on the upper hemisphere of the reference sphere are given on the net.

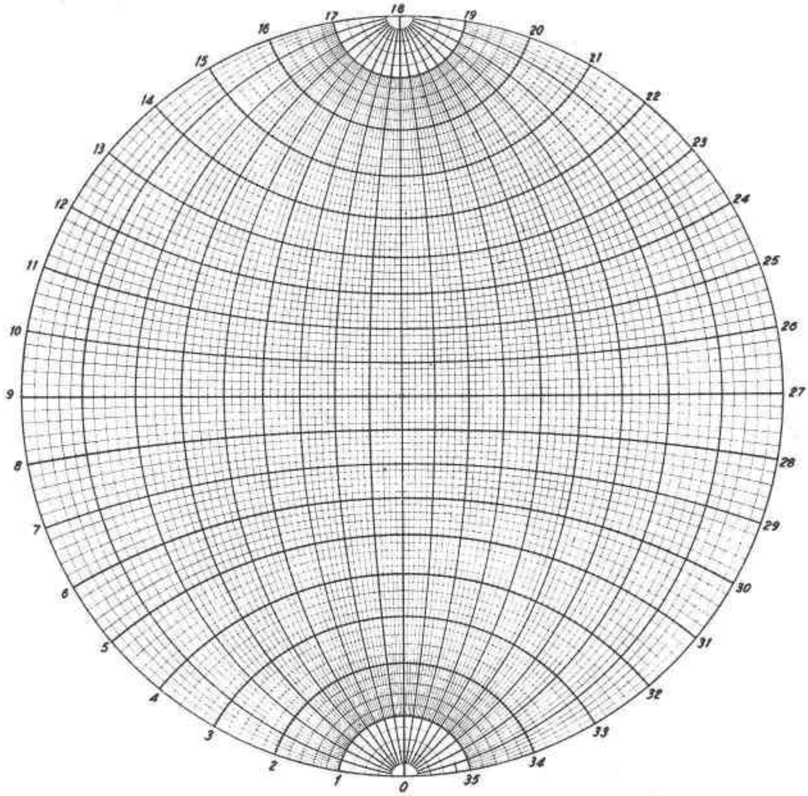

Fig. 2. Wulff net marked at 10° intervals as for use with a standard 4-circle universal stage.

The circumference of the net represents the horizontal great circle corresponding to the equatorial plane of the reference sphere.

The north-south diameter or axis of the net is the projection of a great circle representing a vertical plane containing the north and south poles of the reference sphere. All inclined great circles on the net have the same azimuth. Hence, they pass through the same two diametrically opposite points on the equatorial plane. These two points on the net are the points of intersection of the north-south diameter with the net circumference at which loci the great circles converge.

The east-west diameter of the net is also a straight line. It is a limiting case, being the projection of the largest possible vertical small circle.

When the net is used for plotting the coordinates of a crystal plane, the plane of the thin section is conceived to be coincident with the projection plane. That is, the individual grain examined is visualized as being at the center of the reference sphere, and the planes and axes of optical symmetry are projected therefrom.

With use of the net, projected poles and reference planes of an individual grain can be easily rotated so as to bring any desired plane into coincidence with the projection plane. This is of great assistance in clarifying symmetry problems and in bringing certain optical reference directions into conventional orientation. The ease and rapidity with which planes can be thus interchanged gives the Wulff net a further distinct advantage.

Preparation of Net for Use

Wulff nets, commonly printed on heavy paper stock, should be permanently mounted on stiff cardboard, plywood, or bakelite panels. In actual use a sheet of tracing paper, approximately 9×9 inches is a convenient size, is centered over and superposed on the net. All constructions are made by manipulating this sheet; counting off measured inclinations, tracing necessary great circles, and directly plotting poles. If much data are plotted, it is advisable to glue a small piece of tough paper ($\frac{1}{2}\times\frac{1}{2}$ inch) in the center of the sheet. There is then much less chance of enlarging the perforation and introducing errors.

The tracing paper is centered over the net and there fixed, usually with a pin from above. Pins with knurled heads may be procured, but ordinary ones with glass, sealing wax, or cork heads are also quite satisfactory. Alternatively, the center of the net may be pierced from below and the pin or thumb tack fixed with adhesive on the under side.

After superposition of the tracing paper the circumference of the underlying net circle is traced. It is not absolutely necessary to trace the complete circle. Parts of the net circumference only, say small arcs at 120° intervals, may be drawn. This much at least should be done so that the projection may be readily centered, for it will be necessary when making comparisons with standard reference charts as is often required in plagioclase determination.

Before the net can be used the degrees on its circumference must be plainly marked (Fig. 2). The order in which the degrees are numbered, and the position of the zero point, depend upon the type of stage. With a standard four-circle instrument the 0° mark of the A_1 circle[6] is directly in front of the operator and nearest him when the stage is in the rest position. Hence, the south end of the north-south diameter of the net is the 0° point. Counting clockwise at the west point is the 90° mark, at the north end is the 180° mark, and at the east point the 270° mark.

If the current Emmons' model 5-circle stage is used, the 0° mark on the A_1 circle is directly opposite and farthest away from the observer when the instrument is in the rest position. The 0° point then lies at the north end of the north-south diameter; the 90° mark at the east point; the 180° mark at the south; and the 270° mark at the west point.

Degrees are usually marked at 10° intervals, and always outside of the net circumference. Two sets of numbers may be laid out so that the net can be used interchangeably with either type of stage without confusion.

An arrow or similar index mark is now made on the circumference of the circle drawn on the tracing paper. This index mark is placed directly over the 0° mark of the net beneath. If proper orientation during both measurement and plotting is to be maintained, this zero point on the tracing paper must be plainly visible.

The coordinates obtained by measurement are often placed in a corner of the tracing paper itself rather than recorded separately. Sketches of crystal outlines, twinning elements, inclusions, and other structures in the measured grain may also conveniently be made directly on the tracing paper. In this way orientation can be verified at a glance and correlation of morphological features, optical planes, and axes previously plotted can be readily made.

Constructions

The three constructions essential for plotting the customary data secured in mineral determination are as follows:

(A) Construction of the great circle of a plane.
(B) Construction of the pole of a great circle.
(C) Construction of the third plane of optical symmetry.

In addition, it is readily feasible and extremely convenient to

(D) measure the angle between two stereographically projected planes.
(E) measure the angle between two stereographically projected poles.

[6] The axial nomenclature here used is that of Berek. See: Berek, Max, *Mikroskopische Mineralbestimmung mit Hilfe der Universaldrehtischmethoden*, 9–11, Berlin **(1924)**.

To illustrate the essential constructions it will be assumed that the coordinates (Table 1) of two optical symmetry planes and two morphological reference planes in a crystal of plagioclase have been determined.

TABLE 1

Reference Planes	Symbol	Axis		
		A_1	A_2	A_4
Optical symmetry planes				
N_α (constructed)	⊙	209°	56° R(ight)	
N_β	-¦-	258°	24° L(eft)	
N_γ	△	337°	23° R	
Composition plane	◈	136°	13° L	
Cleavage plane	⋕	203°	4° L	
Optic axis reading				241°

A. To construct the great circle of a plane

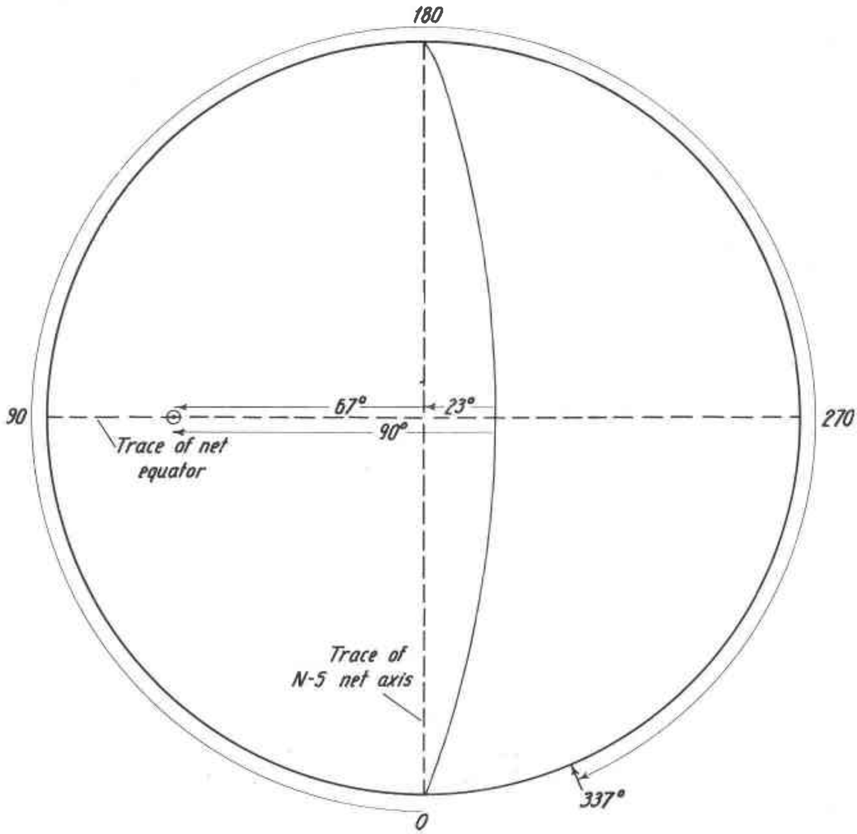

FIG. 3*a*. Showing the method of plotting a great circle and its corresponding pole.

1. Rotate tracing paper until its index mark rests at that degree mark on the net circumference corresponding to the A_1 circle reading.

2. From the net center, along the equator, count off the number of degrees of rotation about the A_2 axis. (The east-west diameter is usually spoken of as the *equator* of the net.) If the reading is taken off the left arc of A_2, count left from the net center; if off the right arc, count right from center.

3. On the transparent paper trace the great circle visible on the underlying net, which passes through the point on the equator thus defined.

Example. To plot the plane of N_γ as given in Table 1. Rotate the tracing paper until its index lies at the 337° mark on the net circumference. From the net center, holding the index mark squarely over the 337° point, count to the right 23° along the equator (Fig. 3*a*). Trace the

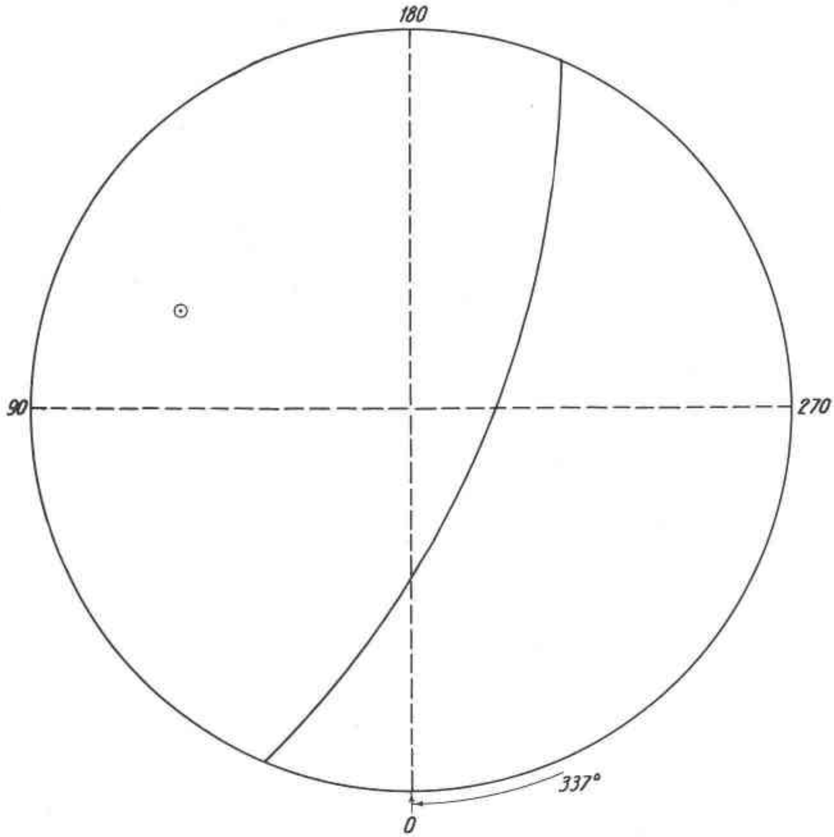

Fig. 3*b*. The great circle and its pole shown in true orientation after rotation of tracing paper back to 0° on the net circumference = rest position of the stage.

entire arc of the great circle cutting the equator at this point. This arc is the projection of the required great circle. If, now, the tracing paper is rotated back until its index mark lies over the 0° mark on the net, the great circle just drawn will indicate the direction and inclination of the measured plane in space (Fig. *3b*).

B. To construct the pole of a great circle

The great circle just constructed represents the stereographic projection of an optical reference plane in the measured crystal. By definition, then, its pole is that point on the reference sphere at which a normal, passing at the same time through the center of the sphere, intersects its surface. The projection of this pole is, therefore, a point 90° from the great circle.

In beginning this construction a plotted great circle must, in all cases, first be brought into coincidence with its corresponding great circle on the net. Hence:

1. Rotate the tracing paper until the index mark lies over that degree mark on the net circumference corresponding to the A_1 reading.
2. Along the equator count 90° to the left or right, depending on the individual case, from the great circle.
3. At the point thus determined prick or mark the tracing paper directly over the net equator. This is the required pole.

In the example the great circle is 23° to the right. Hence, count to the left 67° from the net center and plot the pole (Fig. *3a*).

Were the measured plane vertical, its great circle would, when plotted, be a straight line coinciding with the north-south diameter of the net. Its pole would fall on the circumference of the projection. If the measured plane were horizontal, its great circle would coincide with the projection plane and be represented by a circle corresponding to the net circumference. Its pole would be coincident with the center of the projection.

In actual practice a pole is plotted at the same time the great circle is traced. The reverse operation is to draw the great circle of a given pole. To do this the tracing paper is rotated until the pole lies on the equator and a great circle crossing the equator 90° from the pole is then constructed.

C. Construction of the third plane of optical symmetry

In mineral determination with the universal stage the form of the indicatrix and its orientation with respect to crystallographic reference planes are determined. Biaxial minerals possess three mutually perpendicular planes of optical symmetry. To determine the character of the indicatrix the coordinates of two optical symmetry planes are first measured. The great circles and respective poles of these two planes are plotted. Then, according to the usual methods, the coordinates of the third optical symmetry plane are determined by construction.

Since the three optical symmetry planes are mutually perpendicular, the great circle of any one passes through the pole of both the other planes. Each pole of the two measured planes is 90° distant from P, the intersection point of their respective great circles. The great circle of the third plane is, therefore, that one which joins the poles of the two measured planes. The intersection point P of the great circles representing the two measured planes must also be the pole of the third symmetry plane.

To construct the third plane of optical symmetry:

1. Rotate the tracing paper until the poles of the two measured planes lie on the same great circle of the net (Fig. 4*a*). If both measurement and previous construction were accurate, point P, at the intersection of the two great circles, will lie on the equator 90° from the great circle defined by the two poles.

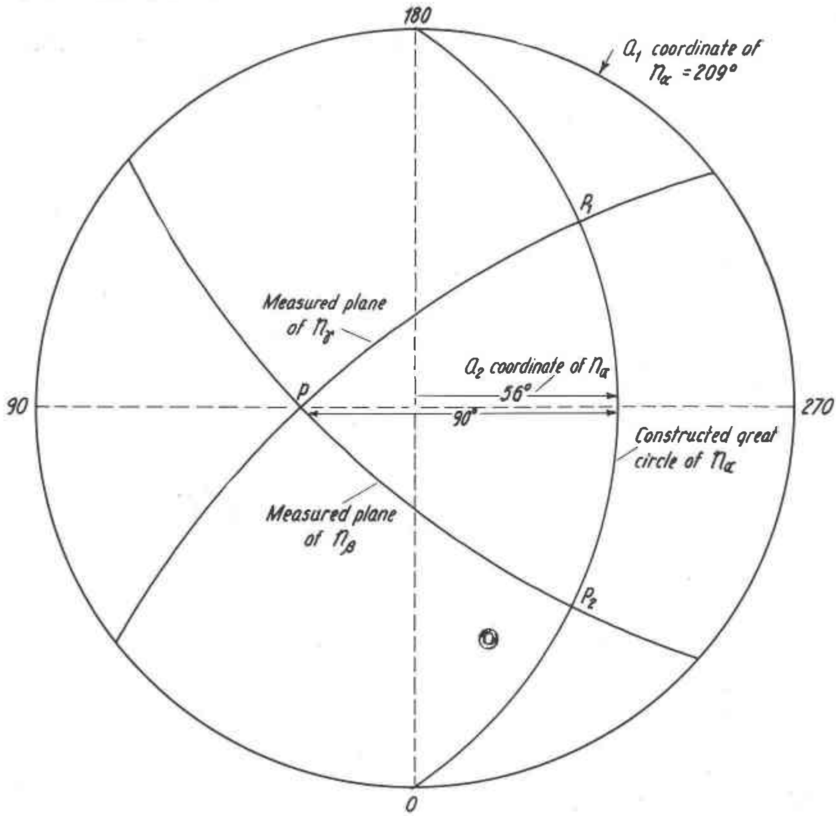

FIG. 4*a*. Method of constructing the great circle of the third optical symmetry plane and derivation of its coordinates.

2. Trace off the great circle defined by the two poles (Fig. 4*a*, P_1 and P_2).

3. The coordinates of the third optical symmetry plane can now be obtained.

(a) Hold the tracing paper so that the great circle of the third optical symmetry plane lies exactly over its corresponding great circle on the net. The A_1 coordinate is found on the net circumference at the point where the index mark of the tracing paper then lies.

(b) The A_2 coordinate is found by counting from the center, along the equator, the degrees of inclination of the constructed plane.

In the example the coordinates of the third plane of symmetry were found to be $N_\alpha 209°–56°$ R (Fig. 4*a*).

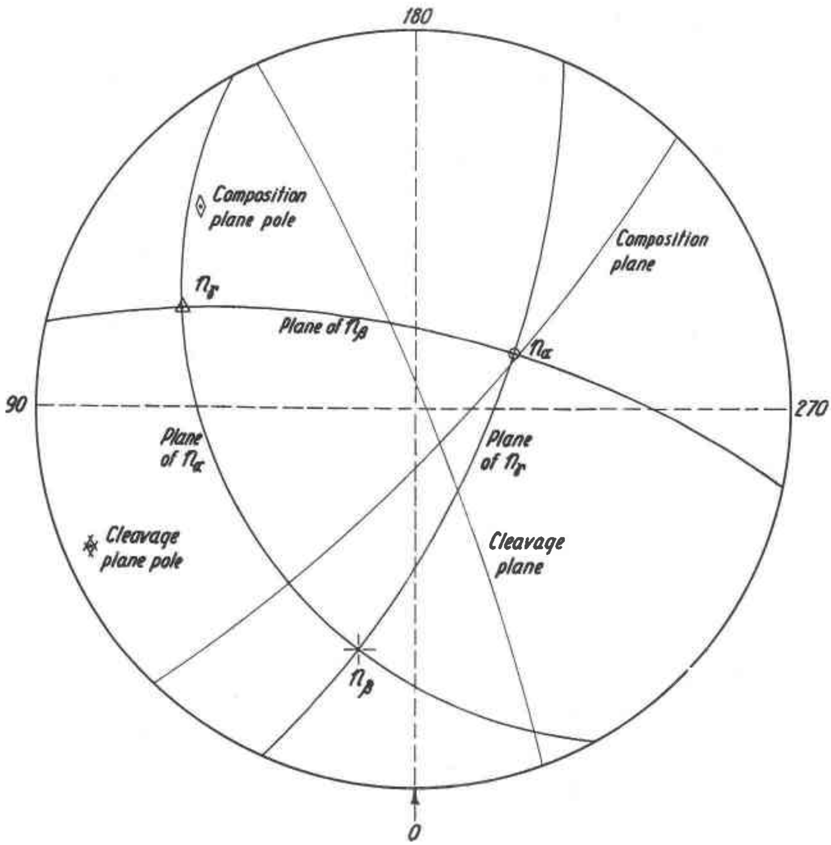

FIG. 4*b*. Completed stereogram with three optical symmetry planes, composition plane and cleavage plane plotted.

D. To measure the angle between two stereographically projected planes

The true angle between two stereographically projected planes is measured along the arc of the great circle perpendicular to the axis of intersection of the two planes.

1. Vertical planes are projected as straight lines, i.e., as arcs of infinite radii, which intersect at the center of the projection. Since the projection plane is the equatorial plane of the reference sphere, the angle between any two vertical great circles can be measured without further construction.

To find this angle count along the net circumference the degrees of arc between the intersection points R and R' of the two meridians (Fig. 5a). Vertical great circles may be thought of as geographical meridians. Hence, this procedure is tantamount to determining longitude by measurement along the earth's equator.

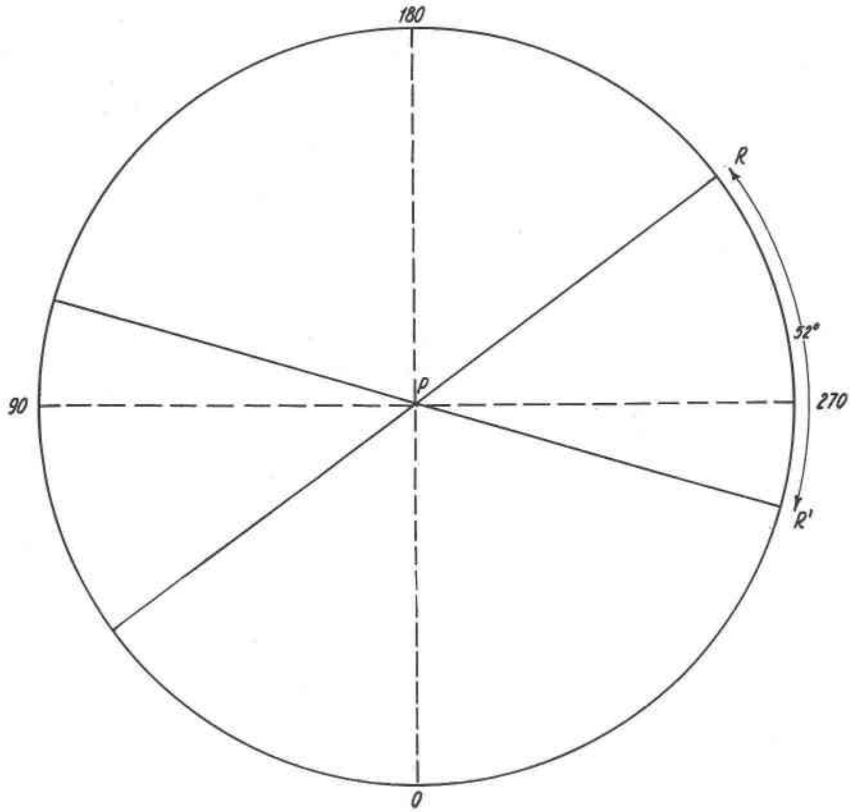

FIG. 5a. Method of determining angle between two vertical planes by measurement along the net circumference.

2. In practice many measured planes will be inclined with respect to the vertical axis of the reference sphere. The great circle of the plane normal to two inclined planes is not coincident with the projection plane. Therefore:

(a) Rotate the tracing paper until P, the intersection point of the two inclined great circles, lies on the equator.

(b) Along the equator count 90° to the left or right of the intersection point P. This will determine the great circle.

(c) Along the arc of this great circle, count the number of degrees between the points of intersection R and R' of the two given great circles. This is the required angle (Fig. 5b).

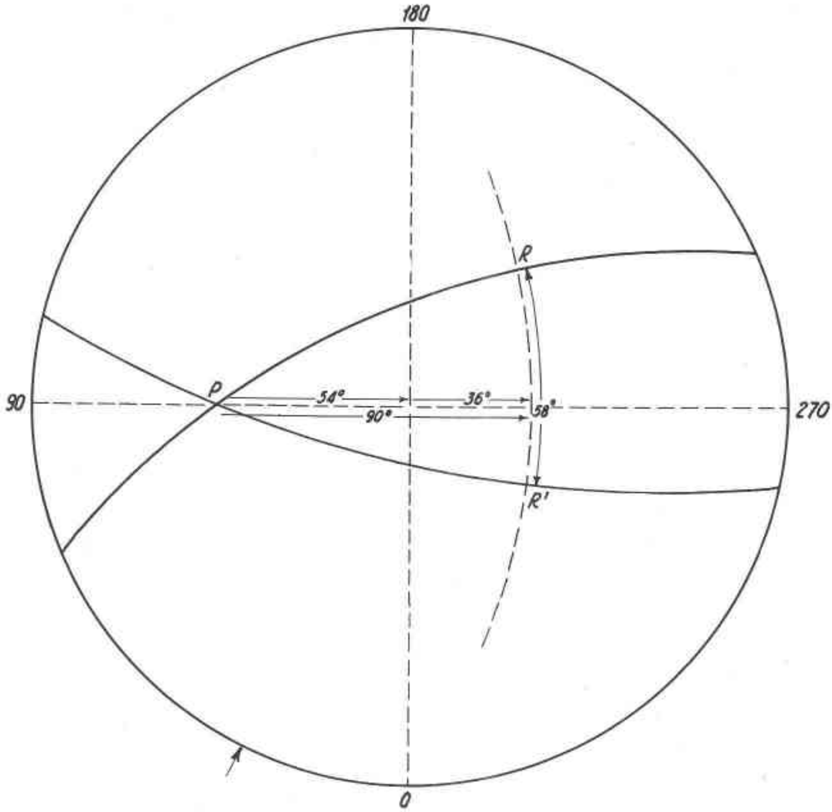

FIG. 5b. Method of determining angle between two inclined planes by measurement along the great circle 90° from their axis of intersection.

The intersection point P of two given great circles may lie on the projection circumference. In this case one plane is horizontal and is, therefore, represented by the fundamental circle of the projection. The plane normal to the two given ones is then a vertical great circle. This latter is projected as a straight line passing through the net center. To determine the angular distance between two planes under these circumstances, the tracing paper is rotated until the intersection point P lies on the equator. The required angle is then read off along the north-south axis of the net.

E. To measure the angle between two stereographically projected poles

The angular distance between two poles is measured along the arc of the great circle passing through them.

1. Rotate the tracing paper until both poles lie on the same great circle of the net.

Along the arc of this great circle count the number of degrees between the two poles (Fig. 6).

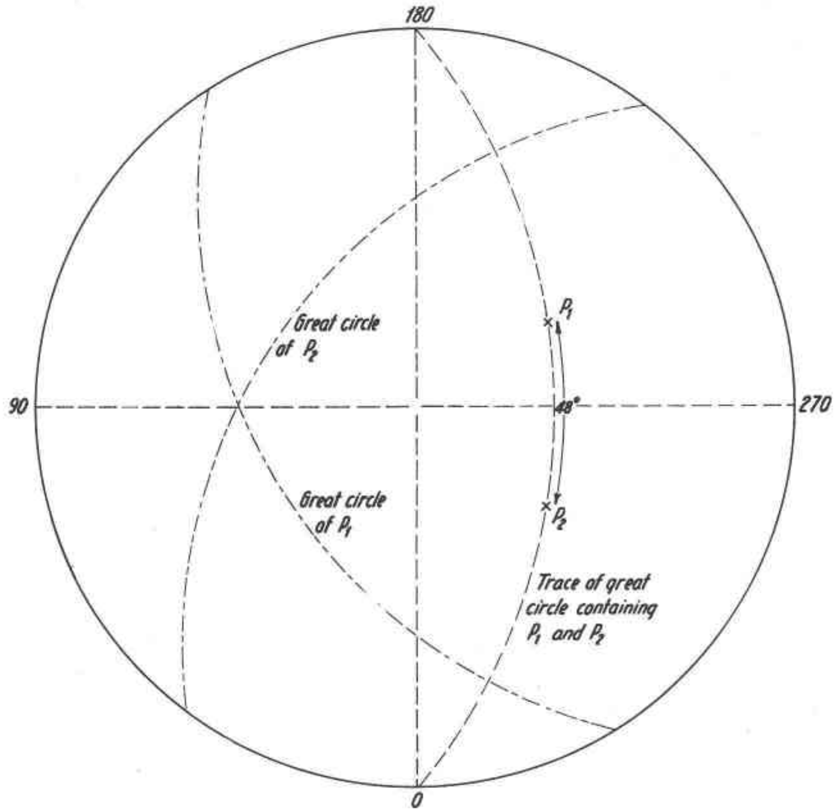

FIG. 6. Illustrating the method of determining the angle between two stereographically projected poles.

If the poles lie on the circumference of the projection, the angle between them is also measured as just indicated. This is so inasmuch as the net circumference represents the horizontal great circle.

In general vertical small circles cannot be used to measure the angle between poles. There is, however, one exception to this. The net equator is the projection of the largest possible vertical small circle and is, therefore, actually a great circle. Hence it may also be used to measure such angular distances.

Transformation of the Projection Plane (Rotation)

It is frequently desirable to bring a particular optical or crystallographic plane in the measured crystal into coincidence with some standard reference plane. Such a procedure is often most helpful in plagioclase determination, especially when the Reinhard method is used. According

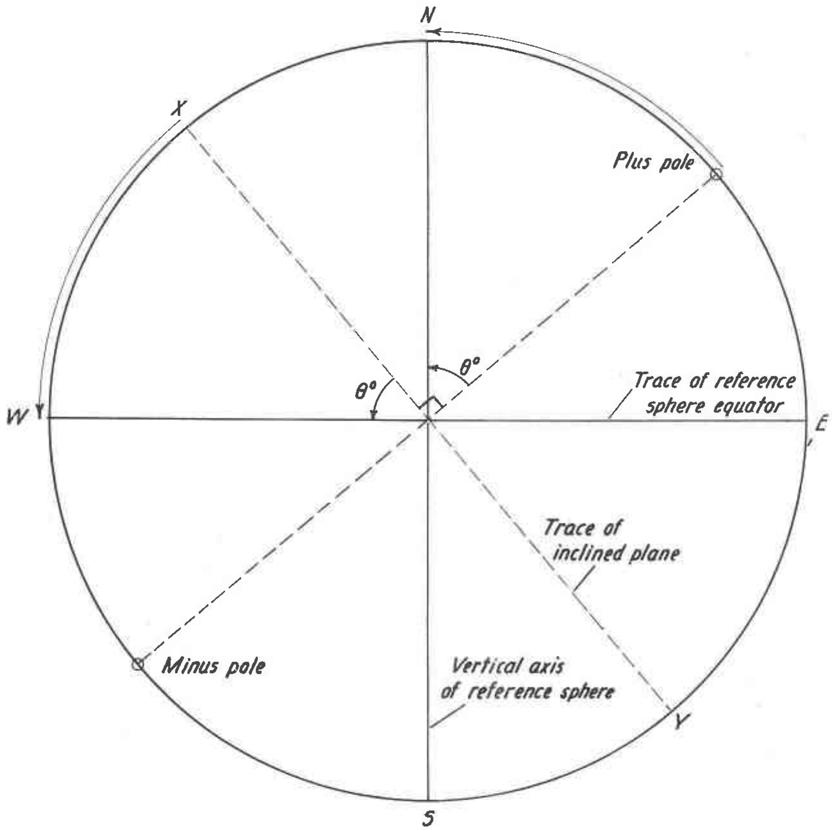

FIG. 7*a*. Vertical section through reference sphere. Pole rotates through angle θ bringing plane XY into coincidence with projection plane.

to this system the observer's measurements are compared with standard projections of optical vectors and important morphological planes in the plagioclase series. Such projections show the migration of the poles of $N_\alpha N_\beta N_\gamma$, for the plagioclase range, with respect to the common composition planes as (001) and (010).

The great circles on the net, with the exception of the fundamental circle, represent planes which are inclined to the equatorial plane of the reference sphere. All these inclined planes intersect the equatorial plane along one diameter. The line on the net representing this diameter is the north-south meridian, which is also the projection of the only vertical great circle.[7]

To bring an inclined plane, such as *ABCD* (Fig. 1), into coincidence with the projection plane, the former must be rotated about the diameter *AC* as an axis. As the plane *ABCD* is rotated, its pole moves a corresponding distance. It traces a path on the reference sphere surface which, if projected, would appear on the net as part of some vertical small circle (Fig. 7*a*).

In practice it is easier to rotate a plane by shifting its pole than to manipulate the great circle itself. To transform the projection plane:

1. Rotate the tracing paper until the pole of the plane in question lies on the equator.

With the pole in this position its great circle is superposed over the corresponding great circle on the net beneath. The great circle of the plane in question intersects the equatorial plane of the reference sphere along the proper diameter *AC* (Fig. 1). If, now, the pole is moved along the equator to the net center, its great circle will then coincide with the projection plane and thereafter be represented by the fundamental circle.

2. Read off on the equator the number of degrees of rotation required to bring the reference pole to the center, and place the pole symbol there.

In rotating the correct angular relationships between the new projection plane and all associated planes must be maintained. To accomplish this the poles of all related planes must be moved in the same direction and the same number of degrees as the pole of the new projection plane. Hence:

3. Shift the poles of all related planes along the arcs of the vertical small circles over which they lie when the pole of the new projection plane lies on the equator as in (1) above (Fig. 7*b*).

After all required poles are thus rotated and their new positions plotted, their corresponding great circles may be drawn.

[7] Wulff, George, Über die Vertauschung der Ebene der stereographischen Projection und deren Verwendung: *Zeits. Krist.*, **21,** 248–254 (1892–93).

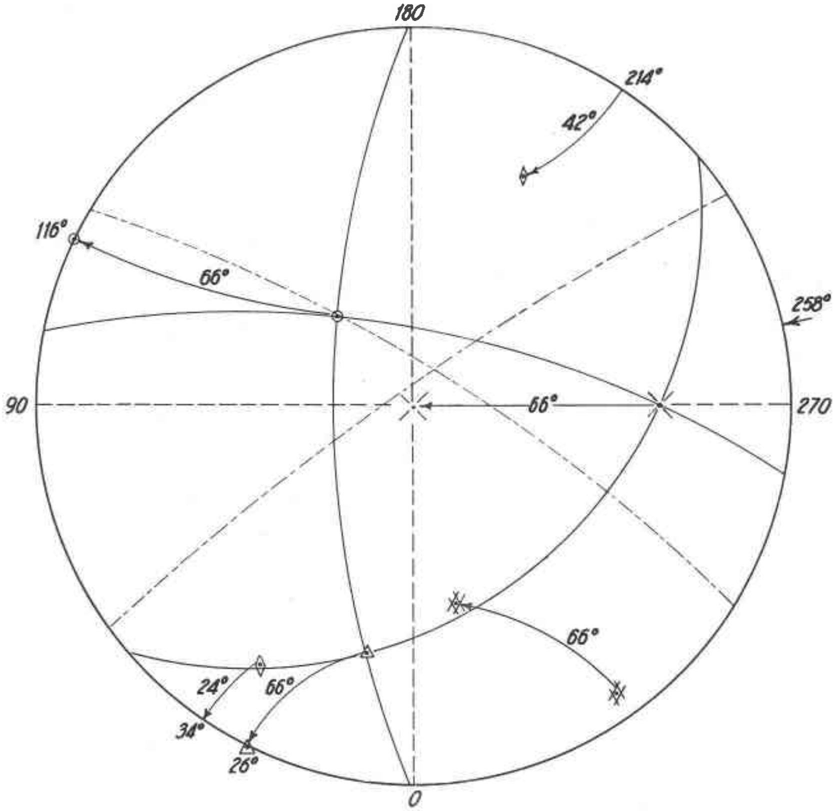

FIG. 7*b*. Illustrating method of keeping proper orientation by shifting poles of related planes a distance equal to that required to bring reference plane pole to projection center.

1. The tracing paper is rotated so that each newly plotted pole in turn is brought over the net equator.

2. The great circle cutting the equator 90° distant from the pole is then traced.

When several poles representing planes of varying orientation are rotated, some usually lie within the net upon completion of rotation. Often, however, other poles will have arrived at the net circumference before rotation is complete. The proper procedure in this case requires explanation. The normal to any plane, which normal passes through the center of the reference sphere, intersects the lower hemisphere surface at a point diametrically opposite the pole on the upper hemisphere. If during rotation a pole is moved along a vertical small circle beyond the net circumference, it will be carried below the equatorial plane into the lower hemisphere. At the same time the pole of the diametrically opposite extremity of this normal will appear on the upper hemisphere.

As has been noted, in stereographic projection only poles on the upper hemisphere are considered. Let these poles be called *positive* and those on the lower hemisphere *negative.* If a positive pole be rotated to some point on the net circumference, its corresponding negative pole will then lie on the net circumference 180 degrees away. Therefore, having rotated a pole along some vertical small circle until the net circumference is reached:

1*a*. Count 180° in either direction around the circumference. The point of emergence of the negative pole lies on the point thus determined.

1*b*. Or, alternatively, a line may be drawn passing through the net center and the point on the circumference where the positive pole lies.

2. After the point of emergence of the negative pole is located, the latter is rotated the remainder of the required angular distance along the arc of that small circle intersecting the circumference at that point (observe composition plane pole, Fig. 7*a*).

The coordinates previously given in Table 1 will again be used for illustration. For comparison with standard projections it may be necessary to transform a projection so that the optic axial plane of the measured crystal becomes the projection plane. If this is done the pole of the optic normal will lie at the center of the projection. In Fig. 7*b* the three optical symmetry planes, the cleavage plane, and the twinning plane, together with their respective poles, have all been plotted. The tracing paper has been rotated until the pole of N_β lies on the equator 66° to the right of center. In this position the index arrow lies on the 258° mark on the net circumference. The poles of N_α and N_γ lie on the same great circle 24° to the left of center.

The pole of N_β must now be moved along the equator 66° to the net center. This will bring the optic axial plane into coincidence with the projection plane. The poles of N_α and N_γ must, likewise, be shifted to the left 66° along their respective vertical small circles. They will then lie on the net circumference. The pole of N_α comes to rest at the 116° point; that of N_γ at the 26° mark on the circumference. They are, it should be noted, 90° apart. The composition plane pole is now moved along its appropriate small circle 66° as indicated. After rotation of 24° it reaches the circumference at the 34° point. It is then moved 42° farther along the corresponding small circle, intersecting the net circumference 180° from the 34° point. This diametrically opposite point is at the 214° mark. After the pole of the cleavage plane is rotated the entire projection is transformed.

If it is required to draw the great circles of these poles in their new positions, the tracing paper is turned until each relocated pole in succession lies on the equator. As before each great circle is drawn by tracing the great circle cutting the equator 90° distant from the pole.

The great circles of N_α and N_γ are now meridians making an angle of 90° with one another. The great circle of the cleavage plane lies 34° to the left of center; that of the composition plane 22° left of center (Fig. 8).

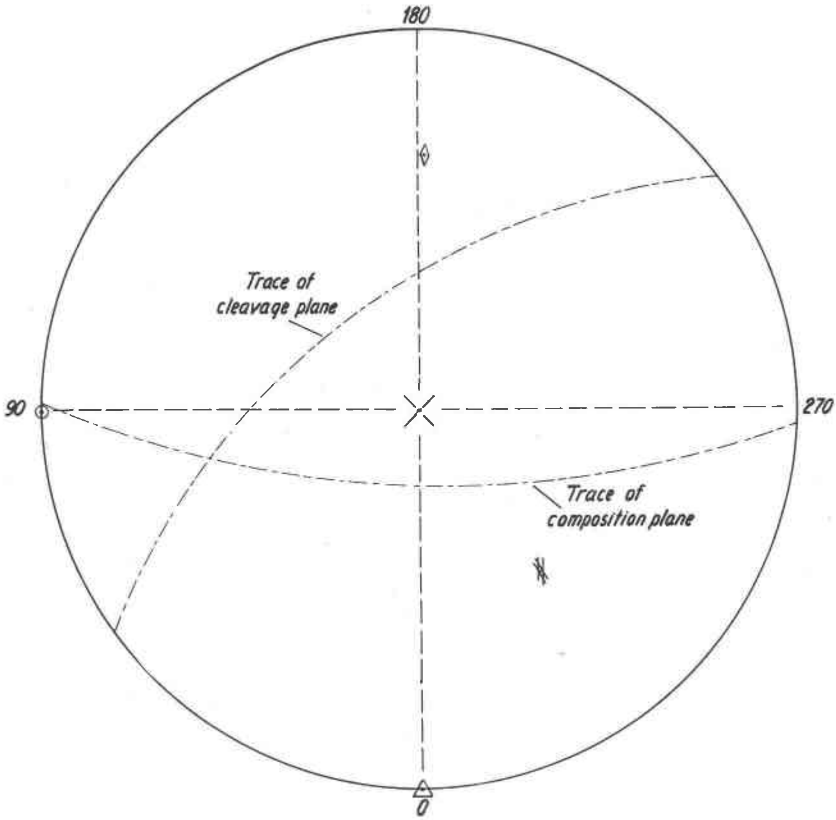

FIG. 8. The transformed stereogram with the optic axial plane coincident with the projection plane.

The stereogram is then completely transformed. The optic axial plane is now the projection plane and all morphological reference planes are in their proper relationship, both with respect to one another and to the planes of optical symmetry.

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