SIMPLE GNOMONIC PROJECTOR FOR
X-RAY LAUEGRAMS

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Abstract

The indexing of Lauegrams requires transposition of the Laue spots (which are at tan 2θ) to tan θ (stereographic trace of plane pole in F-Laue, and gnomonic plane-pole in B-Laue) or to cot θ (gnomonic plane-pole of F-Laue). The geometry involved is simple: to the line through the Laue spot and the center of the Lauegram draw the unit circle (5 cm. radius for $D=5$ cm. in camera), tangent to the line at the center of the Lauegram. From the Laue spot draw a line through the center of the circle. Erect perpendiculars to this line, tangent to each side of the unit circle. It is easily proven that the intersections of these tangents with that of the tangent through the Laue spot and the center of Lauegram are the points required θ, and $90°\text{-}\theta$. A simple transporteur may be constructed by swivelling a carpenter's square to a straight-edge, such that the center of rotation is 5 cm. below the straight edge used to connect Laue spot and center of Lauegram; and the vertical arm moves tangent to it at the required distance. Such a device was described by Clark and Gross in 1937 for plotting $90°\text{-}\theta$. However, the vertical arm can be adjusted to any distance from the center of rotation, giving reductions to 4, 3, 2.5 or 2 cm. as desired. A shorter vertical arm, on the opposite side of the center is used for plotting θ. Since the device is simply an angle bisector, it can be used for converting gnomograms to stereograms.
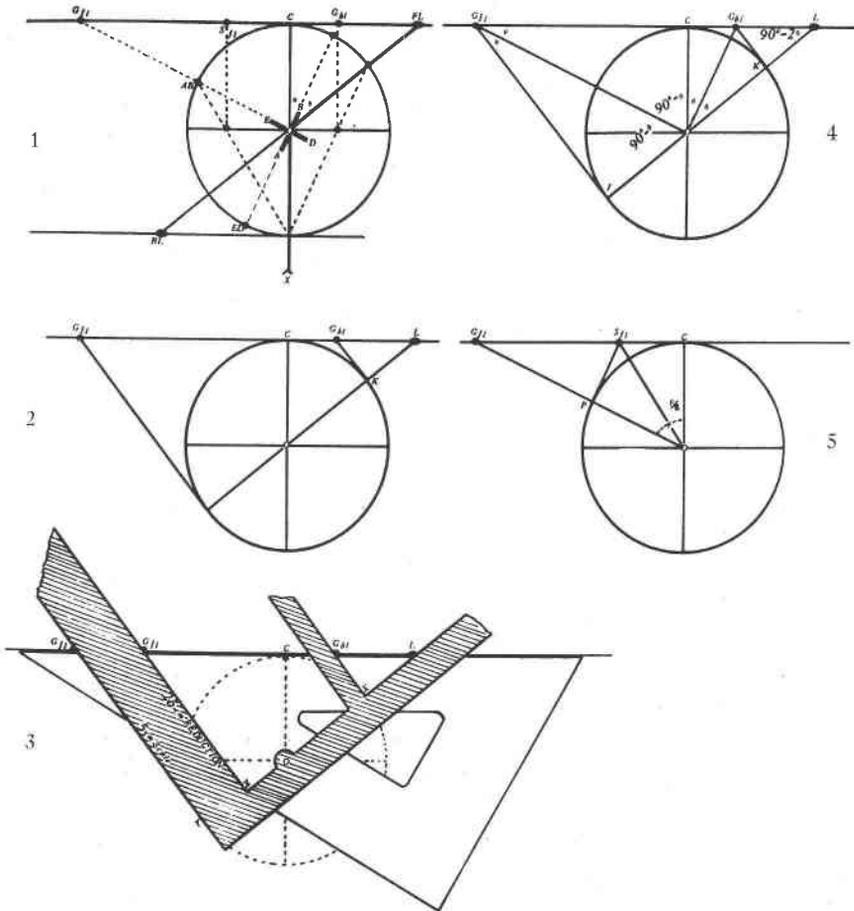
Since the spots on an x-ray Lauegram are at tan 2θ, the indexing of the atomic planes which produced the spots is most rapidly done by making a projection of those planes. The advantage of using the gnomonic projection was long ago demonstrated by Wyckoff.[1] In the gnomonic projection the projected poles are at cot θ (Fig. 1). To facilitate the making of a gnomonic projection from a Forward Lauegram, Wyckoff designed a gnomonic ruler:[1] the position of a Laue spot was read on a scale to the left of the center of the Lauegram (2θ), and the same figure to the right of the center located the gnomonic projection of the pole of the atomic planes (at $90°\text{-}\theta$).

A gnomonic ruler which eliminated the scales was introduced by Clark and Gross[2] in 1937. As described below, modification of this ruler permits reduction of the scale of the projection in Forward Lauegrams, and the addition of another arm extends its usefulness to Back Lauegrams.

[1] Wyckoff, R. W. G., *Am. Jour. Sci.*, **50**, 317 (1920). The gnomonic projection was first used for Lauegrams by Rinne, in 1915 (*Ber. K. Sächs. Ges. Wiss. Leipzig* (Math.-phys. Klasse, **67**, 303, 1915). Wyckoff credits Frederick E. Wright for pointing out the advantage of the gnomonic projection over that of the "modified stereographic projection" introduced by W. L. Bragg (see below) in 1913.

[2] Clark, G. L., and Gross, S. T., *Science*, **17**, 272–273 (1937).

Figure 1 will serve as a key to the simple geometry involved in figures 2 to 5. X-rays (X) impinging upon a set of planes $A–B$ at the reflecting angle will produce a Laue spot at FL (Forward Lauegram), at the Bragg angle 2θ. The pole of the normal to this set of planes is indicated on the spherical projection at AB, and the projection of this pole on the gnomonic plane is at G_{fl}. The problem is to project the position of G_{fl} on the Forward Lauegram from the Laue spot FL.

Another set of planes (drawn at right angles to the set $A–B$) is indicated by the letters $E–D$. They will produce a Back Laue spot at BL. The pole of the set of planes $E–D$ in spherical projection is at ED, and the gnomonic projection of this pole projected to the upper plane will be at G_{bl}.

The problem is to project the position of G_{bl} on the Back Lauegram from the Laue spot BL. Note that, in this case θ is the angle $X–D$.

This figure, for completeness shows the position of the stereographic pole[3] of the set of planes $A–B$, and the stereographic projection of this pole upon the gnomonic plane at S_{fl}.

As indicated in Fig. 2, the graphic solution of this problem is quite simple. Draw a circle, with a radius equal to the distance from crystal to film, tangent to the line between the Laue spot (L) and the center of the Lauegram (C) at this center. Draw a line (LJ) from the Laue spot thru the center of the circle. Erect perpendiculars to this line tangent to the circle, at K and J. The intersection of these tangents, with the tangent (LC) in the gnomonic plane at the points G_{bl} and G_{fl} locate the projection of the poles of the planes of Fig. 1 in the Back Lauegram and the Forward Lauegram respectively.

This immediately suggests a simple projector, by means of which the gnomonic projection of the poles of the planes can be quickly located (Fig. 3). A straight edge, such as that of a draughtman's triangle can be used to connect the Laue spot with the center of the Lauegram: once a center point (C) has been scratched on the edge of the triangle. A square, which can be made from an ordinary carpenter's square, is pivoted to the triangle at a point (O) at right angles to the center point (C) on the edge of the triangle, at a distance $(C$ to $O)$ equal to the distance from crystal to film in the camera (usually 5 cm.). Upright arms are spaced so that their ruling edges (at X and Y) are the same distance from the pivot center, O. The horizontal arm of the square has a ruling edge $Z–X$ which passes through the pivot center.

To use the instrument, the straight-edge of the triangle is placed with its center mark at the center of the Lauegram (or a tracing, or a copy made by pricking points through a print onto a sheet of drawing paper). The horizontal arm of the square is swung on its pivot at O until the ruling edge is upon the Laue spot, L.

The point G_{bl} on the right arm then is the projection of the pole of the planes responsible for the reflection in a Back Lauegram. In the case of

[3] This is the stereographic projection of the pole of the normal to the planes $A–B$, as used by morphological crystallographers. In the early days of x-ray crystallography the literature was ornamented by the "modified stereographic projection" introduced by W. L. Bragg in 1913 (*Proc. Roy. Soc.*, **A 89**, 248, 1913), in which, not the normal to the set of planes, but the projection of the trace of the reflecting planes upon the gnomonic plane was plotted: in Fig. 1, the point G_{bl} (trace of $A–B$) rather than S_{fl} was used in indexing. The zones of which appeared as festoons of spots (ellipses) on the Lauegram projected as circles tangent to the center of the Lauegram.

a Forward Lauegram, the point G_{fl} on the left arm is the projected pole. One can proceed very rapidly over a Lauegram in this fashion. It is convenient to slightly notch the center mark on the triangle: the drawing can be held down with a slightly protruding pin, and the triangle may then be pivoted about the Lauegram from one zone to another. The writer has found it preferable to work counterclockwise around a Lauegram, thus covering up Lauespots that have been projected.

It will be noted that the left arm in Fig. 3 has two ruling edges, one marked 5 cm. and the other 2.5 cm. reduction. The 5 cm. scale is based on a fundamental sphere with a radius of 5 cm. However, on this scale, many poles will lie at considerable distances from the center of the projection, if not beyond the limits of an ordinary sheet of drawing paper. The scale of the projection can be easily reduced by moving the ruling edge towards the pivot: thus a distance of Z to O of 2.5 cm. will produce a projection based on a fundamental sphere of 2.5 cm. This distance was chosen because it was easy to convert tables on hand (of tangents $\times 5$ cm.) to this scale by simply dividing by two. The scale can be made 2 or 3 cm. (or any other desired) by placing the left arm at such a distance from the pivot center at O (the distance, however, from O to C must be equal to the crystal to film distance).

The geometrical proof of Fig. 2 will be obvious from Fig. 4. It is based on the geometrical relation that from an external point only two tangents can be drawn to a circle; that these two tangents are equal; and further, that a line from the intersection of two tangents to the center of the circle bisects the angle between the two tangents.

Therefore $C - G_{bl} = G_{bl} - K = \tan \theta$; and $C - G_{fl} = G_{fl} - J = \cot \theta$.

Since the Lauespot is at 2θ, and it is desired to find θ in the Back Lauegram, the problem is simply one of bisecting this angle. It will be obvious also from Fig. 4, that the location of G_{fl} involves only the bisecting of angle C, J, G_{fl}.

The instrument shown in Fig. 3 is merely an angle bisector and is adaptable to any problem of this kind. One example will suffice (shown in Fig. 5): that of converting a gnomonic projection to a stereographic projection (or vice versa). The pole in gnomonic projection is at the angle ρ; the stereographic pole (upon the same plane) is at $\rho/2$. The angle can be bisected graphically by erecting a tangent at P. The point S_{fl} can be quickly located with the instrument by turning the drawing upside down, placing the center point at C, rotating the horizontal arm to G_{fl} and the point S_{fl} is at the intersection with the right vertical arm.

It may also be pointed out that it is not necessary to use the instrument for every spot in a zone: once two points are located in a zone, a straight

line can be drawn through them. All of the projected points from a festoon will then lie on this line: by connecting each Laue spot in the ellipse with the center of the projection (with a straight edge), then the intersection with the line through the zone will suffice to locate the projected pole.

My thanks are due to Mr. Ben Birchall of Philadelphia, who kindly made the instrument for me.