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Estimation of thermal diffusivity from field observations of temperature as a function of time and depth[1]

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Abstract

Methods have been previously reported for estimating the thermal conductivity from field observations; application of several such methods gives results that differ significantly from one another. The cause of the discrepancy appears to be that each method only utilizes some of the information contained in the data. Therefore, a method has been developed that uses all of the information, giving a "best" answer, in this sense. The method has been applied to data taken from beneath Lake Waiau, a tarn on Mauna Kea, Hawaii. Observations taken over more than two years at depths in the sediments to about ten meters are used to estimate the thermal diffusivity of the sediment. The thermal conductivity of some sediment samples has been measured in laboratory apparatus for comparison with the field results.

Introduction

The thermal diffusivity is related to the thermal conductivity through the relation given in Kappelmeyer and Haenel (1974, p. 10) as

$$K = D\rho c \tag{1}$$

where K is the thermal conductivity (heat flow per unit time per unit distance per degree of temperature), D is the thermal diffusivity (area per unit time), ρ is the density (mass per unit volume) and c is the specific heat (quantity of heat per unit weight per degree of temperature).

The thermal diffusivity can be estimated from measurements of amplitude decrement and/or phase difference of the temperature waves between various depths in the ground. The decay with depth of the amplitude of the temperature wave is given, theoretically by

$$T_2 = T_1 e^{-(z_2 - z_1)\sqrt{\pi/DP}} \tag{2}$$

where T_1 is the amplitude of the temperature wave in the ground at depth z_1, T_2 is the attenuated amplitude at depth z_2 in the ground, P is the period of the temperature wave, and D is the thermal diffusivity of the ground. The phase shift, ϕ, of the attenuated temperature wave between depths z_1 and z_2 is

$$\phi = (z_2 - z_1)\sqrt{\pi/DP}. \tag{3}$$

From equations (2) and (3) the thermal diffusivity can be expressed as a function of the temperature amplitudes by

$$D = \frac{(z_2 - z_1)^2 \pi/P}{[\ln(T_1) - \ln(T_2)]^2} \tag{4}$$

and as a function of phase shift by

$$D = (z_2 - z_1)^2 \pi / P\phi^2. \tag{5}$$

This method is discussed in more detail by Kirkham and Powers (1972) and by Kappelmeyer and Haenel (1974, p. 87).

The temperature profile curve (temperature versus depth at a given time) can be obtained by measuring the temperature at several depths within the ground. If this temperature profile curve is acquired a number of times during the penetration of the temperature wave into the ground, the measurement of the depth at the crossover point of any two of the temperature-profile curves within the same periodic cycle provides input data for a method of calculating thermal diffusivity. The relationship provided by Lovering and Goode (1963, p. 27) is: (the minus-plus was erroneously given as plus-minus.)

$$D = \frac{4z_c^2\pi/P}{[(t_1 + t_2)2\pi/P \mp (2n + 1)\pi]^2} \tag{6}$$

where t_1 and t_2 are the times the measurements were taken from the beginning of the driving function, z_c is the depth at the crossover of these two curves, and n

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is 0, 1, 2, *etc.*, representing the corresponding first, second, third, *etc.* crossover of the two curves. The annual temperature wave is assumed to be a periodic driving function. The thermal diffusivity, D, is the average thermal-diffusivity value of the test material between the crossing points and the level providing the driving function. The thermal conductivity can be indirectly obtained from thermal diffusivity measurements (determined from periodic methods) by either measuring or estimating the density and the specific heat of the test material, thus introducing more measurement error into the thermal-conductivity determination.

We now show our attempts to apply these conventional techniques to a practical problem and how the inconsistent results of the different methods prompted us to develop an improved method. We then describe that method and its application.

Estimation of thermal diffusivity from field observations

Lake Waiau is situated in the Waiau cone, near the summit of the inactive volcano Mauna Kea, on the island of Hawaii. A more specific location is shown in Figure 1. The existence of a negative thermal gradient under Lake Waiau has been determined by Woodcock and Groves (1969). The cause of the anomalous gradient remains uncertain. The answer is contingent on knowledge of the differences in the relative thermal properties of the lake water, the lake sediment, and the cinders and lava surrounding the lake, as well as the thermal regime established in the area by natural processes. The purpose of this study is to determine the thermal conductivity of the lake sediments in which the negative thermal gradient occurs. This will also allow us to obtain an estimate of the heat flux through the sediments.

Woodcock *et al.* (1966) have described the upper two meters of the sediments. This section contains two coarse layers of black ash and several layers of finer gray ash comprising about 5 and 10 percent, respectively, of the section. The remaining 85 percent of the sediments are colorful shades of red and olive-green. These colorful layers consist primarily of very fine particles, less than 0.05 mm in diameter, believed to be windblown from local sources, and about 5 percent is combustible organic materials.

Table 1 lists the thermal-probe data obtained by A. H. Woodcock that were used in the thermal-gradient determination. These data have also been used in this study to make estimates of the thermal diffusivity using the non-steady-state periodic methods

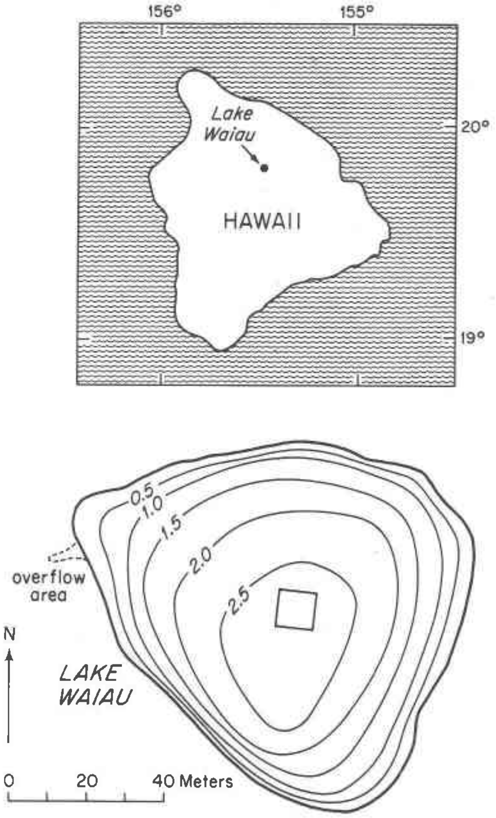

FIG. 1. Map of Lake Waiau showing estimated depth contours in meters. The inner square marks the limits of the area in which the temperature measurements were made, after Woodcock and Groves (1969).

previously described. The estimates were first made without any attempt to smooth the data, therefore large variances are to be expected.

Estimates by amplitude decay and phase lag

Figure 2 is a temperature–time graph of the thermal-probe measurements from Lake Waiau at the 3-meter and the 5-meter depths below the lake surface. The amplitudes T_1 and T_2 are taken as one half of the difference of the maximum and the minimum temperature values in each data set. The amplitude is 2.93°C at the 3-meter level and 0.73°C at the 5-meter level. If we use equation (4) for temperature variation, the estimate of the thermal diffusivity in the sediment layer between 3 and 5 meters below the lake surface is 0.00205 cm²/sec.

We can also plot the data for the 5-meter level on an exaggerated vertical scale so the curve is about the same size as the curve for the 3-meter plot. Then by placing one curve on the other and sliding it along the time axis until the curves match (or by cross-correla-

TABLE 1. Temperature (°C) at standard depths in Lake Waiau sediments as a function of time; Woodcock and Groves (1969)

Depth* (m)	July 7,1965	July 27	Aug. 17	Aug. 29	Sept. 14	Nov. 9	Jan. 6,1966	Feb. 15	Mar. 19	May 1
3.0	8.9	9.2	8.8	9.3	9.6	7.2	3.3	3.4	5.3	6.2
3.5	8.4	8.2	8.1	8.0	8.3	7.4	4.2	4.0	4.8	5.9
4.0	7.6	7.3	7.5	7.4	7.4	7.6	5.2	5.1	4.9	5.8
4.5	7.0	6.8	7.0	7.0	6.8	7.3	6.1	5.6	5.0	5.9
5.0	6.7	6.8	6.7	6.7	6.6	7.0	6.8	6.0	5.4	6.0
6.0	6.3	6.3	6.5	6.35	6.35	6.65	7.0	6.2	6.05	6.2
7.0	6.4	6.3	6.45	6.25	6.25	6.4	6.7	6.15	6.2	6.3

Depth* (m)	June 1	July 2	July 27	Nov. 1	Dec. 1	Dec. 28	Jan. 26, 1967	Mar. 3	Apr. 8	May 4	July 25
3.0	9.0	9.0	7.8	6.4	6.3	3.8	3.5	5.3	6.1	8.0	8.7
3.5	8.0	7.1	7.7	7.1	7.0	5.1	3.7	4.8	5.9	6.5	7.9
4.0	6.7	6.8	7.3	7.5	7.1	6.3	4.7	4.9	5.8	5.9	7.3
4.5	6.0	6.5	6.7	7.2	7.2	6.8	5.6	5.4	5.8	5.9	6.95
5.0	5.9	6.3	6.4	7.0	7.1	6.9	6.1	5.8	6.0	6.1	6.6
6.0	6.0	6.3	6.2	6.65	6.85	6.9	6.4	6.15	6.3	6.3	6.35
7.0	6.1	6.2	6.1	6.3	6.8	6.55	6.2	6.35	6.4	6.3	6.3

*Depth below water surface; for depth below sediment surface subtract ~ 2.85 m.

tion), we can estimate the phase lag by measuring the time difference between the zero-time axes. The estimated phase shift between these two curves is 60 days. And with equation (5) based on the phase lag, we obtain an estimate of 0.00374 cm²/sec for the thermal diffusivity of the same 2-meter layer. There is a difference of 45 percent between these two methods for this layer of sediments, even though the results are based on the very same set of data.

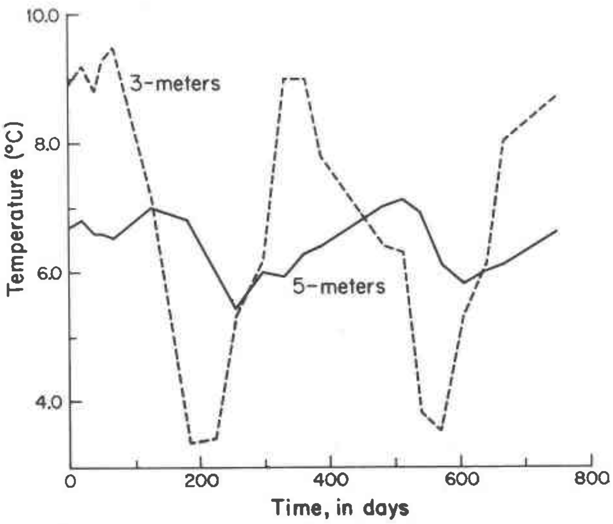

FIG. 2. Temperature, from thermal probes in Lake Waiau sediment, plotted against time for measurements taken at three and five meters below the water surface.

The data in Table 1 have been evaluated for each level of measurements combined with every other level. The results are shown in Table 2. The differences between the amplitude estimates and the phase estimates have a mean value of 50 percent. If we assume that these sediments are isotropic and homogeneous in the layer between the 3-meter and the 7-meter depths, then the thermal diffusivity should be the same for each of the above estimates. The mean thermal-diffusivity value from the amplitude computations is 0.00298 cm²/sec with a standard deviation of 0.00212, and the mean of the phase computations is 0.00410 cm²/sec with a standard deviation of 0.0103. The difference between these phase and amplitude estimates is 38 percent! This difference was attributed to the methodology being inadequate to resolve the unsmoothed data, rather than the assumptions; an alternate methodology was used to try to obtain a more reliable value for the thermal diffusivity.

Estimate by crossover of temperature profiles

Figure 3 is a graph of the temperature–depth profiles for the measurements made on 27 July 1966 and 28 December 1966 in the sediments of Lake Waiau.

TABLE 2. Thermal-diffusivity computations from amplitude decay and phase lag of the thermal-probe data taken in Lake Waiau, 1965–1967

Amplitude Diffusivity (cm^2/sec)	Phase Diffusivity (cm^2/sec)	Upper Depth (cm)	Lower Depth (cm)	Upper Amplitude (°C)	Lower Amplitude (°C)	Phase Lag (days)
0.00284	0.00840	300	350	2.93	2.18	10
0.00175	0.00350	300	400	2.93	1.38	31
0.00204	0.00391	300	450	2.93	1.02	44
0.00205	0.00374	300	500	2.93	0.73	60
0.00264	0.00241	300	600	2.93	0.46	112
0.00307	0.00282	300	700	2.93	0.30	138
0.00118	0.00429	350	400	2.18	1.38	14
0.00176	0.00309	350	450	2.18	1.02	33
0.00186	0.00197	350	500	2.18	0.73	62
0.00260	0.00202	350	600	2.18	0.46	102
0.00311	0.00233	350	700	2.18	0.30	133
0.00289	0.00259	400	450	1.38	1.02	18
0.00243	0.00140	400	500	1.38	0.73	49
0.00336	0.00205	400	600	1.38	0.46	81
0.00387	0.00264	400	700	1.38	0.30	107
0.00208	0.00146	450	500	1.02	0.73	24
0.00355	0.00197	450	600	1.02	0.46	62
0.00412	0.00278	450	700	1.02	0.30	87
0.00495	0.00328	500	600	0.73	0.46	32
0.00512	0.00174	500	700	0.73	0.30	88
0.00529	0.02778	600	700	0.46	0.30	10

The crossover depth of these curves occurs at 447 cm. If we assume the temperature wave detected at the 3-meter depth to be a periodic driving function with a period of 365 days, then the thermal diffusivity of the sediments between 300 and 417 cm can be estimated by equation (6) to be 0.0138 cm^2/sec.

Table 3 is a listing of 43 pairs of curves evaluated in a similar manner. The mean thermal diffusivity is estimated to be 0.00765 cm^2/sec with a standard deviation of 0.0135, and the distribution was very strongly skewed toward the lower values. The large values are due to poor determination of the crossover depth, which occurs when the curves cross at a small angle instead of at nearly right angles—the ideal case depicted in Figure 3. A better statistic for this situation is the mode, rather than the mean. The mode of this distribution is 0.00130 cm^2/sec with a standard deviation of 0.00030. The computations are made under the assumption that the sediment layer between 300 and 682 m below the lake surface is isotropic and homogeneous.

Such large differences among the various ways of estimating the thermal diffusivity make evident that the answer was more dependent upon the method of analysis than on the data! If each method were theoretically correct and used the same data base, then the results should be identical. Each method did seem to have a correct theoretical basis but used different portions of the data, *e.g.*, the temperature amplitudes, the phase difference between depths, and only the crossover points. The large difference between the results thus indicated the need to use all of the data available.

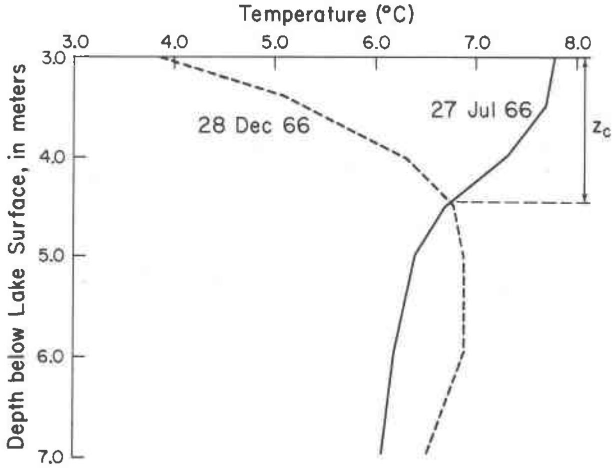

FIG. 3. Temperature profile of two thermal-probe measurements made in the sediments of Lake Waiau.

Estimate by an improved method: the least-squares technique

Figure 4 is a map of the temperature-probe measurements on the time-depth plane. Two distinct features are expressed in the character of the isotherms. The sloping of the troughs and ridges from the left at the top toward the right at the bottom is an indication of the phase lag throughout the layer. And the isotherm gradient decreases markedly from the top to the bottom, indicating the decay of the temper-

TABLE 3 Thermal-diffusivity computations from temperature-profile crossings of thermal probes in Lake Waiau

Thermal Diffusivity (cm^2/sec)	Crossover Depth (cm)	1st Time (days)	2nd Time (days)	Summer Cycle (days)	Intersecting Pair of Measurements
0.00110	317	247	287	366	6Jan66-15Feb66
0.00115	337	181	366	365	1Nov66-4May67
0.00099	339	211	366	365	1Dec66-4May67
0.00122	342	238	303	365	28Dec66-3Mar67
0.00085	343	189	425	366	9Nov65-2Jul66
0.04982	350	59	181	365	2Jul66-1Nov66
0.02314	363	59	211	365	2Jul66-1Dec66
0.00116	371	189	394	366	9Nov65-1Jun66
0.00091	375	189	450	366	9Nov65-27Jul66
0.00125	381	238	339	365	28Dec66-8Apr67
0.00134	383	247	320	366	6Jan66-19Mar66
0.06573	384	105	133	366	17Aug65-14Sep65
0.02493	387	84	189	366	27Jul65-9Nov65
0.02959	387	84	181	365	27Jul66-1Nov66
0.00114	390	287	320	366	15Feb66-19Mar66
0.00115	390	238	366	365	28Dec66-4May67
0.01067	391	133	189	366	14Sep65-9Nov65
0.00104	391	181	447	365	1Nov66-25Jul67
0.01699	394	105	189	366	17Aug65-9Nov65
0.01830	415	84	211	365	27Jul66-1Dec66
0.00106	423	211	447	365	1Dec66-27Jul67
0.00162	425	267	303	365	26Jan67-3Mar67
0.00142	437	247	362	366	6Jan66-1May66
0.01381	447	84	238	365	27Jul66-28Dec66
0.00129	448	247	394	366	6Jan66-1Jul66
0.00620	450	181	211	365	1Nov66-1Dec66
0.00116	466	238	447	365	28Dec66-25Jul67
0.00113	471	247	450	366	6Jan66-27Jul66
0.00126	473	247	425	366	6Jan66-2Jul66
0.00093	475	362	394	366	1May66-1Jul66
0.00176	484	267	339	365	26Jan67-8Apr67
0.00834	490	133	247	366	14Sep65-6Jan66
0.00114	490	267	447	365	26Jan67-25Jul67
0.01161	496	105	247	366	17Aug65-6Jan66
0.00155	500	287	362	366	15Feb66-1May66
0.00166	500	267	366	365	26Jan67-4May67
0.00675	530	181	238	365	1Nov66-28Dec66
0.00594	531	189	247	366	9Nov65-6Jan66
0.00177	600	339	366	365	8Apr67-4May67
0.00123	600	362	450	366	1May66-27Jul66
0.00148	633	339	447	365	8Apr67-25Jul67
0.00156	650	362	425	366	1May66-2Jul66
0.00194	682	303	447	365	3Mar67-25Jul67

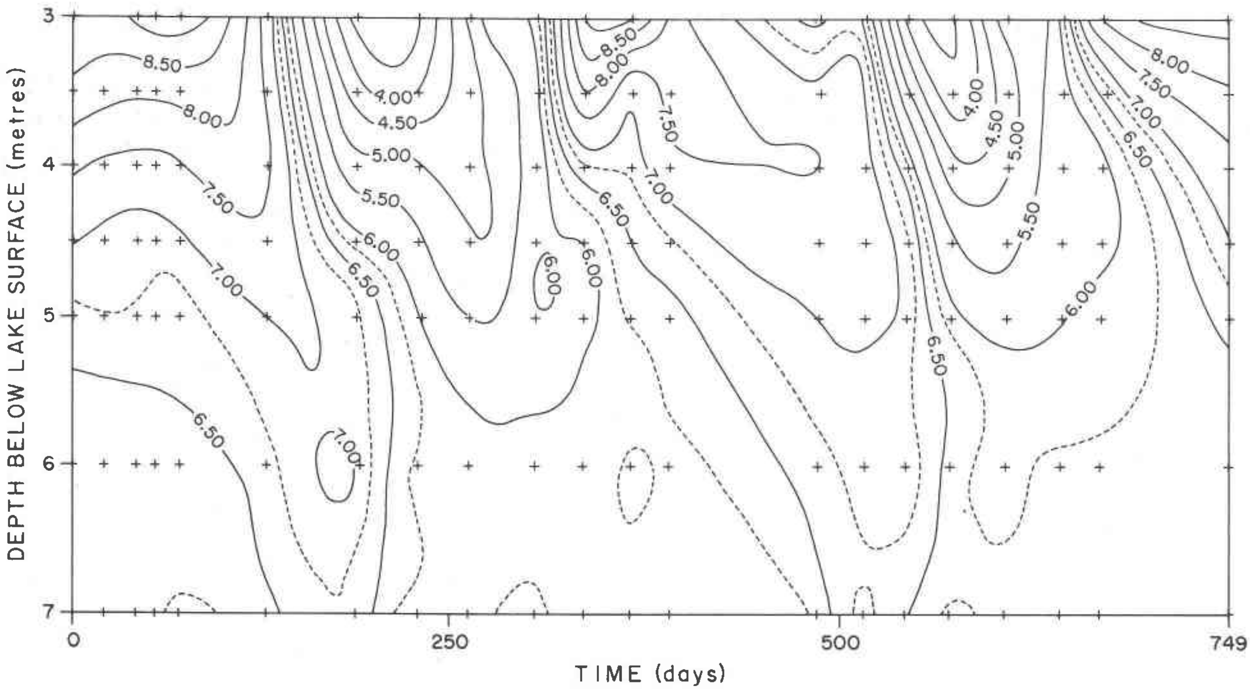

FIG. 4. Map of the temperature-probe measurements on the time-depth plane. Isothermal contours are shown.

ature fluctuation with depth in the layer. Thus, the isotherm map is a more continuous and total representation of the propagation of the annual temperature wave through the sediment layer over the time span of the measurement than any two selected depth or time sections through this map.

A well-defined trough on the right side of the map in Figure 4 shows a phase shift of 94 days in the four-meter layer. The thermal diffusivity is estimated to be 0.00609 cm^2/sec using the phase-lag computation. However, this particular phase lag is not representative of the entire map, and there is no other well-defined ridge or trough that would be a more representative phase lag. If we look at the amplitude decay of the temperature range with depth, the computations give an approximate thermal-diffusivity value of 0.0023 cm^2/sec throughout the sediment layer. Another isotherm map, similar to Figure 4, was constructed for the idealized case of a constant thermal diffusivity of 0.0023 cm^2/sec.

The temperature, T, at any point in the sediment layer can be represented by the following relation from Lovering and Goode (1963)

$$T = T_m + T_r e^{-z\sqrt{\pi/DP}} \sin [(t + \phi)2\pi/P - z\sqrt{\pi/DP}] \quad (7)$$

where

T_r —temperature range at driving level,
T_m—mean temperature of sediments,
D —thermal diffusivity of sediment layer,
z —depth of temperature measurement,
t —time of temperature measurement, and
ϕ —phase displacement in time.

This equation is then used as a model to obtain a least-squares fit of the observed data in Table 1, as represented by the isothermal map in Figure 4, to various idealized data sets.

The temperature, T, the depth, z, and the time, t, are taken to be the known parameters in this model. The period, P, is set to 365 days, and the mean temperature, T_m, is set to 6.3°C from values obtained in the previous calculations. This leaves three parameters, the temperature range, T_r, the phase displacement, ϕ, and the thermal diffusivity, D, to be fitted in the estimation.

The program for calculating least squares was checked and debugged using an artificial data set. Values at the points $T(t, z)$ of the observed data in Figure 4 were taken from the idealized map with D = 0.0023 cm^2/sec by superimposing the two maps. When the least-squares program was run on the artificial data set, it converged on the expected values of the input parameters, including D = 0.0023 cm^2/sec.

After the general region of the least-squares minimum was determined, it was possible to determine the extreme value for the sum of the squared differences between the observed and the idealized data

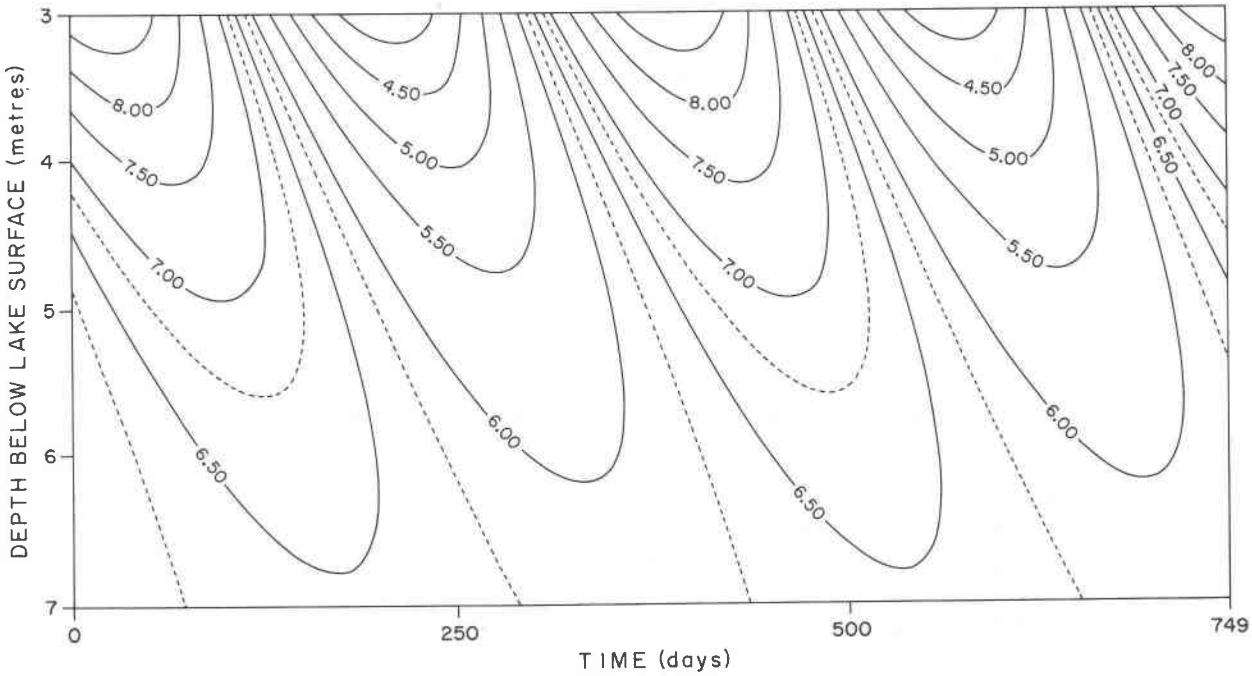

FIG. 5. Idealized map on the time-depth plane, using the values of parameters that best-fit the data in Fig. 4.

sets by varying the values of the input parameters, T_r, ϕ and D, by small amounts. The minimum value for the sum of the squares was found to be 30.3462 (°C) for 147 data points. The corresponding parameters at this minimum are $T_r = 2.655$°C, $\phi = 70.67$ days and $D = 0.00212$ cm²/sec. This calculation then represents the fit of the best regression surface of temperature to the observed temperature surface on the same time–depth plane. An idealized map of the temperature over the same time interval and layer thickness was constructed using these parameters, including $D = 0.00212$ cm²/sec, shown in Figure 5. For the computer program used in this estimate see Watts (1975) Appendix.

We now describe laboratory measurements made to obtain values of thermal conductivity for comparison with the values for thermal diffusivity from the field measurements.

Thermal conductivity of sediments under a Hawaiian alpine lake

A 2-meter core sample from the sediment layer between 3 and 5 meters below Lake Waiau's surface was obtained at the temperature-probe measurement site by A. H. Woodcock on 5 May 1967. A more specific location of the selection site is shown in Figure 1 (see the square). Two specimens were sliced from the ends of this core, one from the 3-meter and the other from the 5-meter level. Each sample was placed inside a cast acrylic annulus in order to be able to use the steady-state apparatus (Watts, 1975). The total heat loss in the measurement of the sediments is approximately 8.5 percent (see Watts, 1975, Chapter 3). The heat-flux values computed for the thermal-conductivity measurements have been adjusted to compensate for this loss. The results of the thermal conductivity measurements in the laboratory are listed in Tables 4 and 5. The measurements of the 5-meter sample are considered unreliable for the later times. The mean of the first four measurements is 0.0029 cal/sec cm °C and is considered the thermal-conductivity measurement of the 5-meter sample. The mean value of the thermal-conductivity measurements for the 3-meter sample is 0.0024 cal/sec cm °C. The difference between the steady-state measurements of the 3-meter and the 5-meter levels is 20 percent. The mean particle density was found to be 2.40 gm/cm³ and the moisture content of the sample from the 3-meter sample was 76.7 percent, following procedures of Lambe (1951). The bulk density for the 3-meter sample is computed to be 1.36 gm/cm³.

The core sample was not stored in a sealed container over the past eight years, therefore, the moisture-content value cannot be representative of the "in situ" situation. Woodcock and Groves (1969) report values of moisture content at the 4-meter and the 6-

TABLE 4. Thermal-conductivity measurements of a sediment sample from 3 meters below the surface of Lake Waiau

Date	Time of Day	Ambient Air Temperature (°C)	Sample Mean Temperature (°C)	Thermal Conductivity (cal/sec cm °C)
4Apr75	2045	25.2	50.32	0.0025
5Apr75	0845	22.8	50.95	0.0023
5Apr75	1300	25.0	52.96	0.0023
5Apr75	1705	25.9	52.94	0.0022
5Apr75	2115	24.8	49.78	0.0026
6Apr75	0640	23.0	33.47	0.0023
6Apr75	0950	23.4	34.14	0.0025
6Apr75	1300	25.1	32.90	0.0023
6Apr75	1600	25.8	32.69	0.0025
6Apr75	1915	26.0	31.70	0.0024

Regression equation: k = 2.5 - 0.001 T mcal/sec cm °C; Standard error of estimate = 0.12 mcal/sec cm °C; Mean reproducibility error = 3.6%; Sample disc thickness = 1.240 cm; Sample surface area = 15.459 cm^2.

meter levels to be 74 and 48 percent, respectively. This indicates that the moisture content decreases with increasing depth. The varying amounts of water with respect to the sediment particles will cause the bulk density and the specific heat of the sediment to also vary with depth. These parameters are used in the conversion of thermal diffusivity to thermal conductivity through the relation $K = D\rho c$.

If we take the thermal diffusivity to be 0.00212 cm^2/sec, as measured, the particle density as 2.40 gm/cm^3, as measured, and assume the specific heat of the particles to be 0.22 cal/gm °C, then the corresponding values of bulk density, specific heat, and thermal conductivity are obtained and are given in Table 6 for three values of moisture content (weight percent). This table shows that for decreasing moisture content, the bulk density increases, and the specific heat and thermal conductivity both decrease. However, within the moisture content limits of 48–74 percent, the thermal conductivity varies about the value 0.00205 cal/sec cm °C, differing by no more than 4.3 percent.

The measurements made by the steady-state laboratory apparatus differ from the above estimate by 18

TABLE 5. Thermal-conductivity measurements of a sediment sample from 5 meters below the surface of Lake Waiau

Date	Time of Day	Ambient Air Temperature (°C)	Sample Mean Temperature (°C)	Thermal Conductivity (cal/sec cm °C)
28Mar75	1620	23.5	58.29	0.0031
29Mar75	1025	23.0	57.85	0.0028
29Mar75	1550	25.1	53.66	0.0029
29Mar75	2130	24.6	54.65	0.0029
30Mar75	0715	23.4	43.42	0.0021
30Mar75	1030	24.9	42.12	0.0020
30Mar75	1305	26.8	40.80	0.0020
30Mar75	1625	27.7	42.70	0.0020
30Mar75	2120	27.3	50.82	0.0024
30Mar75	2330	25.5	40.93	0.0022

Regression equation: k = - 0.33 + 0.06 T mcal/sec cm °C; Standard error of estimate = 0.12 mcal/sec cm °C; Mean reproducibility error = 3.9%; Sample disc thickness = 1.648 cm; Sample surface area = 15.459 cm^2.

TABLE 6. For fixed thermal diffusivity and constant particle density, variation of thermal conductivity versus moisture content

Moisture Content (%)	Bulk Density (gm/cm^3)	Specific Heat (cal/gm °C)	Thermal Conductivity (cal/sec cm °C)
74*	1.81	0.522	0.00212
61	1.87	0.516	0.00205
48*	1.95	0.4373	0.00196

*Moisture-content values reported by Woodcock and Groves (1969).

and 41 percent. This difference can be accounted for, in part, by the remolding that was necessary to fit the sample into the annuluses. Also, small slices from within the core sample are probably not representative of the entire sediment layer. The most representative value for the "in situ" thermal conductivity of the sediments of Lake Waiau found in this study is 0.00205 cal/sec cm °C.

The negative thermal gradient determined by Woodcock and Groves (1969) is 0.052 °C/m. If we combine this value with the thermal-conductivity estimate above, we obtain a heat flux of 0.0107 cal/sec m^2 downward through the sediments at the area of the lake from which the measurements were taken.

Discussion

The estimate of the thermal diffusivity of the Lake Waiau sediments could be improved upon by incorporating another parameter into the new least-squares method. The annual temperature wave was assumed to be a sine wave with a period of 365 days, which is not the case in reality. A more realistic driving function could be modeled into the method from the continuous temperature data that are monitored at the Mauna Kea weather station.

Conclusions

The objective of this study was to obtain a value for the thermal conductivity of the sediments under Lake Waiau. This was accomplished by estimating the thermal diffusivity to be 0.00212 cm^2/sec using a least-squares method applied to temperature data collected by A. H. Woodcock. The thermal conductivity of these sediments is derived from this estimate to be 0.00204 cal/sec cm °C. This result combined with the results of Woodcock and Groves (1969) indicates a heat flux of 1.07 μcal/sec cm^2, downward, flows through the 4-meter layer of sediments in the center of the lake.

References

KAPPELMEYER, O. AND R. HAENEL (1974) Geothermics with special reference to application. *Geoexploration Monographs Series 1-No. 4*, Gebruder Borntraeger, 238 p.

KIRKHAM, DON AND W. L. POWERS (1972) *Advanced Soil Physics*. Wiley-Interscience.

LAMBE, W. T. (1951) *Soil Testing for Engineers*. John Wiley and Sons, Inc.

LOVERING, T. S. AND H. D. GOODE (1963) Measuring geothermal gradients in drill holes less than 60 feet deep, East Tintic District, Utah. *U.S. Geological Survey Bulletin No. 1172*.

WATTS, GEORGE P. (1975) *Methods for measuring the thermal conductivity of earth materials with application to anistropy in basalt and heat flux through lake sediments*. M. S. Thesis University of Hawaii, Honolulu, Hawaii.

WOODCOCK, A. H. AND GORDON W. GROVES (1969) Negative thermal gradient under an alpine lake in Hawaii. *Deep-Sea Research, Supplement to Vol. 16*, p. 393–405.

———, MEYER RUBIN AND R. A. DUCE (1966) Deep layer of sediments in an alpine lake in the tropical mid-Pacific. *Science*, **154**, 647–648.