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Empirical evaluation of fracture toughness: the toughness of quartz

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Abstract

A toughness related parameter, *Asv*, of monocrystalline, amorphous, and finely polycrystalline brittle materials is derived from standard sedimentologic size analysis of a crushed sample. *Asv* is numerically equal to the area under the cumulative-frequency size distribution curve and is related to the fracture toughness, *Kc*, of the material by the relationship, $Asv = 202 + 383 \log Kc$.

A study of quartz shows that toughness generally increases with decreasing grain size and with an increasing degree of interlocking of the grains or fibers.

Introduction

Brittle materials generally break by catastrophic propagation of Griffith flaws. Toughness, a parameter often applied in context to structural metals, has become a valuable measure of the resistance of these materials to fracture. The development of Griffith–Irwin fracture mechanical principles has given rise to several specific parameters for measuring toughness. Of these, fracture toughness, *Kc*, a critical value of the stress intensity factor, has gained wide acceptance. *Kc* is a material constant characterizing the inherent difficulty of crack growth in a material. A discussion of the significance of *Kc* is given by Tetelman and McEvily (1967) and conventional procedures for deriving toughness parameters are discussed in American Society for Testing Materials (1979).

Quantitative aspects of toughness have received little attention from geologists. This stems partly from the elaborate procedures for measurement of toughness parameters and the consequent dearth of toughness data for natural materials, and partly from the lack of obvious significance of conventional toughness parameters to natural processes.

In the present study an empirical method for evaluating fracture toughness is based on a procedure of crushing a specific starting size fraction of material and standard sedimentologic analysis of the resulting products. The method is amenable to rapid analysis, involves no prior knowledge of physical constants of the material except density, and is conceptually readily appreciated.

Historically the process benefits from the treatment of Rosin's Law by Geer and Yancy (1938) who demonstrate the existence of a type of probability distribution for the size fractions of crushed materials. Protodyakonov (1962) used size distribution data of crushed material to obtain a strength index for the material, and Hobbs (1964) and Evans and Pomeroy (1966) have related similarly derived strength indices to compressive strength. Kwong *et al.* (1949) have related amount of new surface area produced by crushing brittle materials to fracture surface energy.

Method

A toughness related parameter, *Asv*, derives from a unique analysis of the size distribution of the crushed product of a material.

Initial preparation of a sample consists of hand-crushing and sieving to obtain approximately 30 g of starting material in the −2 phi to −1.5 phi size range (4 mm to 2.83 mm). An amount of starting material equivalent in weight to a 6.666 cm^3 standard volume (numerically equal in grams to the density multiplied by 6.666) is placed in the 39 mm by 59 mm cylindrical crushing chamber of a SPEX shaker type mixer/mill with three 15.875 mm hardened steel balls and crushed for 45 seconds. The crushed product is sieved in eight inch U.S. Standard Series sieves of one phi interval between −1.5 phi and 4.5 phi for 15 minutes on a Combs gyratory sifting machine. The amount retained on each sieve is weighed to 0.01 g.

Weight data may be plotted as a cumulative-frequency plot with arithmetic ordinate in which cumulative percent is plotted against phi for the seven one-phi intervals from −1.5 phi to 4.5 phi. A quantitative toughness parameter, *Asv*, is numerically equal to the area beneath the cumulative-frequency curve. *Asv* may be compared visually on the graph or obtained directly from the data by the trapezoidal approximation rule in the form

$$Asv \simeq 0.5 \left(P\phi_{-1.5} + \sum_{i=-0.5}^{3.5} 2P\phi_i + P\phi_{4.5} \right)$$

where $P\phi_i$ represents the cumulative percent at phi *i*.

Reproducibility of *Asv* is generally within five percent of the mean for multiple runs on separates of a single homogenous starting sample. The sieving process discriminates on the basis of effective (least) cross-sectional area of the grains and, therefore, variation in grain shape (equidimensional, tabular, fibrous, etc.) must have an important, but undetermined influence on *Asv*. Values of *Asv* obtained with other crushing and sieving apparatus must be standardized with results from this study.

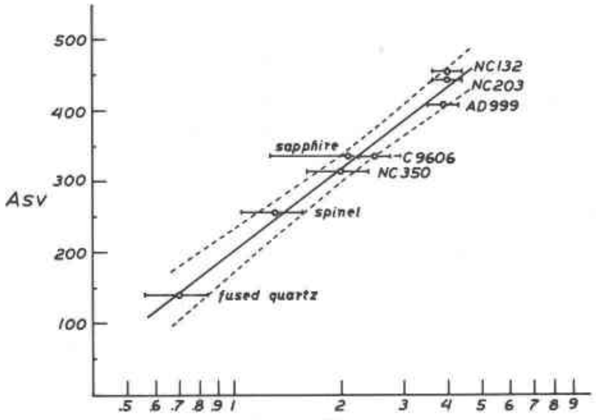

Fig. 1. Relationship between *Asv* and fracture toughness, *Kc*. Regression line and 95 percent confidence bands for *Asv* (based on "*t*" distribution) are derived from the arithmetic mean for each sample except sapphire. Values of *Kc* are from references in Table 1.

Results and discussion

A theoretical basis for relating *Asv* to traditional fracture parameters probably lies in the analysis of cracks produced by spherical indenters (Frank and Lawn, 1967) in which a relationship between the production of a Hertzian crack system and material toughness is derived from Griffith–Irwin fracture mechanics.

An empirical relationship between *Asv* and fracture toughness, *Kc*, is demonstrated in Figure 1 and Table 1 for several standard materials. The regression line represents the statistically significant relationship, $Asv = 202 + 383 \log Kc$. Values of *Kc* reported for the standards were derived by conventional testing methods of fracture mechanics as indicated in the references in Table 1. Although there is a distinct correlation between *Asv* and *Kc* for the standard materials and experimental conditions in this study, fracture toughness is strongly influenced by atmospheric moisture (Dunning *et al.*, 1980 and Schuyler *et al.*, 1981). Inability to control humidity in the experimental situation precludes analysis of this effect on the value of *Asv*.

The approximately 20 g of starting sample necessary to obtain *Asv* ensures that the value obtained represents a nearly average value of toughness for the sample. The use of *Asv* as an indicator of *Kc* is particularly valuable when toughness disparity within the sample renders measurements by conventional methods, which employ a restricted portion of the sample, susceptible to large variations. On the other hand, *Asv* cannot be used as a measure of toughness anisotropy and provides only an indication of average toughness for anisotropic monocrystalline materials. For this reason the data for sapphire shown in Figure 1 are not used in the regression line calculations. Application of the method to polycrystalline material is limited to those whose maximum grain dimensions are considerably less than the minimum diameter of the starting size fraction (2.83 mm).

Toughness of quartz

To demonstrate an application of the method, the toughness parameter, *Asv*, for several quartz materials has been determined and is shown in Table 2. Quartz materials fall into two basic groups by toughness. A group with relatively low toughness of

Table 1. Toughness data for standard materials

	Source	Character	Asv (mean)	Kc (MPa $m^{0.5}$)	Source of data
Fused quartz	G.E.	Amorphous	140	0.7	Wiederhorn (1969)
Spinel	Unknown, synthetic	Monocrystal	257	1.3	Evans and Charles (1976)
Si_3N_4 (NC350)	Norton	Polycrystal	314	2.0	Anstis et al. (1981)
Sapphire	Natural	Monocrystal	337	2.1	Evans and Charles (1976)
C-9606	Corning	Polycrystal	334	2.5	Anstis et al. (1981)
Al_2O_3 (AD999)	Coors	Polycrystal	408	3.9	Anstis et al. (1981)
SiC (NC203)	Norton	Polycrystal	443	4.0	Anstis et al. (1981)
Si_3N_4 (NC132)	Norton	Polycrystal	455	4.0	Anstis et al. (1981)

Asv = 124 to 153 comprises more coarsely crystalline aggregate, monocrystalline, and vitric varieties. A second, tougher, group with *Asv* = 243 to 296 comprises finely polycrystalline varieties with highly sutured grains and interlocking fibers. The greater toughness of agate, with interlocking fibrous texture, compared to flint and chert with granular texture bears an obvious similarity to the toughness relationship between the jade minerals, nephrite and jadeite. Fibrous nephrite is generally tougher than the more granular jadeite (Bradt *et al.*, 1973). The greater value of *Asv* for agate compared to *Asv* for non-banded chalcedony indicates that *Asv* is apparently sensitive to the amount of interlocking of fibers with similar morphology.

A study of the toughness of many natural materials will provide data for a number of attendant applications and investigations. A compilation of *Asv* toughness data for lapidary use, the relationship of toughness to crystal structure and to various natural polycrystalline textures, and the role of toughness (as opposed to hardness and chemical alteration) in weathering processes are examples that have occurred to us.

Table 2. Toughness of quartz

	Grain morphology	Texture	Asv (mean)
Agate, banded	0.02mm by 0.1mm, acicular	Small interlocking bundles of radiating fibers make-up larger interlocking bundles	296
Flint	0.005mm to 0.01mm, equidimensional	Highly sutured, granoblastic	263
Chert	0.01mm to 0.02mm, equidimensional	Highly sutured, granoblastic	253
Chalcedony, non-banded	0.02mm by 0.1mm, acicular	Equidimensional 1cm domains of radiating fibers. Fibers are less interlocking than agate	243
Quartz crystal	Monocrystal		153
Fused quartz	Amorphous		143
Wood opal	Amorphous (?)	Cellular microstructure of wood preserved	136
Tiger eye	0.08mm by 10mm, acicular	Parallel aggregates of highly fractured acicular grains	135
Aventurine	0.1mm to 0.5mm equidimensional quartz 0.4mm by 0.4mm by 0.04mm mica	Lepidoblastic texture from mica. Little suturing of quartz	124

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