

*The dispersion phenomena of some monoclinic Felspars.*

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## 1. INTRODUCTION.

IN the present paper the dispersion of the refractive indices of monoclinic felspars from four different localities is given, likewise that of the optic axial angle and of the optic axes in some of them.

The method of investigation and instruments for these studies have been already mentioned in my previous paper.<sup>1</sup> As the refractive indices and dispersion of the glass hemisphere of the total-reflectometer have not so far been given, I shall in the first place state the way in which they have been determined and their values and the degree of accuracy attainable, for the dispersion of the glass hemisphere differs to an appreciable degree from that of other instruments of the same kind.

*Refractive Indices and the Dispersion of the Glass Hemisphere.*

Hilger's wave-length spectrometer was used to select from the Nernst lamp, or the arc-light, waves of the definite length. These were checked by employing the bright lines of lithium ( $671\mu\mu$ ), sodium ( $589\mu\mu$ ), thallium ( $535\mu\mu$ ), given by a Bunsen burner, and the bright blue line of hydrogen obtained by sparking through a tube. The latter line was, however, read in the instrument as  $485.2$  instead of the correct number  $486\mu\mu$ . The error from this cause has been neglected, and the light was taken to have the wave-length  $486\mu\mu$ . As stated in this volume on p. 189, the dispersion of the hemisphere was compared with that of a glass prism, the dispersion of which was found by the method of minimum deviation. The goniometer used for the measurement

<sup>1</sup> S. Kôzu, 'The dispersion phenomena of albite from Alp Rischuna, Switzerland.' *Mineralogical Magazine*, 1915, vol. xvii, pp. 189-192.

of the angle and the minimum deviation of the glass prism was the large model made by the Fuess Company. The horizontal divided circle has a diameter of 25 cm. and is divided to 10 minutes. By means of the sliding micrometer placed in the eye-piece, readings to 10 seconds can be made.

The angle of the prism was measured in eight different positions of the goniometer-circle, three readings being taken in each position. The angles varied from  $60^{\circ} 0' 10''$  to  $59^{\circ} 59' 30''$ ; and the mean of the twenty-four readings is  $59^{\circ} 59' 48''$ .

For the determinations of the dispersion of the prism, twelve different wave-lengths were used: 431, 455.5, 486, 508.5, 527, 535, 554, 589 (Na), 610, 644, 671, and 700 in  $\mu\mu$  units as read in Hilger's spectrometer.

The minimum deviation of the prism for sodium-light was measured in four different positions of the circle; and in each case five readings were made on both sides of the fixed direction of the collimator. The results are given in Table I.

Table I.

*Minimum Deviation of the Glass Prism for Sodium-light.*

(Temperature  $20^{\circ}$  C., nearly.)

	Reading on right.	Reading on left.	Mean.
1	38° 3' 5''	38° 2' 20''	38° 2' 42''
2	38 3 10	38 2 40	38 2 55
3	38 2 45	38 3 3	38 2 54
4	38 2 33	38 3 17	38 2 55
Mean			38° 2' 51.5''

The refractive index of the prism for sodium-light is therefore 1.51000.

In determining the minimum deviations for the other wave-lengths only one position of the circle was used, five readings being made, as in the case of sodium-light, on both sides of the collimator. The observed minimum deviations  $\delta$  and the corresponding refractive indices  $\mu$  are given in Table II.

In order to increase the illumination in the above experiments a condenser was placed between the source of light and the spectrometer's collimator, for the opening of the slit was as narrow as possible. A second condenser was also placed in a suitable position between the spectrometer's telescope and the goniometer's collimator. This simple

Table II.

*Minimum Deviations and Refractive Indices of the Glass Prism.*

(Temperature 20° C., nearly.)

$\lambda$ in $\mu\mu$	$\delta$	$\mu$
431	38° 58' 18"	1.52057
455.5	38 46 26	1.51325
486	38 32 54	1.51572
508.5	38 24 54	1.51420
527	38 19 1	1.51308
535	38 16 13	1.51255
554	38 11 24	1.51163
589	38 2 51.5	1.51000
610	37 59 4	1.50928
664	37 52 46	1.50808
671	37 48 7	1.50719
700	37 43 50	1.50637

arrangement was very useful, for the first condenser increased the illumination, whilst the second enabled one to adjust any desired wave-length, since a distinct image of the pointer in the spectrometer's telescope could then be seen in that of the goniometer superposed on the bright line used.

Using the refractive indices thus obtained, those of the glass hemisphere of the total-reflectometer were measured, the precautions indicated by Viola<sup>1</sup> being carefully attended to. The results are given in Table III.

Table III.

*Critical Angles and Refractive Indices of the Glass Hemisphere.*

(Temperature 20° C., nearly.)

$\lambda$ in $\mu\mu$	$\theta$	$\mu$	Viola's determinations.
455.5	51° 45' 30"	1.9331	...
486	52 10 29	1.9190	1.91951
508.5	52 25 14	1.9106	...
527	52 35 29	1.9049	1.90572
535	52 39 26	1.9025	...
554	52 48 20	1.8976	...
589	53 1 30	1.8901	1.89040
610	53 8 41	1.8863	...
644	53 18 36	1.8807	...
671	53 24 48	1.8771	...
700	53 30 18	1.8738	...

<sup>1</sup> C. Viola, Zeits. Kryst. Min., 1900, vol. xxxii, pp. 311-314.

Of the wave-lengths employed by Viola only three agree with mine. Judging from the differences in these cases, the glass hemisphere of his instrument differed considerably from that used by me.

Gifford's<sup>1</sup> determination of the refractive indices of quartz by the minimum deviation method are extremely accurate, and they have been used by me to check my determinations of the indices of the hemisphere. For this purpose I used a quartz plate cut perpendicular to the optic axis, and the refractive indices obtained by the use of the hemisphere are given in column 3 of Table IV. I have only carried these values to the

Table IV.

*Critical Angle  $\theta$  and Principal Refractive Indices of Quartz and Fluorite.*  
(Temperature 20° C., nearly.)

$\lambda$ in $\mu\mu$	$\theta$	$\omega$ and $\epsilon$	Gifford.	$\Delta$
Quartz. <sup>2</sup>				
486	$\left\{ \begin{array}{l} \omega \ 58^{\circ}51'52'' \\ \epsilon \ 54 \ 20 \ 7 \end{array} \right.$	$\left\{ \begin{array}{l} 1.5498 \\ 1.5591 \end{array} \right.$	$\left\{ \begin{array}{l} 1.54970 \\ 1.55899 \end{array} \right.$	$\left\{ \begin{array}{l} +0.00010 \\ +0.00011 \end{array} \right.$
508.5	$\left\{ \begin{array}{l} \omega \ 54 \ 7 \ 34 \\ \epsilon \ 54 \ 35 \ 15 \end{array} \right.$	$\left\{ \begin{array}{l} 1.5482 \\ 1.5575 \end{array} \right.$	$\left\{ \begin{array}{l} \dots \\ \dots \end{array} \right.$	$\left\{ \begin{array}{l} \dots \\ \dots \end{array} \right.$
527	$\left\{ \begin{array}{l} \omega \ 54 \ 18 \ 52 \\ \epsilon \ 54 \ 47 \ 45 \end{array} \right.$	$\left\{ \begin{array}{l} 1.5472 \\ 1.5565 \end{array} \right.$	$\left\{ \begin{array}{l} 1.54718 \\ 1.55639 \end{array} \right.$	$\left\{ \begin{array}{l} +0.00002 \\ +0.00011 \end{array} \right.$
589 (Na)	$\left\{ \begin{array}{l} \omega \ 54 \ 47 \ 50 \\ \epsilon \ 55 \ 16 \ 26 \end{array} \right.$	$\left\{ \begin{array}{l} 1.5444 \\ 1.5535 \end{array} \right.$	$\left\{ \begin{array}{l} 1.54426 \\ 1.55337 \end{array} \right.$	$\left\{ \begin{array}{l} +0.00014 \\ +0.00013 \end{array} \right.$
644	$\left\{ \begin{array}{l} \omega \ 55 \ 5 \ 45 \\ \epsilon \ 55 \ 35 \ 7 \end{array} \right.$	$\left\{ \begin{array}{l} 1.5424 \\ 1.5515 \end{array} \right.$	$\left\{ \begin{array}{l} 1.54231 \\ 1.55135 \end{array} \right.$	$\left\{ \begin{array}{l} +0.00009 \\ +0.00015 \end{array} \right.$
671	$\left\{ \begin{array}{l} \omega \ 55 \ 12 \ 26 \\ \epsilon \ 55 \ 41 \ 26 \end{array} \right.$	$\left\{ \begin{array}{l} 1.5415 \\ 1.5504 \end{array} \right.$	$\left\{ \begin{array}{l} 1.54146 \\ 1.55047 \end{array} \right.$	$\left\{ \begin{array}{l} +0.00004 \\ -0.00007 \end{array} \right.$
Fluorite.				
486	48° 29' 5''	1.4369	1.43707	-0.00017
527	48 54 7	1.4355	1.43556	-0.00006
589	49 20 37	1.4339	1.43385	+0.00005
671	49 44 51	1.4326	1.43226	+0.00034

<sup>1</sup> J. W. Gifford, Proc. Roy. Soc., London, 1902, vol. lxx, p. 329; 1910, ser. A, vol. lxxxiv, p. 193.

<sup>2</sup> The refractive indices of quartz for different wave-lengths, determined by Mülheims and Viola by the method of total reflection, are given in Zeits. Kryst. Min., 1888, vol. xiv, p. 224, and 1900, vol. xxxii, p. 314, respectively. For comparison, the indices determined by them for three wave-lengths employed by me are cited below:—

	486 $\mu\mu$ (F)		527 $\mu\mu$ (E)		589 $\mu\mu$ (D)	
	$\omega$	$\epsilon$	$\omega$	$\epsilon$	$\omega$	$\epsilon$
Mülheims	1.54965	1.55896	1.54708	1.55639	1.54423	1.55328
Viola	1.54992	1.55910	1.54751	1.55657	1.54435	1.55342

fourth place of decimals, for a greater degree of accuracy is not to be expected from the method. Gifford gives the indices to six places of decimals, but in column 4 I have given only five, the fifth figure being increased by unity if his sixth exceed 5. The differences between his and my values are given in column 5. As a further check the refractive indices of a fluorite plate determined by me are compared with those given by Gifford for this mineral.

Owing to several circumstances the same order of accuracy cannot in general be attained with the feldspars, for the transparency of the crystals, the size and perfection of the polished plane surface, and the intensity of the light all affect the distinctness of the shadows. The feldspars examined were:—

Adularia from St. Gotthard, Switzerland (Cambridge Museum).

Adularia from Riedertobel, Bristenstock, Switzerland (Cambridge Museum).

Moonstone from Ceylon (Cambridge Museum).

Orthoclase from Madagascar.

## 2. ADULARIA FROM ST. GOTTHARD, SWITZERLAND.

(Tables V–VIII.)

The crystal is a triple twin according to the Baveno-law, and forms a rectangular prism bounded by three rough faces (001), and a dull double face (010). The most conspicuous terminal faces are  $x$  (101). The basal plane is traversed by numerous crevices parallel to the edge (001, 101). The material is, as far as examination under the microscope enables one to judge, pure, transparent, and almost free from inclusions. The extinction-angle on (010) makes  $5^\circ$  with the edge (010, 001) in the obtuse angle  $X \wedge Z$ .

### *Dispersion of the Principal Indices.*

The plate prepared for this determination was tested in white light in the polarizing microscope, and found to be practically perpendicular to the acute bisectrix.<sup>1</sup> The crystal is negative, and the optic axial planes for all colours are perpendicular to that of symmetry. Hence the obtuse bisectrix ( $Bx_0$ ) coincides with the dyad axis of the crystal, which is a fixed direction. It is therefore only necessary to find this direction, or

<sup>1</sup> It is clear that the plate need not be perpendicular to the acute bisectrix, as long as it is at right angles to the plane of symmetry of the crystal. When the plane of observation is perpendicular to that of symmetry, two principal indices for each  $\lambda$  are determined with accuracy; and when the two planes coincide, the third index is accurate, but not necessarily the other.

that of the symmetry-plane, in the plate to be able to determine  $\alpha$ ,  $\beta$ , and  $\gamma$ . For (1) when the plane of observation is perpendicular to that of symmetry  $\beta$  and  $\alpha$  are found, and (2) when the two planes coincide  $\gamma$  and  $\alpha_1$  are found. The first three values are correct, for the shadows are due to vibrations parallel to principal axes. The last,  $\alpha_1$ , is due to vibrations perpendicular to the plate, and is not for all colours a principal index. Hence it is not strictly accurate, though the error in a case such as this is so small as to be negligible. Position (2) of the plate was fixed and noted when the latter had been shifted so as to give the critical angle  $\theta_\gamma$  for sodium-light; and in a like manner position (1) was fixed by the determination of  $\theta_\beta$  for the same colour. The positions so fixed served for the measurement of the critical angles for other colours. The critical angles observed for a number of wave-lengths, and their variations from those for sodium-light, are given in Table V; the refractive indices corresponding to the critical angles and the birefringencies in Table VI.

Table V.

## Observed Critical Angles.

(Temperature 19° C., nearly.)

$\lambda$ in $\mu\mu$	$\theta_\alpha$	$\Delta\theta_\alpha$	$\theta_\beta$	$\Delta\theta_\beta$	$\theta_\gamma$	$\Delta\theta_\gamma$
486	52° 34.39'	-52.98'	52° 46.86'	-52.55'	52° 51.32'	-52.49'
508.5	52 49.47	-37.90	53 1.86	-37.55	53 6.52	-37.29
535	53 5.16	-22.21	53 16.56	-22.85	53 21.32	-22.49
554	53 13.89	-13.48	53 25.26	-14.15	53 30.07	-13.34
589 (Na)	53 27.37	0 0	53 39.41	0 0	53 43.81	0 0
610	53 34.84	+ 7.37	53 46.66	+ 7.25	53 51.29	+ 7.43
644	53 43.44	+ 16.07	53 56.31	+ 16.90	54 0.55	+ 16.74
671	53 51.02	+ 23.65	54 2.81	+ 23.40	54 7.00	+ 23.19

Table VI.

## Refractive Indices and Birefringencies.

$\lambda$ in $\mu\mu$	$\alpha$	$\beta$	$\gamma$	$\gamma - \alpha$	$\gamma - \beta$	$\beta - \alpha$
486	1.5239	1.5232	1.5297	0.0058	0.0015	0.0043
508.5	1.5224	1.5264	1.5281	0.0057	0.0017	0.0040
535	1.5211	1.5249	1.5265	0.0054	0.0016	0.0038
554	1.5201	1.5239	1.5255	0.0054	0.0016	0.0038
589 (Na)	1.5185	1.5225	1.5239	0.0054	0.0014	0.0040
610	1.5178	1.5217	1.5232	0.0054	0.0015	0.0039
644	1.5162	1.5203	1.5217	0.0055	0.0014	0.0041
671	1.5154	1.5195	1.5208	0.0054	0.0013	0.0041

*Dispersion of the Optic Angle, Optic Axes, and Optic Axial Plane.*

The dispersion of the axial angle was measured by the Hutchinson universal goniometer, the plate being immersed in a liquid of known refractive index for each wave-length. The observed value  $2H$ , the deduced value  $2V$ , and the corresponding refractive indices— $\mu$  of the liquid and  $\beta$  of the plate—are given in Table VII.

The dispersion of the optic axes for each wave-length was determined by measurements made along two directions—one along the optic axial plane and the other perpendicular to it, the measurements in both cases being made with reference to the position of the axes for sodium-light. The former angular distance (which for the present purpose may be termed horizontal angular distance of the optic axes) was directly measurable by the horizontal circle of the goniometer, for it is merely the difference of  $H$  for different colours; and the second angular distance (vertical angular distance, say) was obtained by a cross-hatched eye-piece, one

Table VII.

*Dispersions of the Optic Axial Angles.*

(Temperature 18.5° C.)

$\lambda$ in $\mu\mu$	$2H$	$V$	$\mu$ (of liquid)	$\beta$
486	59° 57.2'	60° 45.7'	1.5468	1.5282
508.5	60 12.5	60 55.6	1.5419	1.5264
535	60 30.5	61 8.3	1.5393	1.5249
589	61 1.4	61 30.4	1.5333	1.5225
644	61 27.5	61 50.9	1.5290	1.5203
671	61 39.5	61 59.2	1.5275	1.5195

division of which corresponds to 16' of arc. The results obtained are given in Table VIII (p. 260).

In this table the figures given in columns *A* and *B* are the horizontal angular distances of the two optic axes, measured separately from the corresponding axes for sodium-light taken as zero points. The signs + and - denote increment and decrement of the angles, respectively. The values of *A* and *B* do not agree very well; and the differences must be due to experimental errors, the principal source of which is the lack of distinctness of the isogyres. One isogyre, to which *A* refers, was

Table VIII.

*Dispersion of the Optic Axes.*

$\lambda$ in $\mu\mu$	Horizontal angular distance.					Vertical angular distance.	
	<i>A</i>	<i>B</i>	Mean of <i>A</i> and <i>B</i>	$\Delta V$	<i>H</i>	$\Delta I$	$\Delta U$
486	-33.3'	-30.5'	-31.9'	-22.3'	-34.35'	-32.0'	-20.9'
508.5	-29.5	-22.0	-25.3	-17.4	-29.45	-24.0	-15.7
585	-16.5	-14.0	-15.3	-11.0	-15.25	-16.0	-10.5
589	0	0	0	0	0	0	0
644	+15.0	+11.0	+13.0	+10.3	+13.05	+12.8	+ 8.5
671	+19.0	+16.0	+17.5	+14.4	+19.05	+16.0	+10.5

better defined than the other, and therefore the values of *A* may be taken as the more accurate. In column 6,  $\Delta H$  for each wave-length is computed from the data ( $\Delta V$ ,  $\beta$ , and  $\mu$ ) of Table VII by the formula  $\Delta H = \frac{\beta}{\mu} \Delta V$ . It will be seen that these figures agree well with those given in column *A*, which were the values derived from the distinct isogyre.

The observed vertical angular distances,  $\Delta I$  (say), of the optic axes for the same wave-lengths are given in column 7, and the corresponding value,  $\Delta U$ , in the crystal given in column 8, has been found by the equation  $\Delta U = \frac{\Delta I}{\beta}$ . The sign attached to  $\Delta I$  indicates an increase or decrease in the angle made by the optic axial plane with (001).

### 3. ADULARIA FROM RIEDERTOBEL, BRISTENSTOCK, SWITZERLAND.

(Tables IX and X.)

The crystal used was a simple one bounded by the prism (110) and by (101) and (001); the habit was as shown in fig. 7 of Dana's 'System of Mineralogy,' 1892 (p. 316). The section, which was cut nearly parallel to (100), was transparent and colourless. The examination was made with light of five different wave-lengths only. The observed critical angles, and the corresponding refractive indices and birefringencies are given in Tables IX and X, respectively.

*Table IX.*  
*Observed Critical Angles.*  
(Temperature 19.4° C.)

$\lambda$ in $\mu\mu$	$\theta_\alpha$	$\Delta\theta_\alpha$	$\theta_\beta$	$\Delta\theta_\beta$	$\theta_\gamma$	$\Delta\theta_\gamma$
508.5	52° 54.19'	-37.66'	53° 5.93'	-37.72'	53° 9.81'	-37.49'
535	53 9.37	-22.48	53 20.44	-23.21	53 24.00	-22.80
589	53 31.85	0	53 43.65	0	53 46.80	0
644	53 48.31	+16.46	54 0.12	+16.47	54 3.81	+17.01
671	53 55.31	+23.46	54 7.43	+23.78	54 10.87	+24.07

*Table X.*  
*Refractive Indices and Birefringencies.*

$\lambda$ in $\mu\mu$	$\alpha$	$\beta$	$\gamma$	$\gamma - \alpha$	$\gamma - \beta$	$\beta - \alpha$
508.5	1.5240	1.5279	1.5290	0.0050	0.0011	0.0039
535	1.5226	1.5262	1.5274	0.0048	0.0012	0.0036
589	1.5200	1.5238	1.5249	0.0049	0.0011	0.0038
644	1.5177	1.5215	1.5227	0.0050	0.0012	0.0038
671	1.5171	1.5209	1.5220	0.0049	0.0011	0.0038

It will be noticed that these refractive indices are all higher than the corresponding ones of the St. Gotthard specimen—about 0.0015 for  $\alpha$ , about 0.0013 for  $\beta$ , and about 0.0010 for  $\gamma$ .

#### 4. MOONSTONE FROM CEYLON.

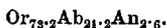
(Tables XI–XIII.)

This mineral has been regarded by mineralogists as a variety of adularia showing schillerization. The specimen studied in the Cambridge Museum differs remarkably in its optical properties from the preceding specimens of adularia. This difference<sup>1</sup> may be a consequence of the

<sup>1</sup> A chemical analysis of the Ceylon moonstone, recently made by E. H. Kraus (Zeits. Kryst. Min., 1901, vol. xxxiv, p. 175), is as follows:—

	SiO <sub>2</sub>	Al <sub>2</sub> O <sub>3</sub>	CaO	K <sub>2</sub> O	Na <sub>2</sub> O	Loss of Ignition.	Total.
%	65.87	19.45	0.51	11.91	2.57	0.18	100.49
Mol. prop.	1.098	0.190	0.009	0.127	0.042		

From this analysis, the molecular formula computed by me is given as



presence of much soda, for it is well known that the refractive indices increase with the soda. The samples used were fragments bounded by the perfect basal cleavage and the less perfect one parallel to the side pinakoid (010), and in some cases by prismatic faces, which were identified as (110) and (100). The angles between the cleavage planes parallel to (001) and (010) were measured on three pieces by the reflection goniometer. They were  $89^{\circ} 58'$ ,  $90^{\circ} 1'$ , and  $90^{\circ} 2'$ , and thus it appears that this crystal can be taken as belonging to the monoclinic system. The material is sometimes discoloured (of a pale brown colour) along the cleavage or parting face, but otherwise it is fresh. Its characteristic schillerization can be distinctly seen on the two different parting faces—one near the front pinakoid (100), and the other almost parallel to (110). The parting planes are generally uneven and curved, but are sometimes fairly even. According to Des Cloizeaux, the plane of maximum schillerization is nearly coincident with (100) and is inclined to the base at angles varying from  $65^{\circ}$  to  $69^{\circ}$ ; but I have found the angle to be much more variable, and on the three fragments measured the angles were  $62^{\circ}$ ,  $70^{\circ}$ , and  $75^{\circ}$ . The other schillerization-plane makes with the basal cleavage angles varying between  $80^{\circ}$  and  $86^{\circ}$ —the angles measured on five fragments were  $80\frac{1}{2}^{\circ}$ ,  $81^{\circ}$ ,  $82\frac{1}{2}^{\circ}$ ,  $84^{\circ}$ , and  $86^{\circ}$ . The transparency varies according to the direction in which light passes through the crystal. Fairly thick sections parallel to the base and the side pinakoid are transparent. For instance, an incandescent gas-light was in a dark room seen through a plate, 6 mm. thick, cut and polished parallel to the base; but no distinct image of the light could be seen through a similar plate, 2 mm. thick, when the polished surfaces were parallel to the maximum schillerization-face.

*Dispersion of the Refractive Indices and the Axial Angle.*

The section employed for these determinations was practically perpendicular to (010), and made an angle of  $68^{\circ} 59'$  with (001). It was inclined at nearly  $16^{\circ}$  to the acute bisectrix, but since the section is perpendicular to the symmetry-plane, the critical angles for the different principal indices are not affected by the inclination of the bisectrix to the plane of section. The method of observation is the same as that already described.

Three observations, given below, of each of the critical angles for sodium-light were made, and the arithmetical mean was employed to give the indices.

	$\theta_\alpha$	$\alpha$	$\theta_\beta$	$\beta$	$\theta_\gamma$	$\gamma$
1	53° 40·20'	1·5227	53° 53·50'	1·5271	53° 59·30'	1·5289
2	53 39·92	1·5226	53 53·13	1·5269	53 59·20	1·5289
3	53 39·82	1·5226	53 53·55	1·5271	53 59·27	1·5289
Mean	53 39·98	1·5226	53 53·39	1·5270	53 59·27	1·5289

The axial angle computed from the principal refractive indices is  $2V = 66^\circ 19.7'$ , and the observed value, given in Table XIII, is  $66^\circ 24'$ —an agreement so close that it can only be regarded as accidental (see page 273).

Table XI.

*Observed Critical Angles.*(Temperature  $19^\circ\text{C}$ ., nearly.)

$\lambda$ in $\mu\mu$	$\theta_\alpha$		$\theta_\beta$		$\theta_\gamma$	
	I	II	I	II	I	II
486	52° 47·62'	52° 47·39'	53° 1·06'	53° 0·75'	53° 6·67'	53° 6·39'
508·5	53 2·63	53 1·86	53 16·38	53 16·28	53 21·56	53 21·85
535	53 17·13	53 16·86	53 30·55	53 30·55	53 36·81	53 36·92
554	—	53 25·93	—	53 39·65	—	53 45·54
589 (Na)	53 40·20	53 39·82	53 53·50	53 53·55	53 59·30	53 59·27
610	—	53 47·04	—	54 0·85	—	54 6·47
644	53 57·25	53 56·36	54 10·81	54 10·17	54 17·14	54 16·52
671	54 4·08	54 3·14	54 17·06	54 17·11	54 23·31	54 23·24

Table XII.

*Refractive Indices and Birefringencies.*

$\lambda$ in $\mu\mu$	$\alpha$	$\beta$	$\gamma$	$\gamma - \alpha$	$\gamma - \beta$	$\beta - \alpha$
486	1·5283	1·5328	1·5347	0·0064	0·0019	0·0045
508·5	1·5265	1·5313	1·5332	0·0067	0·0019	0·0048
535	1·5250	1·5296	1·5316	0·0066	0·0020	0·0046
554	1·5241	1·5286	1·5305	0·0064	0·0019	0·0045
589 (Na)	1·5226	1·5270	1·5289	0·0063	0·0018	0·0045
610	1·5218	1·5263	1·5281	0·0063	0·0018	0·0045
644	1·5203	1·5248	1·5268	0·0065	0·0020	0·0045
671	1·5196	1·5240	1·5260	0·0064	0·0020	0·0044

The critical angles— $\theta_\alpha$ ,  $\theta_\beta$ , and  $\theta_\gamma$ —for other wave-lengths were with few exceptions measured on two different occasions and are given in columns I and II of Table XI (p. 263). The table shows that the results obtained at different times agree very well, for the greatest differences ( $\theta$  for 644 and 671  $\mu\mu$ ) do not exceed one minute, which corresponds to about 3 units in the fourth decimal place of the corresponding refractive index.

The refractive indices given in Table XII (p. 263) were computed from the mean values of I and II in Table XI, with the exception of those for sodium-light, in which three observations were taken owing to the greater precision attainable with this colour.

Table XIII gives the axial angles for different wave-lengths, the notation used being the same as before. As already stated, the plate was not perpendicular to the acute bisectrix, and to avoid error arising from this cause a liquid was used with a refractive index equal to  $\beta_{\text{Na}}$  of the crystal. Hence the angle  $2H$  is correct for sodium-light, but not so for other colours. The error, however, arising from the difference of dispersion of the liquid and the index  $\beta$  is in all cases small.

Table XIII.

*Dispersion of the Axial Angles.*

(Temperature 18° C., nearly.)

$\lambda$ in $\mu$	$2H$	$2V$	$\mu$ (liquid)	$\beta$
486	65° 14.5'	65° 29.5'	1.5380	1.5328
508.5	65 34.0	65 43.6	1.5346	1.5313
535	65 51.0	65 56.9	1.5316	1.5296
589	66 24.0	66 24.0	1.5271	1.5271
644	66 48.8	66 44.1	1.5232	1.5248
671	66 56.5	66 50.0	1.5219	1.5240

5. ORTHOCLASE FROM MADAGASCAR.

(Tables XIV to XVII.)

The crystals examined by me were bought in London, and the locality is given as Fianarantsoa, Madagascar. They are transparent and of pale yellow colour, with rather depolished faces. According to Prof. A. Lacroix,<sup>1</sup> yellow crystals of felspar are found in the gneissic region of

<sup>1</sup> A. Lacroix, *Compt. Rend. Acad. Sci. Paris*, 1912, vol. clv, p. 672; *Minéralogie de la France*, 1913, vol. v, pp. 28-29.

Itrongahy, between Betroka and Benenitra, south of Fianarantsoa, and occur scattered in the soil, or embedded in calcareous tufa, together with large transparent crystals of diopside, apatite, and quartz. In his opinion these minerals have been derived from a pegmatite with miarolitic structure. He says: 'Ce feldspath est souvent jaune et parfois même d'un jaune d'or foncé qui ne disparaît pas à la suite d'un chauffage prolongé au rouge . . . Les variations de densité (2.558 à 2.597) . . .'

Some of my specimens showed crystallographic faces which are more or less corroded and depolished, suggesting that the mineral has suffered erosion: but the interior seemed quite unaltered, and no want of transparency was observed, nor any change in the optic character of different parts of the plate. Most of the crystals were tabular with the side pinakoid (010) largely developed, one was prismatic and elongated along the vertical axis, another also prismatic but elongated along the  $X$  axis. The faces observed on different specimens are:  $c$  (001),  $b$  (010),  $m$  (110),  $z$  (130),  $y$  (201),  $x$  (101),  $a$  (100), and  $n$  (021).

Through the courtesy of Mr. A. Hutchinson, the specific gravity of the specimen examined optically was determined by four students in the Cambridge Mineralogical Laboratory by the suspension method. The mean of their results is  $2.564 \pm 0.002$ .

#### *Dispersion of the Principal Indices.*

The plate was cut practically perpendicular to the acute bisectrix, the optic axial plane being parallel to (010). The critical angles for sodium-light were measured on two occasions in the way already described, and the mean values were adopted. They gave the principal indices:<sup>1</sup>  $a = 1.5216$ ,  $\beta = 1.5259$ , and  $\gamma = 1.5261$ . Computing the axial angle from these indices by the tangent formula, we obtain  $2V = 24^\circ 12.8'$ . This value is greater by  $3^\circ 55.7'$  than that computed from  $\beta$  and  $2E$ , the angle measured with the universal goniometer (see Table XVI).

When the value of  $\gamma - \beta$  differs very little from zero, very small variations in  $\gamma$  and  $\beta$ , lying within the limits of error in the observations, give rise to great differences in  $2V$ . Thus, for example, leaving  $a$  and  $\gamma$  unchanged, but taking  $\beta$  to be 1.5260 (a difference in the fourth decimal place of one unit, which corresponds to  $23''$ , or  $0.4'$ , in the critical angle  $\theta_\beta$ ) we find  $2V$  to be  $17^\circ 18'$ . It is obvious that in such a case

<sup>1</sup> The indices given by Lacroix are  $\alpha = 1.5197$ ,  $\beta = 1.5248$ , and  $\gamma = 1.5253$ ; the specimen having the density 2.565.

the true axial angle is hardly obtainable by computation from its refractive indices determined by the total-reflectometer.

As before, the critical angles for other wave-lengths were measured along the directions corresponding to the principal indices for sodium-light; these values are given in Table XIV and the corresponding refractive indices and birefringencies in Table XV.

Table XIV.

*Observed Critical Angles.*

(Temperature 19° C., nearly.)

$\lambda$ in $\mu\mu$	$\theta_\alpha$	$\Delta\theta_\alpha$	$\theta_\beta$	$\Delta\theta_\beta$	$\theta_\gamma$	$\Delta\theta_\gamma$
486	52° 44.0'	-52.9'	52° 57.7'	-52.3'	52° 58.8'	-51.7'
508.5	52 59.3	-37.6	53 12.9	-37.1	53 13.6	-36.9
585	53 14.1	-22.8	53 27.3	-22.7	53 28.3	-22.2
589	53 36.9	0	53 50.0	0	53 50.5	0
644	53 53.3	+16.4	54 7.1	+17.1	54 7.6	+17.1
671	53 59.1	+22.2	54 14.0	+24.0	54 14.5	+24.0

Table XV.

*Refractive Indices and Birefringencies.*

$\lambda$ in $\mu\mu^*$	$\alpha$	$\beta$	$\gamma$	$\gamma-\alpha$	$\gamma-\beta$	$\beta-\alpha$
486	1.5272	1.5318	1.5322	0.0050	0.0004	0.0046
508.5	1.5257	1.5302	1.5305	0.0048	0.0003	0.0045
535	1.5241	1.5285	1.5288	0.0047	0.0003	0.0044
589	1.5216	1.5259	1.5261	0.0045	0.0002	0.0043
644	1.5193	1.5238	1.5240	0.0047	0.0002	0.0045
671	1.5184	1.5230	1.5232	0.0048	0.0002	0.0046

*The Dispersion of the Axial Angle  $\alpha$  of the Optic Axes.*

The axial angles for different wave-lengths are given in Table XVI, the angle  $2E$  being measured in air. The columns 3 and 4, containing  $2V$ , show clearly the differences in the angles given by the two methods. The angles  $2E$  are the arithmetical mean of four readings for each wave-length, the temperature of the room varying between 19.8° and 20.3° C.

The trend of the dispersion curve of the angle  $E$  is noteworthy, for if the curve is drawn on millimeter paper, in the same way as those of the Eifel sanidine given in Plate X, fig. 1, it is found to be concave down-

wards, whilst those of the sanidine when the axial plane coincided, as in this crystal, with that of symmetry were concave upwards. Therefore, if we suppose that at 20° C. the axial angle of the Madagascar sanidine vanishes for a greater wave-length, there must be two inflection points on the curve. This is extremely improbable, and we conclude that *E* must have a minimum value which may occur for a definite or infinite wave-length. One of the Eifel sanidines in the Cambridge Museum has its axial plane coincident with that of symmetry, and the dispersion curve of *E* shows a like trend, but the minimum angle, like that of the

Table XVI.

*Dispersion of the Optic Axial Angles.*

Madagascar orthoclase ( <i>t</i> = 19.8°—20.3° C.)					Eifel sanidine ( <i>t</i> = 19° C.)
$\lambda$ in $\mu\mu$	<i>2E</i>	<i>2V</i> computed from <i>E</i> and $\beta$ .	<i>2V</i> computed from the refractive indices.	$\beta$	<i>2E</i> (see p. 268).
486	38° 53'	25° 9.5'	32° 44.7'	1.5318	33° 16.0'
508.5	36 54.6	23 42.7	28 56.0	1.5302	31 54.0
535	34 46.0	22 32.7	29 7.5	1.5285	30 34.0
589	31 10.5	20 17.1	24 12.8	1.5259	28 4.6
644	28 7.5	18 21.0	23 15.2	1.5238	25 58.0
671	26 43.5	16 27.7	22 46.2	1.5230	24 53.0

Table XVII.

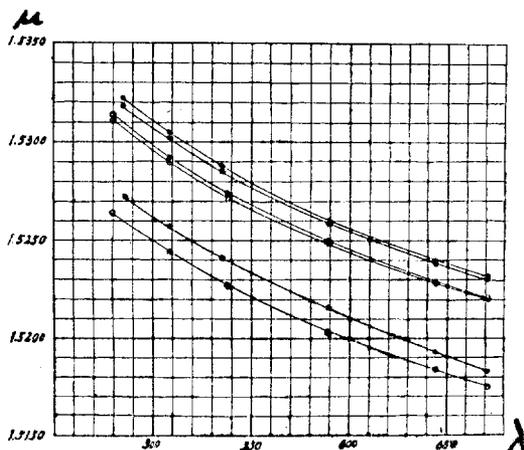
*Dispersion of the Optic Axes and of the Acute Bisectrix  $Bx_a$ .*

(Temperature 17.5°—18.4° C.)

$\lambda$ in $\mu\mu$	$\Delta H$ observed.		$\Delta V$ computed from $\frac{\mu}{\beta} \Delta H$ .		Dispersion of $Bx_a$ .	Refractive index of liquid ( $\mu$ ).
	<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>		
486	+2° 5.0'	+2° 46.7'	+2° 7.3'	+2° 49.2'	-21.0' (?)	1.5359
508.5	+1 27.5	+2 9.7	+1 29.1	+2 11.2	-21.0	1.5326
535	+0 55.75	+1 23.2	+0 56.6	+1 23.9	-13.7	1.5297
589	0 0	0 0	0 0	0 0	0	1.5252
644	-0 45.0	-1 2.8	-0 45.6	-1 3.5	+ 9.0	1.5216
671	-1 2.25	-1 30.0	-1 3.0	-1 30.7	+13.9	1.5202

Madagascar specimen, does not fall within the limits of the wave-lengths employed. The data are set out in Table XVI (p. 267).

Since the position of the optic axes in the symmetry-plane is not fixed by any line of symmetry, it is convenient to refer their dispersions,  $\Delta H$ , to the position of the two axes  $A$  and  $B$  for sodium-light; the axis  $B$  being that lying in the acute angle between  $Bx_a$  and the vertical axis  $Z$ . The observed values  $\Delta H$  are given under the headings  $A$  and  $B$  in columns 2 and 3 of Table XVII (p. 27). The corresponding values of  $\Delta V$  of columns 4 and 5 are computed by the formula  $\Delta V = \frac{\mu}{\beta} \Delta H$ . The dispersion of  $Bx_a$ , referred to the position of that for sodium-light, is clearly one-half the difference between  $A$  and  $B$  of columns 4 and 5, and is taken as negative when the bisectrix moves towards  $B$ , and positive when it moves towards  $A$ .



The Dockweiler sanidine examined by Offret<sup>1</sup> shows relations similar to those of the Madagascar specimen, for in both the birefringence  $\gamma - \beta$  is extremely small and increases as  $\lambda$  diminishes. Consequently the optic axial angle increases rapidly as  $\lambda$  diminishes. Though his refractive indices, determined at 22° C., are considerably lower than those of the Madagascar specimen measured at 20° C., the dispersion curves of the refractive indices shown in the accompanying text-figure are extremely similar. The black dots give the actual values found for the Madagascar

<sup>1</sup> A. Offret, Bull. Soc. franç. Min., 1890, vol. xiii, p. 645.

felspar. The circlets on the corresponding Dockweiler curves give the values in Offret's tables for the wave-lengths used by him.

From what has been stated above, the Madagascar felspar is to be compared with the sanidine group by reason of its small optic axial angle and very low birefringence ( $\gamma - \beta$ ), but its refractive indices are much higher than those of the other members of the group.

## 6. CRITICISM OF THE DISPERSION FORMULAE.

The dispersion formulae of the axial angle:—

$$\sin E = \sqrt{P + \frac{Q}{\lambda^2}}, \quad . . . . . (1)$$

and

$$\tan V = \sqrt{P_t + \frac{Q_t}{\lambda^2}}, \quad . . . . . (2)$$

which have been proved to be applicable to the Eifel sanidine, were tested for that from Madagascar, and also for the St. Gotthard adularia and the Ceylon moonstone. For the first of these both equations were examined, but for the two others only the tangent formula was applied, as their axial angles were not directly observed in air.

### *The Madagascar Orthoclase.*

The two sets of coefficients— $P$ ,  $Q$  and  $P_t$ ,  $Q_t$ —in equations (1) and (2) were computed, in a way similar to that described in the paper on the Eifel sanidine (p. 246), from the observed  $E$ , and from the  $V$  which was computed from  $E$  and  $\beta$ . The results are:—

$$\begin{aligned} P &= -0.00981, & Q &= 28460.6. \\ P_t &= -0.00506, & Q_t &= 12886.5. \end{aligned}$$

Table XVIII gives the observed  $E$ , and the  $E'$  computed by introducing the values of  $P$  and  $Q$  in equation (1); and also the values of  $V$  computed from  $E$  and  $\beta$ , and those of  $V'$  deduced from equation (2) by the introduction of the values of  $P_t$  and  $Q_t$ .

The table shows that the differences  $\Delta E$  and  $\Delta V$  are very small. Hence it follows that equations (1) and (2) may be accepted as giving the dispersion formulae of  $E$  and  $V$  for this crystal.

Similarly as before, the variations of

$$x = \frac{a}{\sqrt{\gamma^2 - a^2}}, \quad \text{and of } z = \frac{a}{\gamma \sqrt{\beta^2 - a^2}},$$

Table XVIII.

$\lambda$ in $\mu\mu$	$E$ observed in air.	$E'$ computed from formula (1).	$\Delta E$	$V$ computed from $E$ and $\beta$ .	$V'$ computed from formula (2).	$\Delta V$
486	19° 26.5'	19° 26.0'	+ 0.5'	12° 34.8'	12° 32.5'	+ 2.3'
508.5	18 27.3	18 27.5	- 0.2	11 56.3	11 56.9	- 0.6
535	17 23.0	17 25.2	- 2.2	11 16.3	11 18.3	- 2.0
589	15 35.2	15 35.5	- 0.3	10 8.5	10 9.3	- 0.8
644	14 3.7	14 1.5	+ 2.2	9 10.5	9 9.7	+ 0.8
671	13 22.0	13 21.7	+ 0.3	8 43.9	8 43.7	+ 0.2

within the given range of wave-lengths were determined, and they were found to be negligible in the computation of  $E$  and  $V$ , for the former is 0.0068, and the latter 0.0701.

Using the observed  $\beta$ , we get  $\sqrt{\gamma^2 - \beta^2} = 0.0329$ ; and using  $\beta$  deduced from observed  $E$ ,  $\gamma$ , and  $a$ ,  $\sqrt{\gamma^2 - \beta^2} = 0.0268$ .

Therefore in the first case:—

$$\Delta E_I = \frac{0.0068 \times 0.0329}{\cos E_0},$$

and in the second case:—

$$\Delta E_{II} = \frac{0.0068 \times 0.0268}{\cos E_0}.$$

When  $E_0$  is 20°,  $\Delta E_I$  and  $\Delta E_{II}$  are both less than 1'.

Again,  $\Delta V_I = 0.0701 \times 0.0329 \times \cos^2 V_0$ ,

and  $\Delta V_{II} = 0.0701 \times 0.0268 \times \cos^2 V_0$ .

When  $V_0 = 8^\circ$ , the larger of these is less than 9', and the smaller is less than 7'.

*The St. Gotthard Adularia and the Ceylon Moonstone.*

The values of  $P_t$  and  $Q_t$  computed as before are:—

$$P_t = 0.38024, \quad Q_t = - 8859.0 \text{ for adularia.}$$

and  $P'_t = 0.46005, \quad Q'_t = - 11054.5$  for moonstone.

In Table XIX, the angles  $V$  and  $V_t$  computed from  $H$ ,  $\mu$ , and  $\beta$  (see Tables VII and XIII) are given in columns 2 and 5, and in columns 3 and 6 are those,  $V'$  and  $V'_t$ , obtained by introducing the above values

Table XIX.

$\lambda$ in $\mu\mu$	Adularia, St. Gotthard.			Moonstone, Ceylon.		
	$V$ computed from $H, \mu,$ and $\beta$ (see Table VII).	$V'$ computed from formula (2).	$\Delta V$	$V_1$ computed from $H, \mu,$ and $\beta$ (see Table XIII).	$V_1'$ computed from formula (2).	$\Delta V_1$
486	30° 22.9'	30° 20.9'	+ 2.0'	32° 44.8'	32° 44.1'	+ 0.7'
508.5	30 27.8	30 27.9	- 0.1	32 51.8	32 51.7	+ 0.1
535	30 34.2	30 35.0	- 0.8	32 58.5	32 59.3	- 0.8
589	30 45.2	30 46.4	- 1.2	33 12.0	33 12.0	+ 0.0
644	30 55.5	30 55.5	0.0	33 22.1	33 21.5	+ 0.6
671	30 59.6	30 59.0	+ 0.6	33 25.0	33 25.4	- 0.4

of  $P_t, P_t'$  and  $Q_t, Q_t'$  in equation (2). The differences  $\Delta V$  and  $\Delta V_1$  between them are very small, and are practically negligible.

The changes of  $z = \frac{\alpha}{\gamma \sqrt{\beta^2 - \alpha^2}}$  for both adularia and moonstone, and their effects on  $V$  were also tested.

The results are:—

for adularia,  $\Delta V = \Delta z \sqrt{\gamma^2 - \beta^2} \cdot \cos^2 V = 0.1372 \times 0.06403 \times \cos^2 V,$

if  $V = 30^\circ,$   $\Delta V = 22.6';$

for moonstone,  $\Delta V = 0.02754 \times 0.24535 \times \cos^2 V,$

if  $V = 33^\circ,$   $\Delta V = 16.3'.$

These results are unsatisfactory; but, as shown later on, small errors in the refractive indices cause appreciable changes in the axial angle, and also in  $\Delta z$ . For example, suppose that in the calculation of  $\Delta z$  and  $\Delta V$  for adularia, the  $\alpha$  for 486  $\mu\mu$  to be alone changed, and this by 0.0001. Making then  $\alpha = 1.5240$  instead of 1.5239,  $\Delta z$  becomes 0.0549 instead of 0.1372, and  $\Delta V$  becomes = 9.3'. It follows therefore that, when  $\sqrt{\gamma^2 - \beta^2}$  is not very small, there will be considerable uncertainty as to the

effect of the variation of  $z = \frac{\alpha}{\gamma \sqrt{\beta^2 - \alpha^2}},$  as long as the indices are liable to errors in the fourth decimal place.

7. THE EFFECT ON  $2V$  OF ERRORS IN THE REFRACTIVE INDICES.

The effect on  $2V$  (measured about the axis of least elasticity) of errors in  $a$ ,  $\beta$ , and  $\gamma$  are given by differentiation of the formula:—

$$\tan V = \frac{\gamma \sqrt{\beta^2 - a^2}}{a \sqrt{\gamma^2 - \beta^2}}.$$

The difference  $dV$  is given by:—

$$dV = \frac{\partial V}{\partial a} da + \frac{\partial V}{\partial \beta} d\beta + \frac{\partial V}{\partial \gamma} d\gamma,$$

where

$$\frac{\partial V}{\partial a} = - \frac{\gamma \sqrt{\gamma^2 - \beta^2}}{(\gamma^2 - a^2) \sqrt{\beta^2 - a^2}},$$

$$\frac{\partial V}{\partial \beta} = \frac{a\gamma}{\beta \sqrt{(\beta^2 - a^2)(\gamma^2 - \beta^2)}},$$

and

$$\frac{\partial V}{\partial \gamma} = - \frac{a \sqrt{\beta^2 - a^2}}{(\gamma^2 - a^2) \sqrt{\gamma^2 - \beta^2}}.$$

$\Delta V_1$  about the other bisectrix can only differ from  $\Delta V$  in sign, and consequently the partial differentials must also change sign.

The errors to which the refractive indices, determined by the Abbé-Pulfrich total-reflectometer, are liable lie, with few exceptions, within the limits of  $\pm 0.0002$ . If we investigate the effects of an error of  $\pm 0.0001$  in one or more of the indices, we have twenty-six values, some or all of which differ from  $V$ . Computing for sodium-light from

Tables VI and XII the values of  $\frac{\partial V}{\partial a}$ ,  $\frac{\partial V}{\partial \beta}$ , and  $\frac{\partial V}{\partial \gamma}$ , and assuming that the refractive indices are each affected by a difference of 0.0001, we obtain:—

	$\frac{\partial V}{\partial a} da$	$\frac{\partial V}{\partial \beta} d\beta$	$\frac{\partial V}{\partial \gamma} d\gamma$
Adularia	18.9'	-1°12.5'	53.5'
Moonstone	17.9	-59.3	41.4

Hence we see that, if  $da$ ,  $d\beta$ , and  $d\gamma$  all have the same sign,  $dV = 0$ . On the other hand, when  $d\beta$  is opposite in sign to the other two,  $dV$  has a maximum value:  $\pm 2^\circ 24.9'$  for adularia, and  $\pm 1^\circ 58.6'$  for moonstone. Therefore  $2V$  may vary, according to the possible magnitudes (double  $\pm 0.0001$ ) and signs of the errors in  $\alpha$ ,  $\beta$ , and  $\gamma$ , by as much as  $9^\circ 47.6'$  for the first, and  $7^\circ 54.4'$  for the second.

We see then that even very small errors in  $\alpha$ ,  $\beta$ , and  $\gamma$  may affect  $2V$  to an extent out of all proportion to the error possible in a more direct determination of the angle. Let us suppose the value,  $2V'$ , found directly to be affected by an error of  $\pm 10'$ . The least error due to one of  $\pm 0.0001$  in any of the indices for adularia or moonstone is seen to occur when  $da$  is the only one which is not zero, and to be  $\pm 19' \times 2$  for one, and  $\pm 36'$  for the other, very nearly four times the error assumed possible in  $2V'$ . Therefore, if  $2V$  agrees with  $2V'$  within the limits of  $\pm 10'$ , we conclude that there is no error in any of the indices exceeding  $\pm 0.0001$ , except when all three have the same sign and  $dV = 0$ .

If, however, the angle  $V$  be determined from the approximate formula  $\tan^2 V = \frac{\beta - \alpha}{\gamma - \beta}$ , it is easy to show that the angle is unchanged by any equal errors of  $\alpha$ ,  $\beta$ , and  $\gamma$ . For on differentiation it is found that  $\frac{\partial V}{\partial \alpha} + \frac{\partial V}{\partial \beta} + \frac{\partial V}{\partial \gamma} = 0$ .

For moonstone, the angle given by the exact tangent formula is  $33^\circ 10'$ , by the approximate one  $33^\circ 19'$ , and by direct observation  $33^\circ 12'$ . This shows that in such a case the last formula gives a satisfactory result. Trial, however, shows that serious divergencies arise when the angle  $V$  approaches  $45^\circ$  or zero.<sup>1</sup>

<sup>1</sup> See F. E. Wright, 'Graphical methods in microscopical petrography.' Amer. Journ. Sci., 1918, vol. xxxvi, p. 509.